The Distributional Implications of Climate Policies Under Uncertainty

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Abstract

Promoting the decarbonization of economic activity through climate policies raises many questions. From a macroeconomic perspective, it is important to understand how these policies perform under uncertainty, how they affect short-run dynamics and to what extent they have distributional effects. In addition, uncertainties directly associated with climate policies, such as uncertainty about the carbon budget or emission intensities, become relevant aspects. We study the implications of emission reduction schemes within a Two-Agent New-Keynesian (TANK) model. This quantitative exercise, based on data for the German economy, provides various insights. In the light of frictions and fluctuations, compared to other instruments, a carbon price (i.e. tax) is associated with lower volatility in output and consumption. In terms of aggregate welfare, price instruments are found to be preferable. Conditional on the distribution of revenues from climate policies, quantity instruments can exert regressive effects, posing a larger economic loss on wealth-poor households, whereas price instruments are moderately progressive. Finally, we find that unexpected changes in climate policies can induce substantial aggregate adjustments. With uncertainty about the carbon budget, the costs of adjustment are larger under quantity instruments.

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1 Introduction

The early work of Nordhaus (1977) has already stressed the fact that greenhouse gas (GHG) emissions evolving as an external effect of economic activity affect atmospheric dynamics and spur global warming. Today, most policymakers and academics reached a consensus that reducing GHG emissions in order to alleviate global warming is a fundamental challenge the global community is facing at the moment. Due to the potential consequences of temperature increases, such as floods, droughts and extreme weather events, the goal is to limit the increase in global temperatures by meeting a certain carbon budget. This ambition is reflected in the increasing number of national and supra-national initiatives that seek to reduce emissions. As of today, the Carbon Pricing Dashboard of the World Bank reports over 50 different initiatives either already implemented or scheduled for implementation. In general, these initiatives seek to achieve emissions reductions through policy instruments that attach a price to emissions in order to incentivize and promote a timely decarbonization of economic activity. Clearly, the transition towards a carbon-neutral economy takes time, and the required structural adjustment is associated with costs. These economic costs differ between policy instruments and are not necessarily uniformly distributed across economic agents.

Therefore, in order to implement climate policies efficiently and to avoid adverse distributional consequences, it is important to understand how these policies affect economic activity and gauge their distributional implications. In this context, Weitzman (1974) pointed out that economic uncertainty has direct implications for overall welfare and the optimal choice of policy instruments. Based on this, a strand of macroeconomic research developed to assess the implications of climate policies for short-run macroeconomic dynamics under uncertainty. As documented by Doda (2014), GHG emissions fluctuate at business cycle frequencies and tend to move procyclical with GDP. This implies that policies that reduce emissions can also exert effects on short-run macroeconomic dynamics. Using a real business cycle model, Heutel (2012) examines the effects of climate policies for macroeconomic dynamics and welfare. He finds that emission policies should optimally behave procyclically. In a similar framework, Fischer and Springborn (2011) compare taxes and quotas to intensity targets with regard to their implications for macroeconomic volatility and welfare. They find that an intensity target exerts a comparatively small effect on output, but implies larger volatility in labor markets relative to other instruments. Their results suggest that from a welfare perspective, a tax is the preferable instrument. Dissou and Karnizova (2016) compare a cap-and-trade system to a carbon tax within a multi-sectoral business cycle model and find that a cap-and-trade system provides a more stable macroeconomic environment, but is deficient in terms of welfare compared to a tax.

Subsequent research on the macroeconomic implications of climate policy design has extended the agenda in several directions. Annicchiarico and Dio (2015) examine climate policies within a New-Keynesian business cycle model, taking nominal and real rigidities into account. Their results confirm that taxes and intensity targets yield higher welfare but also imply a higher volatility in macroeconomic aggregates compared to a cap-and-trade system. They also point out that the welfare effects of policy instruments crucially depend on the degree of nominal frictions. In a similar framework, Annicchiarico and Di Dio (2017) study the interactions between climate policy and monetary policy. They highlight that these interactions have important implications for macroeconomic dynamics and the effectiveness of climate policies. Another aspect studied by Annicchiarico and Diluiso (2019) within a two-country
New-Keynesian model are international spillover effects of climate policies. Their findings suggest that a tax regime tends to amplify fluctuations while a cap-and-trade scheme alleviates spillover effects. They highlight that the welfare effects of policy instruments depend on the type of shock and thus provide no clear-cut ranking between the different policies.

