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A MODEL OF THE STOCHASTIC CONVERGENCE BETWEEN EURO AREA BUSINESS CYCLES

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Abstract

A new non-linear parametric model, the Stochastic Cyclical Convergence Model (SCCM), is used for measuring the convergence of business cycles between euro area countries and the euro area aggregate. The model combines unobserved component models with time-varying parameter models. The convergence between the two cycles is characterised by two time-varying parameters, the phase-shift and a weight, which is related to the phase-adjusted correlation. A Kalman filter-based iterative procedure is used for the model estimation. SCCM models are applied to the GDP of euro area countries, the United Kingdom and of the euro area aggregate over the period 1963:1-2002:4. When the euro was launched, the convergence was already achieved for most of euro area countries, but Finland, Greece and Ireland had still not converged in 2002:4. The British cycle is also divergent with a lead equal to 3 quarters in 2002:4 and a weight equal to 0.6 in 2002:4. UK shocks have asynchronous asymmetric effects and this suggests that it would be delicate for the UK to join the euro area.

Keywords: convergence, synchronisation, business cycles, multivariate unobserved components models, time-varying parameter models, Kalman filter.

JEL *Classification*: C13, C32, E32.

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Contents

1 Introduction

Empirical studies simply based on correlation coefficients (e.g. Artis and Zhang, 1997, Angeloni and Dedola, 1999 and Wynne and Koo, 2000) have generally established the existence of an increased convergence between euro area business cycles during the Exchange Currency Mechanism (ECM) period. Belo (2001) has confirmed the existence of convergence within the euro area using annual data from 1960 to 1999. He has also shown a persistent lead of the UK cycle over that of the euro area and a strong association between the two cycles, after correcting for this lead. Now, we would like to adress such an issue for the recent period, which follows the launch of the euro area. Has the convergence between business cycles been reinforced since 1999? To answer to this question, we try to improve on sub-sample correlation analysis, by modelling the convergence dynamics with time-varying parameters. Such techniques have the interest to measure the recent evolution of the cyclical convergence.

We use a stochastic cyclical convergence model (SCCM) developed in Lemoine (2005) to measure the degree of convergence between cycles of euro area countries and of the rest of the euro area. By analogy with the main concepts of cross-spectral analysis, i.e. the phase and the gain, this time-domain parametric model descibes the convergence between two cycles with two time-varying (tv) parameters: the tv-phase and the tv-weight. A cycle is converging toward the other one if the tv-phase converges toward 0 and the tv-weight converges toward 1: in such a case, cycles get synchronised, their amplitudes converge toward each other and the correlation of their innovations converge toward 1.

Cycles are estimated with Baxter-King (BK) filters applied to quarterly GDP series on the period 1960:1-2005:4. Each bivariate SCCM model is applied to the cycles of a euro area country and to the one of the rest of the euro area, considered as an aggregate. As three years are truncated by the BK filter at the beginning and at the end of the cycle, SCCM models are estimated on the period 1963:1-2002:4. This analysis is supplemented by a comparison with the convergence of UK and euro area business cycles, in order to try to verify the first test proposed by the Chancellor of the Exchequer for UK entry into the euro area.

Section 2 reviews the literature related to the cyclical convergence. Section 3 contains a description of the unobserved component models of the phase-shifted cycles, the SCCM model and its estimation procedure. Empirical results are presented in Section 4.

2 Literature overview

Many growth cycle models are available. In this paper, growth cycles are modelled with multivariate unobserved components models even if, for limiting the number of parameters, they are pre-estimated with the band-pass Baxter-King (BK) filter. Other growth cycle models, like Beveridge-Nelson decomposition or Structural Vector Auto-Regressive models (SVAR), are not considered, since their results are more difficult to interpret: the transitory components do not show enough persistence, a property that is generally expected from a cycle, and are sometimes negatively correlated with the capacity utilization rate, which is assumed to be closely related to the business cycle (Camba-Mendez and Rodriguez-Palenzuela, 2003).

As shown by Belo (2001), the phase-shift among business cycles of different countries might change the diagnosis concerning the correlation. Rünstler (2004) extends the multivariate unobserved component model, in order to take into account such phase-shifts and to measure phase-adjusted correlation coefficients. This extension is presented in Section **??** and will be used in this paper.

The association degree between business cycles in a group of countries has first been defined by the presence of common components in Vector Auto-Regressive (VAR) models. Cofeature tests proposed by Engle and Kozicki (1993) and Vahid and Engle (1993) have been applied, for example, by Mills and Holmes (1999) to European countries over different exchange-rate-regime time periods. Because such an association criterion is very restrictive, Kose, Prasad, and Terrones (2003), Stock and Watson (2003) and Bordo and Helbling (2003) have preferred to use factor models (initially developed by Stock and Watson, 1991) and their extended versions for measuring the share of variance explained by a common factor. As this seems to be an interesting way to formulate an association indicator in a multivariate model, the Rünstler model is reformulated here in a common factor framework (Section 3.1).

Cyclical convergence is generally studied with *ad hoc* indicators (for example correlation coefficients in Artis and Zhang, 1997) applied to estimated cycles on various sub-samples. As far as we know, few models have been developed for studying the cyclical convergence dynamics. Recently, Koopman and Azevedo (2004) have proposed a multivariate unobserved components model that tries to incorporate these dynamics: the convergence is associated with a progressive reduction of the phase-shift and an increase in the correlation between the two cycles. Koopman and Azevedo (2004) have modelled convergence dynamics using logistic functions, by extending a model initially developed by Rünstler (2004). But logistic functions are monotonous and do not allow transitory divergences, thus they do not allow convergence and divergence movements to occur successively. Indeed, each turning point is an opportunity for divergence, for example some countries might have begun their recoveries, while others remain in deep recession.

Such recurrent divergences require a stochastic model of phase-shift and correlation coefficients, for example a random walk model. Models with stochastic variation of regression parameters were first proposed in Cooley and Prescott (1976). In the unobserved component framework, a stochastic covariance model is proposed by Harvey, Ruiz, and Shepard (1994). Boone (1997) uses time-varying parameter (TVP) regressions for measuring convergence of supply and demand shocks, in a SVAR model. Engle (2002) formalises this mechanism in a multivariate Generalised Auto-Regressive Conditional Heteroskedastic (GARCH) model of dynamic conditional correlation. Although they do not take into account any phase-shift, Hallett and Richter (2004) propose an interesting approach of the convergence between US and European business cycles: they use TVP models for studying the evolution of the coherence (a frequency-domain concept).

In Lemoine (2005), a new bivariate model is proposed, the *Stochastic Cyclical Convergence Model* (SCCM), in which the phase-shift and the correlation between the two cycles follow random walk dynamics. Contrary to the logistic function specification of Koopman and Azevedo (2004), the random walk processes allow for successive convergence and divergence movements. As this model is non-linear, a local version of the *Iterative Extended Kalman Filter* (IEKF) is used for its estimation. This estimation method has been checked on simulated data.

3 A model of stochastic cyclical convergence

In this section, the bivariate stochastic cycle model with phase-shifts (Section 3.1) is presented before introducing the SCCM model (Section 3.2).

3.1 Bivariate stochastic cycle model with phase-shifts

To model cyclical dynamics of a stationary time series vector $\mathbf{y}_t$ and to take into account possible phase-shifts, we first consider the *multivariate stochastic cycle model with phase-shifts* developed by Rünstler (2004). This model is an extension of the multivariate model developed in Harvey and Koopman (1997). It is interesting to reformulate the Rünstler model in a *single common factor model*[1]. Both models are statistically equivalent (Koopman and Azevedo, 2004), but such a formulation allows to distinguish common from idiosyncratic shocks. The aim of this section is only to describe the main properties of the Rünstler model. Their proofs are provided in Rünstler (2004) and Koopman and Azevedo (2004).

The bivariate case will be sufficient for the analysis carried out in this article. Each series $y_{i,t}$ can be decomposed into a common cycle (a transformation of the vector $\bar{\boldsymbol{\psi}}_t^c$), a specific cycle (a transformation of the vector $\bar{\boldsymbol{\psi}}_{i,t}^*$) and an irregular component ($\varepsilon_{i,t}$). The state equations have a similar iterative form for both cyclical components ($\bar{\boldsymbol{\psi}}_t^c$ and $\bar{\boldsymbol{\psi}}_{i,t}^*$). For $i = 1, 2$ and $t = 1, ..., n$, measurement and state equations are written as follows:

$$\begin{cases} y_{i,t} = a_i \left[\cos(\lambda\xi_i), \sin(\lambda\xi_i)\right] \bar{\boldsymbol{\psi}}_t^c + [1,0]\bar{\boldsymbol{\psi}}_{i,t}^* + \varepsilon_{i,t} \\ \bar{\boldsymbol{\psi}}_t^c = \phi T_\lambda \bar{\boldsymbol{\psi}}_{t-1}^c + \bar{\boldsymbol{\kappa}}_t^c \\ \bar{\boldsymbol{\psi}}_{i,t}^* = \phi T_\lambda \bar{\boldsymbol{\psi}}_{i,t-1}^* + \bar{\boldsymbol{\kappa}}_{i,t}^* \end{cases} \tag{1}$$

with

$$\bar{\boldsymbol{\psi}}_{i,t} = \begin{bmatrix} \psi_{i,t} \\ \psi_{i,t}^+ \end{bmatrix}, \bar{\boldsymbol{\kappa}}_{i,t} = \begin{bmatrix} \kappa_{i,t} \\ \kappa_{i,t}^+ \end{bmatrix} \text{ and } T_\lambda = \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix}.$$

Processes $\varepsilon_{i,t}$, κ_t^c, κ_t^{+c}, $\kappa_{i,t}^*$, $\kappa_{i,t}^{+*}$, $\gamma_{i,t}$ and $\delta_{i,t}$ are white independent Gaussian noises, with standard errors $\sigma_{\varepsilon,i}, \sigma_{\kappa,c}, \sigma_{\kappa,c}, h_i, h_i, \sigma_{\gamma,i}$ and $\sigma_{\delta,i}$. Parameters $\lambda \in [0;\pi]$ and $\phi \in [0;1[$ are the frequency and the damping factor of both cycles. The damping factor ϕ and the frequency λ are the same in both series. Parameters a_i and ξ_i are respectively the weights and the phases of the common cycle in each series $y_{i,t}$.

The only difference between this model and the model of Harvey and Koopman (1997) consists in defining the cyclical component as a function of hidden cycles $\psi_{2,t}$ and $\psi_{2,t}^+$, since $\xi_2 \neq 0$. As

[1]Factor models were initially used in a SVAR framework (Stock and Watson, 1991).

shown below, $\psi_{1,t}$ and $\psi_{2,t}$ are synchronised, but the transformation of $\psi_{2,t}$ and $\psi_{2,t}^{+}$ creates a "phase-shift" equal to $\xi_2 - \xi_1$ periods between the second cycle and the first one. The constraints $a_i \geq 0$ and $-\pi < \lambda\xi_2 < \pi$ are assumed[2].

For identifying the model, the normalisation constraints ($\xi_1 = 0, a_1 = 1, h_1 = 0$) are imposed. Thus, the two series play different roles in the model: conventionally, a *cycle of interest* is distinguished from a *reference cycle*. The properties of the cycle of interest are studied in comparison with the reference cycle. The relationship between the two cycles is characterised by their phase-shift ξ_2 and the weight a_2. The phase-shift is the lag relative to the reference cycle. The weight is a sort of amplitude ratio of the cycle of interest in comparison with the reference cycle. The relation between cycles can equivalently be characterised by the phase-shift and the phase-adjusted correlation ρ, which can be computed with the following formula:

$$\rho = a_2 / \sqrt{a_2^2 + h_2^2/\sigma_{\kappa,c}^2}. \tag{2}$$

The correlation ρ between cyclical innovations is called the phase-ajusted correlation, because it relates the series $y_{1,t}$ to the series $y_{2,t}$ adjusted from its phase-shift. The contemporaneous correlation between both cycles is equal to $\rho\cos(\lambda\xi_2)$. As explained in Rünstler (2004), the phase-shift, the weight and the phase-adjusted correlation concepts are closely related to frequency-domain concepts (the phase, the gain and the coherence). But the model-based approach avoid potential distortions of the filter-based approach, as documented by Harvey and Trimbur (2003).

The cross-covariance function allow to determine identification conditions and to show that the covariance between the two cycles is maximised when the second series is shifted $floor(\xi_2)$ times. The cycles are stationary processes with the following cross-covariance function[3]:

$$\Gamma(\tau) = \frac{\phi^{|\tau|}}{1-\phi^2} \begin{bmatrix} \sigma_{\kappa,1}^2 \cos(\lambda\tau) & \rho\sigma_{\kappa,1}\sigma_{\kappa,2}\cos[\lambda(\tau+\xi_2)] \\ \rho\sigma_{\kappa,1}\sigma_{\kappa,2}\cos[\lambda(\tau-\xi_2)] & \sigma_{\kappa,2}^2\cos(\lambda\tau) \end{bmatrix} \tag{3}$$

If $\phi = 0$, $\lambda = 0$, $\lambda = \pi$ or $\rho = 0$, the cross-covariance does not depend on ξ_2 and the phase-shift ξ_2 is not identifiable. On the contrary, if the *identification conditions* ($0 < \phi < 1, 0 < \lambda < \pi, 0 < \rho$) are assumed, the cross-covariance between the series $y_{1,t}$ and $y_{2,t-\tau}$ is maximised when the lag operator is applied $floor(\xi_2)$ times on the second cycle[4].

Finally, it can be noticed that both series can share a common cycle in the sense of Engle and Kozicki (1993), i.e. it is possible in some cases to find a linear combination of both series that would be equal to a white noise. The conditions are a perfect synchronisation ($\xi_2 = 0$) and the absence of specific shocks ($h_2 = 0$). In this case, the correlation between the two phase-adjusted cycles is equal to one ($\rho = 1$).

3.2 Stochastic Cyclical Convergence Model (SCCM)

The previous bivariate model (i.e. a multivariate stochastic cycle model with phase-shifts expressed in a single common factor framework) is extended in Lemoine (2005), in order to incorporate the

[2] Using basic trigonometric identities, it can be shown that such constraints are equivalent to the constraint $-\pi/2 < \lambda\xi_2 < \pi/2$ assumed in Rünstler (2004).

[3] Proof of the auto-covariance expression is given by Rünstler (2004).

[4] For a real x, the $floor$ function returns the largest integer not greater than x.

convergence dynamics. This extension differs from that of Koopman and Azevedo (2004). Each property of the cycle of interest, regard to the reference cycle, is modelled with a time-varying (tv) parameter: the convergence is characterised by the *tv-phase* $\xi_{2,t}$ and the *tv-weight* $a_{2,t}$. These tv-parameters are random walk processes. The stochastic cyclical convergence model (SCCM) is written as follows:

$$\begin{aligned} y_{i,t} &= a_{i,t}\left[\cos(\lambda\xi_{i,t}), \sin(\lambda\xi_{i,t})\right]\bar{\boldsymbol{\psi}}_t^c + [1,0]\,\bar{\boldsymbol{\psi}}_{i,t}^* + \varepsilon_{i,t} \\ \bar{\boldsymbol{\psi}}_t^c &= \phi T_\lambda \bar{\boldsymbol{\psi}}_{t-1}^c + \bar{\boldsymbol{\kappa}}_t^c \\ \bar{\boldsymbol{\psi}}_{i,t}^* &= \phi T_\lambda \bar{\boldsymbol{\psi}}_{i,t-1}^* + \bar{\boldsymbol{\kappa}}_{i,t}^* \\ a_{i,t} &= a_{i,t-1} + \gamma_{i,t} \\ \xi_{i,t} &= \xi_{i,t-1} + \delta_{i,t} \end{aligned} \tag{4}$$

$\varepsilon_{i,t}$, κ_t^c, κ_t^{+c}, $\kappa_{i,t}^*$, $\kappa_{i,t}^{+*}$, $\gamma_{i,t}$ and $\delta_{i,t}$ are white independent Gaussian noises, with standard errors $\sigma_{\varepsilon,i}, \sigma_{\kappa,c}, \sigma_{\kappa,c}, h_i, h_i, \sigma_{\gamma,i}$ and $\sigma_{\delta,i}$. Using the normalisation constraints,

$$a_{1,t} = 1, \xi_{1,t} = 0, h_1 = 0, \tag{5}$$

in this particular formulation, the second cycle is generated by the propagation of the first cycle. The propagation is characterised by the tv-phase ($\xi_{2,t}$) and the tv-weight ($a_{2,t}$), which should again verify $-\pi < \lambda\xi_{2,t} < \pi$ and $a_{2,t} \geq 0$. But it can equivalently be characterised by the tv-phase ($\xi_{2,t}$) and the phase-adjusted tv-correlation (ρ_t), by computing:

$$\rho_t = a_{2,t}/\sqrt{a_{2,t}^2 + h_2^2/\sigma_{\kappa,c}^2} \tag{6}$$

In this case, the auto-covariance function[5] of the cycles at a lag τ and a date $t > \tau$, conditional on a realisation of the tv-correlation time series $\boldsymbol{\rho}_{1:t} = [\rho_{2,1}, ..., \rho_{2,t}]'$ and of the tv-phase one $\boldsymbol{\xi}_{1:n} = [\xi_{2,1}, ..., \xi_{2,n}]'$ is:

$$\Gamma_{a,\xi}(\tau;t) = \frac{\phi^\tau}{1-\phi^2}\begin{bmatrix} \sigma_{\kappa,1}^2\cos(\lambda\tau) & \tilde{\rho}_{\tau,t}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau+\xi_{2,t})\right] \\ \tilde{\rho}_{\tau,t}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau-\xi_{2,t-\tau})\right] & \sigma_{\kappa,2}^2\cos(\lambda\tau) \end{bmatrix}, \tag{7}$$

with $\tilde{\rho}_{\tau,t}$ the exponential smoother (with a parameter ϕ^2) of $\rho_{t-\tau}$ or ρ_t below the sign of τ:

$$\tilde{\rho}_{\tau,t} = \begin{cases} (1-\phi^2)\sum_{i=0}^{t-\tau}\phi^{2i}\rho_{t-\tau-i} \text{ if } \tau > 0 \\ (1-\phi^2)\sum_{i=0}^{t}\phi^{2i}\rho_{t-i} \text{ if } \tau < 0 \end{cases}.$$

At time t, if $\phi = 0$, $\lambda = 0$, $\lambda = \pi$ or ($\boldsymbol{\rho}_{1:t} = \mathbf{0}_t$), the autocovariance matrix $\Gamma_{\rho,\xi}(\tau,t)$ does not depend on $\xi_{2,t}$ and the phase-shift $\xi_{2,t}$ is not identifiable. Thus, the identification conditions $\phi > 0$, $0 < \lambda < \pi$ and ($\rho_t > 0, \forall t$) are assumed.

The cyclical convergence at a date t is defined in this paper by the two following conditions: perfect synchronisation ($\xi_{2,t} = 0$) and perfect correlation ($\rho_t = 1$). In the case of a perfect and permanent synchronisation, the phase-shift is assumed constant ($\sigma_{\delta,2} = 0$) and equal to zero ($\xi_{2,0} = 0$). For the perfect symmetric case, the correlation is assumed constant and equal to one,

[5] Proof of the auto-covariance expression is given in Appendix A.

which is equivalent to assuming $(\sigma_{\gamma,2} = 0, h_2 = 0)$. Finally, if $\sigma_{\delta,2} = 0$, $\xi_{2,0} = 0$, $\sigma_{\gamma,2} = 0$ and $h_2 = 0$ simultaneously, cycles are synchronised and symmetric. This is the common cycle case, initially defined by Engle and Kozicki (1993): a linear combination of the cycles is a white noise; the only difference between the two cycles is their amplitude.

3.3 Estimation methodology

As the issue raised does not directly concern the trend/cycle decomposition, if series of interest $x_{i,t}$ and $x_{2,t}$ have a trend, they are detrended and smoothed with band-pass BK filters before the iterative estimation procedure:

$$y_{i,t} = BK_{\alpha,\beta}(x_{i,t}) \tag{8}$$

where the operator BK transforms a series x into its trend, using a Baxter-King filter. The choice of the parameter of the operator (here α or β) depends on the considered series. For the empirical application in section 4, where series of interest are quarterly GDP time series, periods between 1.5 and 8 years are filtered by setting $\alpha = 6$ and $\beta = 32$.

The parameter vector of the SCCM model, denoted by $\mathbf{p} = \left[\phi, \lambda, h_2, \sigma_{\kappa,c}, \xi_{2,0}, \sigma_{\delta,2}, \theta_{2,0}, \sigma_{\gamma,2}\right]'$ for convenience[6] contains: the damping factor (ϕ), the frequency (λ), the specific and common standard deviations $(h_2, \sigma_{\kappa,c})$, the tv-shift parameters $(\xi_{2,0}, \sigma_{\delta,2})$ and the tv-weight parameters $(\theta_{2,0}, \sigma_{\gamma,2})$. The tv-parameters $(\xi_{2,t}, a_{2,t})$ and $\mathbf{p}$ are transformed, in order to facilitate their estimation and to take into account constraints. The transformed parameters and tv-parameters are called $\boldsymbol{\theta}_{\mathbf{p}} = [\theta_\phi, \theta_\lambda, \theta_h, \theta_{\kappa,c}, \theta_{\xi,0}, \theta_\delta, \theta_{a,0}, \theta_\gamma]'$ and $(\theta_{\xi,t}, \theta_{a,t})$. The parameters can then be calculated by the following formula:

$$\begin{aligned}
\phi &= \frac{|\theta_\phi|}{\sqrt{1+\theta_\phi^2}}, \lambda = \frac{2\pi}{2+|\theta_\lambda|}, h_2 = \theta_h, \sigma_{\kappa,c} = \theta_{\kappa,c}, \\
\xi_{2,0} &= \frac{2+|\theta_\lambda|}{2\pi}\theta_{\xi,0}, \sigma_{\delta,2} = \frac{2+|\theta_\lambda|}{2\pi}\frac{\theta_\delta}{\sqrt{n}}, a_{2,0} = \theta_{a,0}, \sigma_{\gamma,2} = \frac{\theta_\gamma}{\sqrt{n}}.
\end{aligned}$$

and the tv-parameters by

$$\xi_{2,t} = \frac{2+|\theta_\lambda|}{2\pi}\theta_{\xi,t}, a_{2,t} = \theta_{2,t}.$$

The damping factor (ϕ) is not allowed to be negative or higher than 1. The tv-phase parameters ($\xi_{2,0}$ and σ_δ) are rescaled with the frequency λ. This transformation implies a simpler constraint $-\pi < \theta_{\xi,t} < \pi$, instead of $-\pi/\lambda < \xi_{2,t} < \pi/\lambda$. The standard deviations $\sigma_{\delta,2}$ and $\sigma_{\gamma,2}$ are rescaled with the square root of the sample size ($\sqrt{n}$). This transformation, used in Stock and Watson (1998), allows the direct definition of the order of magnitude of $\theta_{\xi,t}$ and $\theta_{a,t}$ (instead of their innovations) by the parameters θ_δ and θ_γ .

The difficulty in estimating the SCCM model with the filtered series $y_{1,t}$ and $y_{2,t}$, then, comes from the non-linear transformation of the tv-parameters $\theta_{\xi,t}$ and $\theta_{a,t}$ in the measure equation. The model cannot be written in a linear state-space form and a linear Kalman filter cannot be

[6]There is no irregular component ($\sigma_{\varepsilon,1} = \sigma_{\varepsilon,2} = 0$), as irregular components are pre-filtered from the real data sets.

directly applied to approximate maximum likelihood estimates. An iterative extended Kalman filter procedure (described in Appendix B and called IEKF), in which the equation is linearly approximated[7] at each step n around previous step estimates, is therefore used to estimate the parameters $\boldsymbol{\theta}_\mathbf{p}$ and the tv-parameters $(\theta_{\xi,t}, \theta_{a,t})$. The procedure is initialised by pre-modelling the business cycles in a common factor Rünstler model. The procedure is iterated by applying the Kalman smoothers with diffuse initialisation at each step, until the estimates are stabilised, i.e. until the iteration error $e^{(n)}$ is lower than a fixed value. The iteration error is computed as the standard error of the difference between estimates at iteration (k) and $(k+1)$.

4 Empirical results

After a short presentation of the data sources, the estimated models are presented and the cyclical convergence is described with its two components, the tv-phase and the tv-weight. In Section 4.2, we use the SCCM models outlined above to assess whether the first OCA criterion is met for euro area countries, that is whether the national business cycles have converged towards the euro area cycle.

4.1 Data sources

The GDP time series are taken from the Eurostat database and have been retropolated to 1960 with the OECD Business Sector Database (BSDB). Countries of interest are Germany (GE), France (FR), Spain (ES), Italy (IT), the Netherlands (NL), Austria (AU), Finland (FI), Greece (GR), Portugal (PR) and Ireland (IR). The United Kingdom (UK) is also considered, as this country could meet the official criteria for entering in the euro area. Luxembourg is not considered, because the economy is very specialised in the banking sector and has a very low weight in the euro area GDP. The series are quarterly and expressed in 1995 value of euro from 1960:1 to 2005:4. They have been seasonally adjusted and, after a logarithm transformation, they have been detrended with band-pass BK filters (parameters equal to 6 and 32). Because of the detrending step, results results are provided from 1963:1 to 2002:4. Estimation of SCCM models has been carried out using algorithms and routines written on EViews.

4.2 On the convergence between euro area cycles

The bivariate SCCM model has been estimated with the IEKF procedure applied to the cycles of euro area member states and the UK. If the reference cycle had been that of the euro area aggregate, a direct bias would affect the comparison of convergence processes. In the case of a big country, a high weight in the euro area GDP would induce an artificial over-estimation of cyclical convergence, regard to that of a little one. Therefore, except for the UK which does not belong to

[7]For estimating non-linear models, Durbin and Koopman (2001) proposed such a method. However, they also recommend improving the estimation with importance sampling techniques. This improvement is let for further research.

the euro area, the euro area cycle is not chosen as reference cycle. For each euro area member, the reference cycle is computed from a euro area aggregate, which excludes the considered member.

Parameter estimates are reported in Table 1 and tv-parameter estimates are presented in the Figure 1. Cycle parameters estimates (ϕ and λ) are standard for BK filtered GDP series. In particular, the estimated frequency implies a period between 18 and 22 quarters, i.e. approximately equal to 6 years.

In a first group that includes Germany, France and Belgium, business cycles converge early toward the euro area business cycle. Their tv-weights stay between 0.7 and 1.4. Since 1980, the tv-phases stay between -1 and 2 quarters in France and Belgium. In Germany, a transitory lag appears equal to 3 quarters in 1991, because the recession takes place after the re-unification.

In a second group that includes Italy, Spain, the Netherlands, Austria and Portugal, business cycles converge only at the end of the sample. The tv-weights still take values lower than 0.5 or higher than 1.5 until 1995, but the tv-phases stay between -3 and 3 quarters since 1980. Since the launch of the euro in 1999, the tv-weights stay between 0.9 and 1.2.

In a third group that includes Finland, Greece and Ireland, convergence has still not occurred at the end of the sample. Tv-weights are equal to 1.7 in Finland, 0.4 in Greece and 2.4 in Ireland. These cycles are quite synchronised with the euro area one: tv-phases stay between 0 and 2 quarters.

The United Kingdom would belong to the third group. Indeed, its tv-weight is equal to 0.6 in 2002:4. Moreover, since 1980, the UK cycle shows a persisting lead, relative to the euro area cycle. This lead is maximal in 1991:3 with a value equal to 6 quarters, because of the German reunification. Since 1994, the lead is stable approximately equal to 3 quarters.

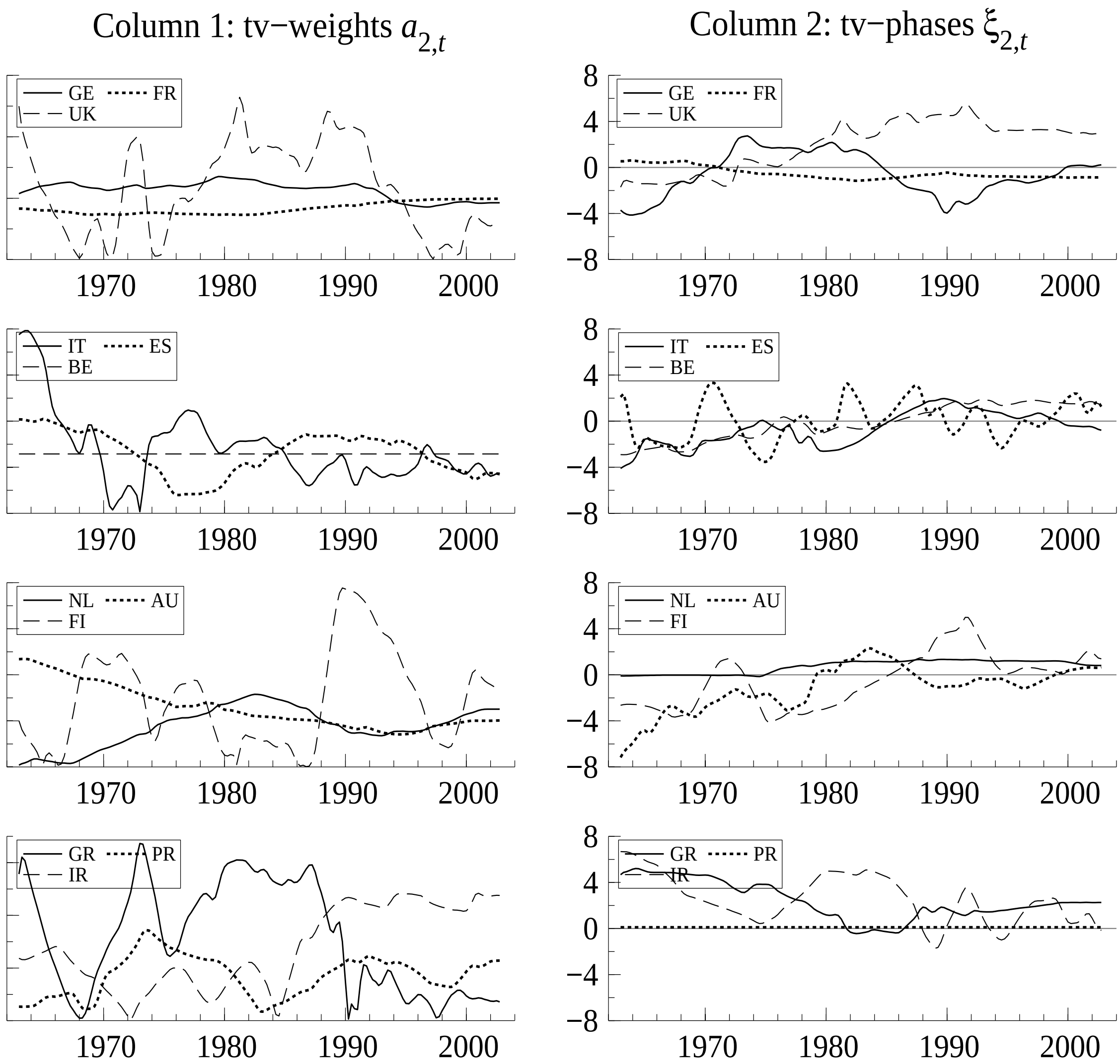
Until the end of the 1990s, the results are generally consistent with the previous papers, which use a similar approach of the cyclical convergence (Belo, 2001 and Koopman and Azevedo, 2004), but the approach is improved at the end of the 1990s and at the beginning of the 2000s. Both papers characterise the cyclical convergence by the evolution of the phase-shift and of the phase-adjusted correlation. Below equation (6), the phase-adjusted correlation should have the same evolution as the tv-weight considered in this section. With simple correlation coefficients, Belo (2001) describes the three same groups, but finds that Greece would belong to the second group. He also shows a persisting lead of the UK cycle relative to that of the euro area and, after a correction for this lead, a strong association between the two cycles. These results have been confirmed by Koopman and Azevedo (2004) with an extended unobserved component model. However, as explained previously, Koopman and Azevedo (2004) model monotonous evolution of the phase and the phase-adjusted correlation. Their model does not allow for a succession of convergence and divergence movements. Thus, in the IT, ES and UK cases, they estimate a global increase of the phase-adjusted correlation since 1970 and do not detect the decreases that appear in Figure 1. The decrease of the correlation between the UK and the euro area cycles is also undetected by Belo (2001). He computes moving-average indicators with a window of 12 years. With the annual sample 1960-1999, such indicators become useless for detecting any variation occurring after 1993 ($12/2 = 6$ years before the end of the sample).

Table 1: Parameters estimates for euro area countries, relative to the rest of the euro area

Cases	$\hat{\phi}$	$\hat{\lambda}$	$\hat{h}_2$	$\hat{\sigma}_{\kappa,c}$	$\hat{\xi}_{2,0}$	$\hat{\sigma}_{\delta,2}$	$\hat{a}_{2,0}$	$\hat{\sigma}_{\gamma,2}$
GE	0.97	0.33	0.18	0.23	0.54	0.75	0.83	0.15
FR	0.96	0.33	0.15	0.23	-4.10	0.11	3.88	0.05
IT	0.97	0.35	0.21	0.22	2.10	0.75	2.04	1.70
ES	0.97	0.30	0.19	0.23	-2.91	1.75	1.29	0.49
BE	0.96	0.36	0.17	0.22	-0.09	0.41	0.04	0.00
NL	0.96	0.32	0.18	0.22	-1.70	0.11	2.34	0.30
AU	0.96	0.35	0.19	0.22	-7.17	0.84	1.00	0.19
FI	0.97	0.29	0.22	0.22	-2.64	0.90	2.79	2.16
GR	0.96	0.35	0.30	0.22	4.67	0.55	0.27	2.12
PR	0.97	0.32	0.32	0.22	0.11	0.00	1.18	0.62
IR	0.97	0.31	0.26	0.22	6.67	1.15	2.50	0.92
UK	0.97	0.31	0.12	0.22	-3.71	0.72	1.07	1.84

Legend: parameter estimates ($\hat{\mathbf{p}}$) are presented for models of euro area countries cycles, relative to the cycle of the rest of the euro area. For the United Kingdom, the reference is the cycle of the aggregated euro area.
Source: Eurostat, OECD and computations of the author.

Figure 1: Tv-parameters estimates for euro area countries, relative to the rest of the euro area



Legend: estimates of the tv-parameters ($\hat{\xi}_{2,t}$,$\hat{a}_{2,t}$) are presented for models of euro area countries cycles, relative to the cycle of the rest of the euro area. For the United Kingdom, the reference is the cycle of the aggregated euro area.

Note: The Belgian tv-weight and the Portugese tv-shift are constant, because the estimates of their innovation variances are not significantly different from zero.

Source: Eurostat, OECD and computations of the author.

5 Conclusion

SCCM models have been applied to the cycle of each euro area country, relative to the rest of the euro area, from 1963:1 to 2002:4. The SCCM model is an extension of the models developed by Rünstler (2004) and Koopman and Azevedo (2004), in which the evolution of the relation between two cycles is characterised by the tv-phase and the tv-weight.

Empirical results are twofold. Firstly, the cycles of the euro area have synchronised: their tv-phases have generally converged toward low values (between -2 and 2 quarters) before the launch of the euro. Concerning the tv-weights, results are more ambiguous. Germany, France and Belgium have converged toward the euro area cycle since 1980. The convergence is more recent for Italy, Spain, the Netherlands, Austria and Portugal. Despite the launch of the euro in 1999, Finland, Greece and Ireland have still not converged. Thus, in terms of business cycle convergence, being in a monetary union might raise problems especially for the third group, which represents 5.8 % of the euro area GDP.

For the United Kingdom, the convergence was not achieved in 2002: the UK cycle is dampened and has the biggest lead (3 quarters), relative to other euro area countries. Thus, if the UK entered into the euro area, it would belong to the third group, which would represent 17.5 % of the euro area aggregated GDP. Our results suggest that it would be delicate for the UK to join the euro area.

Although our convergence model has proved to be an interesting starting point for testing the stochastic convergence of business cycles in a probabilistic framework, some improvements have been left for future research. The estimation procedure of our non-linear model could be made more precise by using importance sampling techniques. In particular, such techniques would make it possible to simulate posterior distribution of the tv-parameters and to estimate their confidence bands. The model could be applied with a higher multivariate dimension. An explanation of the convergence could be given by including other variables, like economic policy indicators (interest rates, exchange rates and fiscal indicators) in the convergence mechanism.

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Appendices

A Auto-covariance of business cycles in a SCCM model

A.1 Simple multivariate form of the SCCM model

The SCCM model (4) is statistically equivalent (has the same cross-autocovariance functions) to the following model:

$$
\begin{aligned}
y_{1,t} &= [1,0]\,\bar{\boldsymbol{\psi}}_{1,t} + \varepsilon_{1,t},\\
y_{2,t} &= \left[\cos(\lambda\xi_{2,t}), \sin(\lambda\xi_{2,t})\right]\bar{\boldsymbol{\psi}}_{2,t} + \varepsilon_{2,t},\\
\bar{\boldsymbol{\psi}}_{i,t} &= \phi T_\lambda \bar{\boldsymbol{\psi}}_{i,t-1} + \bar{\boldsymbol{\kappa}}_{i,t},
\end{aligned}
$$

with $\bar{\boldsymbol{\psi}}_{1,t} = \left[\psi_{i,t}, \psi^{+}_{i,t}\right]'$ and $\bar{\boldsymbol{\kappa}}_{i,t} = \left[\kappa_{i,t}, \kappa^{+}_{i,t}\right]'$. The initial conditions at $t = 0$ are defined by $\bar{\boldsymbol{\psi}}_{i,0} = \bar{\boldsymbol{\kappa}}_{i,0}$. $\boldsymbol{\kappa}_t = [\kappa_{1,t}, \kappa_{2,t}]'$ and $\boldsymbol{\kappa}^{+}_t = \left[\kappa^{+}_{1,t}, \kappa^{+}_{2,t}\right]'$ are bivariate normal disturbances mutually uncorrelated at all time periods and have covariance matrices $\Sigma_{\kappa,t}$:

$$
\begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} \sim NID(0,\Sigma_\kappa),\; \begin{bmatrix} \kappa^{+}_{1,t} \\ \kappa^{+}_{2,t} \end{bmatrix} \sim NID(0,\Sigma_\kappa),\; \text{with } \Sigma_{\kappa,t} = \begin{bmatrix} \sigma^2_{\kappa,1} & \rho_t\sigma_{\kappa,1}\sigma_{\kappa,2} \\ \rho_t\sigma_{\kappa,1}\sigma_{\kappa,2} & \sigma^2_{\kappa,2} \end{bmatrix}.
$$

$\bar{\boldsymbol{\kappa}}_{i,t}$ are bivariate normal disturbances and have 2×2 covariance matrices $\sigma^2_{\kappa,i} I_2$ (with I_2 the 2×2 identity matrix).

Given that

$$
E\left(\psi_{1,0}\psi_{2,0}\middle|\, \boldsymbol{\rho}_{1:t}\right) = E\left(\kappa_{1,0}\kappa_{2,0}\middle|\, \boldsymbol{\rho}_{1:t}\right) = E\left(\kappa^{+}_{1,0}\kappa^{+}_{2,0}\middle|\, \boldsymbol{\rho}_{1:t}\right) = E\left(\psi^{+}_{1,0}\psi^{+}_{2,0}\middle|\, \boldsymbol{\rho}_{1:t}\right),
$$

we can prove recursively that

$$
E\left(\psi_{1,t}\psi_{2,t}\middle|\, \boldsymbol{\rho}_{1:t}\right) = E\left(\psi^{+}_{1,t}\psi^{+}_{2,t}\middle|\, \boldsymbol{\rho}_{1:t}\right) \tag{A.1}
$$

for each t.

A.2 Cross-autocovariance between bivariate cycles

Before considering the covariance matrix of business cycles, conditional on the tv-parameters, since $(T_\lambda)^\tau = T_{\lambda\tau}$ for all integer τ, we compute the cross-autocovariance $\Omega_\rho(\tau,t)$ between the bivariate cycles $\bar{\boldsymbol{\psi}}_{1,t}$ and $\bar{\boldsymbol{\psi}}_{2,t-\tau}$ at a lag $\tau > 0$, conditional on $\boldsymbol{\rho}_{1:t} = [\rho_1, ..., \rho_t]'$:

$$
\begin{aligned}
\Omega_\rho(\tau,t) &= E\left(\bar{\boldsymbol{\psi}}_{1,t}\bar{\boldsymbol{\psi}}'_{2,t-\tau}\middle|\, \boldsymbol{\rho}_{1:t}\right) \qquad\qquad (A.2)\\
&= E\left[\left(\phi^\tau T_{\lambda\tau}\bar{\boldsymbol{\psi}}_{1,t-\tau} + \sum_{i=0}^{\tau-1}\phi^i T_{\lambda i}\bar{\boldsymbol{\kappa}}_{1,t-i}\right)\bar{\boldsymbol{\psi}}'_{2,t-\tau}\middle|\, \boldsymbol{\rho}_{1:t}\right]\\
&= \phi^\tau T_{\lambda\tau}\Omega_\rho(0,t-\tau),
\end{aligned}
$$

with $E\left[\cdot\middle|\, \boldsymbol{\rho}_{1:t}\right]$ the "conditional on $\boldsymbol{\rho}_{1:t}$" expectation. Given that $\bar{\boldsymbol{\psi}}_{1,t-1} \perp \bar{\boldsymbol{\kappa}}_{2,t}$ and $\bar{\boldsymbol{\psi}}_{2,t-1} \perp \bar{\boldsymbol{\kappa}}_{1,t}$, the cross-covariance $\Omega_\rho(0,t)$ between $\bar{\boldsymbol{\psi}}_{1,t}$ and $\bar{\boldsymbol{\psi}}_{2,t}$, conditional on $\boldsymbol{\rho}_{1:t}$, is given by

$$
\begin{aligned}
\Omega_\rho(0,t) &= E\left(\bar{\boldsymbol{\psi}}_{1,t}\bar{\boldsymbol{\psi}}'_{2,t}\middle|\, \boldsymbol{\rho}_{1:t}\right)\\
&= E\left[\left(\phi T_\lambda\bar{\boldsymbol{\psi}}_{1,t-1} + \bar{\boldsymbol{\kappa}}_{1,t}\right)\left(\phi T_\lambda\bar{\boldsymbol{\psi}}_{2,t-1} + \bar{\boldsymbol{\kappa}}_{2,t}\right)'\middle|\, \boldsymbol{\rho}_{1:t}\right]\\
&= \phi^2 T_\lambda\Omega_{\boldsymbol{\rho}}(0,t-1)T'_\lambda + \rho_t\sigma_{\kappa,1}\sigma_{\kappa,2}I_2,
\end{aligned}
$$

Since $\psi^+_{1,t} \perp \psi_{2,t}$, $\psi_{1,t} \perp \psi^+_{2,t}$ and from (A.1), it follows that

$$\Omega_\rho(0,t) = \begin{bmatrix} E\left(\psi_{1,t}\psi_{2,t}\middle|\,\boldsymbol{\rho}_{1:t}\right) & E\left(\psi_{1,t}\psi^+_{2,t}\middle|\,\boldsymbol{\rho}_{1:t}\right) \\ E\left(\psi^+_{1,t}\psi_{2,t}\middle|\,\boldsymbol{\rho}_{1:t}\right) & E\left(\psi^+_{1,t}\psi^+_{2,t}\middle|\,\boldsymbol{\rho}_{1:t}\right) \end{bmatrix} = E\left(\psi_{1,t}\psi_{2,t}\middle|\,\boldsymbol{\rho}_{1:t}\right) I_2$$

Hence, $\Omega_\rho(0,t-1)$ commutes with the matrix T_λ which is orthogonal ($T_\lambda T_\lambda^{'} = I_2$),

$$\Omega_\rho(0,t) = \phi^2 \Omega_\rho(0,t-1) + \rho_t \sigma_{\kappa,1}\sigma_{\kappa,2} I_2. \tag{A.3}$$

Since $\Omega_\rho(0,0) = \rho_0 \sigma_{\kappa,1}\sigma_{\kappa,2} I_2$ and given equation (A.3),

$$\Omega_\rho(0,t) = (1-\phi^2)^{-1} \tilde{\rho}_t \sigma_{\kappa,1}\sigma_{\kappa,2} I_2, \tag{A.4}$$

with $\tilde{\rho}_t = (1-\phi^2)\sum_{i=0}^{t} \phi^{2i}\rho_{t-i}$ the exponential smoother of ρ_t (the parameter is equal to ϕ^2).

From equations (A.2) and (A.4), we derive the cross-autocovariance $\Omega_\rho(\tau,t)$ between bivariate cycles $\bar{\boldsymbol{\psi}}_{1,t}$ and $\bar{\boldsymbol{\psi}}_{2,t-\tau}$ at a lag $\tau > 0$, conditional on $\boldsymbol{\rho}_{1:t} = [\rho_1, ..., \rho_t]'$:

$$\Omega_\rho(\tau,t) = (1-\phi^2)^{-1} \phi^\tau \tilde{\rho}_{t-\tau} \sigma_{\kappa,1}\sigma_{\kappa,2} T_{\lambda\tau}, \tag{A.5}$$

A.3 Auto-covariance matrix between business cycles

From (A.5), the cross-autocovariance $\gamma_{\rho,\xi}(\tau,t)$ between business cycles $y_{1,t}$ and $y_{2,t}$ at a lag $\tau > 0$, conditional on $\boldsymbol{\rho}_{1:t}$ and $\boldsymbol{\xi}_{1:n} = [\xi_1, ..., \xi_n]'$, is given by

$$\begin{aligned}
\gamma_{\rho,\xi}(\tau,t) &= E\left[\left[\begin{array}{cc} 1 & 0 \end{array}\right] \bar{\boldsymbol{\psi}}_{1,t}\bar{\boldsymbol{\psi}}^{'}_{2,t-\tau} \left[\begin{array}{c} \cos\xi_{2,t-\tau} \\ \sin\xi_{2,t-\tau} \end{array}\right]\middle|\, \boldsymbol{\rho}_{1:t}, \boldsymbol{\xi}_{1:n}\right] \\
&= \left[\begin{array}{cc} 1 & 0 \end{array}\right] \Omega_\rho(\tau,t) \left[\begin{array}{c} \cos\xi_{2,t-\tau} \\ \sin\xi_{2,t-\tau} \end{array}\right] \\
&= (1-\phi^2)^{-1}\phi^\tau \tilde{\rho}_{t-\tau}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau - \xi_{2,t-\tau})\right].
\end{aligned}$$

with $E(\cdot|\,\boldsymbol{\rho}_{1:t}, \boldsymbol{\xi}_{1:t})$ the "conditional on $\boldsymbol{\rho}_{1:t}$ and $\boldsymbol{\xi}_{1:n}$" expectation.

By using similar arguments, it might be proved more generally that the autocovariance matrix $\Gamma_{\rho,\xi}(\tau,t)$ of $[\psi_{1,t}; \cos(\lambda\xi_{2,t})\psi_{2,t} + \sin(\lambda\xi_{2,t-\tau})\psi^+_{2,t}]'$ at a lag $\tau \geq 0$, conditional on $\boldsymbol{\rho}_{1:t}$ and $\boldsymbol{\xi}_{1:n}$, is given by

$$\Gamma_{\rho,\xi}(\tau,t) = (1-\phi^2)^{-1}\phi^\tau \left[\begin{array}{cc} \sigma^2_{\kappa,1}\cos(\lambda\tau) & \tilde{\rho}_{t-\tau}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau+\xi_{2,t})\right] \\ \tilde{\rho}_{t-\tau}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau-\xi_{2,t-\tau})\right] & \sigma^2_{\kappa,2}\cos(\lambda\tau) \end{array}\right];$$

at a lag $\tau < 0$, it can also be shown that

$$\Gamma_{\rho,\xi}(\tau,t) = (1-\phi^2)^{-1}\phi^{-\tau} \left[\begin{array}{cc} \sigma^2_{\kappa,1}\cos(\lambda\tau) & \tilde{\rho}_{t}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau+\xi_{2,t})\right] \\ \tilde{\rho}_{t}\sigma_{\kappa,1}\sigma_{\kappa,2}\cos\left[\lambda(\tau-\xi_{2,t-\tau})\right] & \sigma^2_{\kappa,2}\cos(\lambda\tau) \end{array}\right].$$

At time t, if $\phi = 0$, $\phi = 1$, $\lambda = 0$, $\lambda = \pi$ or $\boldsymbol{\rho}_{1:t} = \mathbf{0}_t$, the autocovariance matrix $\Gamma_{\rho,\xi}(\tau,t)$ does not depend on $\xi_{2,t}$ and the phase-shift $\xi_{2,t}$ is not identifiable. Thus, the *identification conditions* ($\phi > 0$), ($0 < \lambda < \pi$) and ($\rho_t > 0$, $\forall t$) are assumed.

B Iterative extended Kalman filter (IEKF)

The local version of the iterative extended Kalman filter (IEKF) is an iterative procedure, designed for estimating a linear approximate of the SCCM model. The notations are defined in Section 3.2 and Section 3.3.

Initialisation $k = 0$

- Estimation of $\bar{\boldsymbol{\psi}}_t^c, \bar{\boldsymbol{\psi}}_{2,t}^*, \hat{\theta}_{\xi}, \hat{\theta}_a$ in the common factor Rünstler model, by applying the Kalman and EM algorithms to the equation system (**??**), where transformed parameters $\boldsymbol{\theta_p}$ have been integrated. The phase-shift θ_ξ and the weight a are also deduced from transformed parameters: $(\xi = \theta_\xi, a = \theta_a)$
- $\hat{\theta}_{\xi,t}^{(0)} = \hat{\theta}_\xi, \hat{\theta}_{a,t}^{(0)} = \hat{\theta}_a$.
- $e^{(0)} = 1$.

Iteration k+1

- If the iteration error $e^{(k)} > 10^{-7}$, Kalman and EM algorithms (with diffuse initialisation) are applied to models (B.1) and (B.2), otherwise the algorithm is stopped. First, $a_t^{(k+1)}$ is estimated conditional to $\hat{\xi}_t^{(k)}$:

$$\begin{cases} y_{1,t} = [1,0]\bar{\boldsymbol{\psi}}_t^c \\ y_{2,t} = \theta_{a,t}^{(k+1)} \left[\cos\hat{\theta}_{\xi,t}^{(k)}, \sin\hat{\theta}_{\xi,t}^{(k)}\right] \bar{\boldsymbol{\psi}}_t^c + [1,0]\tilde{\psi}_{2,t}^{*(k+1)} \\ \bar{\boldsymbol{\psi}}_{2,t}^{*(k+1)} = \frac{|\theta_\phi|}{\sqrt{1+\theta_\phi^2}} T_{\frac{2\pi}{2+|\theta_\lambda|}} \bar{\boldsymbol{\psi}}_{2,t-1}^{*(k+1)} + \tilde{\kappa}_{2,t}^* \\ \theta_{a,t}^{(k+1)} = \theta_{a,t-1}^{(k+1)} + \gamma_{2,t}, \theta_{a,0}^{(k+1)} = \theta_{a,0} \end{cases}. \tag{B.1}$$

  with $\kappa_t^c \sim N(0, \theta_{\kappa,c})$, $\kappa_t^{+c} \sim N(0, \theta_{\kappa,c})$, $\kappa_{2,t}^* \sim N(0, \theta_h)$, $\kappa_{2,t}^{+*} \sim N(0, \theta_h)$, $\gamma_{2,t} \sim N(0, \frac{\theta_\gamma}{\sqrt{n}})$.

- Then, $\xi_t^{(k+1)}$ is estimated conditional to $\hat{a}_t^{(k+1)}$ and $\hat{\xi}_t^{(k)}$:

$$\begin{cases} y_{1,t} = [1,0]\bar{\boldsymbol{\psi}}_t^c \\ \tilde{y}_{2,t} = \hat{\theta}_{a,t}^{(k+1)} \hat{\theta}_{\xi,t}^{(k+1)} \left[-\sin\hat{\theta}_{\xi,t}^{(k)}, \cos\hat{\theta}_{\xi,t}^{(k)}\right] \bar{\boldsymbol{\psi}}_{1,t} + [1,0]\tilde{\psi}_{2,t}^{**(k+1)} \\ \bar{\boldsymbol{\psi}}_{2,t}^{**(k+1)} = \frac{|\theta_\phi|}{\sqrt{1+\theta_\phi^2}} T_{\frac{2\pi}{2+|\theta_\lambda|}} \bar{\boldsymbol{\psi}}_{2,t-1}^{**(k+1)} + \tilde{\kappa}_{2,t}^* \\ \theta_{\xi,t}^{(k+1)} = \theta_{\xi,t-1}^{(k+1)} + \frac{2\pi}{2+|\theta_\lambda|}\delta_{2,t}, \theta_{\xi,0}^{(k+1)} = \theta_{\xi,0} \end{cases}, \tag{B.2}$$

  with $\kappa_t^c \sim N(0, \theta_{\kappa,c})$, $\kappa_t^{+c} \sim N(0, \theta_{\kappa,c})$, $\kappa_{2,t}^* \sim N(0, \theta_h)$, $\kappa_{2,t}^{+*} \sim N(0, \theta_h)$, $\delta_{2,t} \sim N(0, \frac{2+|\theta_\lambda|}{2\pi} \frac{\theta_\delta}{\sqrt{n}})$, $\tilde{\psi}_{2,t}^{**(k+1)}$ the specific cycle of the linearised model (B.2) at the iteration $(k+1)$ and $\tilde{y}_{2,y}$ the adjusted series defined by:

$$\tilde{y}_{2,t} = y_{2,t} - \hat{\theta}_{a,t}^{(k+1)} \left[\cos\hat{\theta}_{\xi,t}^{(k)}, \sin\hat{\theta}_{\xi,t}^{(k)}\right] \bar{\boldsymbol{\psi}}_{1,t} + \hat{\theta}_{a,t}^{(k+1)} \hat{\theta}_{\xi,t}^{(k)} \left[-\sin\hat{\theta}_{\xi,t}^{(k)}, \cos\hat{\theta}_{\xi,t}^{(k)}\right] \bar{\boldsymbol{\psi}}_{1,t}.$$

- Finally, the iteration error $e^{(k+1)}$ is computed:

$$e^{(k+1)} = 0.5 \times \sqrt{\frac{1}{n}\sum_{t=0}^{n}\left(\hat{\theta}_{a,t}^{(k+1)} - \hat{\theta}_{a,t}^{(k)}\right)^2} + 0.5 \times \sqrt{\frac{1}{n}\sum_{t=0}^{n}\left(\hat{\theta}_{\xi,t}^{(k+1)} - \hat{\theta}_{\xi,t}^{(k)}\right)^2}.$$