The present paper contributes to this strand of research in several ways. Primarily, we contribute to the debate regarding the design of climate policies under uncertainty by taking distributional aspects into account. This provides an important additional dimension for the assessment of different policy instruments. While the analysis of Ohlendorf et al. (2020) highlights that the distributional effects of climate policies have already been examined in various constellations, the short-run macroeconomic perspective has largely been neglected. Building on a Two-Agent New-Keynesian (TANK) model, we close this gap in the literature. As emphasized by Debortoli and Gali (2017), the presence of heterogeneous households affects aggregate macroeconomic dynamics, and improves the empirical fit of these models. Furthermore, we enrich the analysis by taking wage rigidites into account. This reflects the empirically observed inertia and downward rigidity in wages and allows us to assess the distributional effects of labor market frictions. Finally, we use this framework to assess uncertainties that directly evolve from climate policies. These uncertainties fundamentally depend on our scientific knowledge and technological possibilities. On the one hand, as argued by Fujimori et al. (2019), the estimates of the remaining carbon budget, as published by the Intergovernmental Panel on Climate Change (IPCC), differ substantially across studies and are subject to ongoing research. Advances in the understanding of atmospheric dynamics can thus lead to revisions and updates of the carbon budget, which requires alignments of climate policies. On the other hand, the technological abilities of the economy to reduce emissions vary over time. Conditional on external factors such as weather or internal factors like the utilization of factor inputs, the emission intensity of production can change. In order to comply with emissions reduction policies, these fluctuations require economic adjustments.

We calibrate the numerical model to match stylized aspects of the German economy and conduct a comparison between price instruments, a cap-and-trade scheme and an intensity target policy. While we find that those instruments do not alter the dynamics of macroeconomic aggregates qualitatively, we observe quantitative differences in terms of volatility and aggregate welfare. In the present model, a cap-and-trade system is associated with a larger volatility in output and consumption than other instruments. The price instrument is preferable from a welfare perspective. Furthermore, for the baseline specification, price instruments are found to exert neutral or even slightly progressive effects, while the intensity target and the cap-and-trade system are found to be regressive. This finding results from the presence of frictions and also depends on the utilization of the revenues from climate policy. When revenues are redistributed via transfers, climate policy becomes moderately progressive. In terms of aggregate welfare, the documented annual welfare costs associated with climate policy in Germany vary between 43 euros per capita under a price instrument with full redistribution, and 317 euros per capita under a cap-and-trade scheme with full absorption of revenues. Furthermore, uncertainty associated with climate policies alters macroeconomic dynamics and welfare and should not be neglected in the assessment of policy instruments. A 10% change in the carbon budget requires substantial economic adjustments associated with consumption and output losses. Through a staggered adjustment of policy instruments these transitional costs can be alleviated. Nevertheless, the results of stochastic simulations highlight that already the presence of carbon budget uncertainty is relevant for aggregate welfare.
The remainder of this paper is structured as follows. The second section presents the theoretical model and briefly discusses the set of policy instruments. The third section presents the calibration and the numerical analysis of the model. Finally, the last section concludes the paper.

2 Model

2.1 Structure of the Model

In the following model, we focus on short-run macroeconomic dynamics and do not model the potential adverse effects of climate change and the necessary long-run adjustments of production structures explicitly. The model economy can then be interpreted in two ways. As emphasized by Annicchiarico and Dio (2015), one view is that the economy has already accomplished the transition towards a carbon-neutral stationary equilibrium. Alternatively, one can assume that the model economy moves along a deterministic balanced decarbonization path. During the transition process, the economy is subject to aggregate uncertainty, where different types of shocks induce fluctuations around the pathway and require economic adjustments.

Formally, the model integrates carbon dioxide emissions that emerge as a by-product of production as in Fischer and Springborn (2011) into a one-sector New-Keynesian model. As it is common in the literature, we assume that firms operate under monopolistic competition and face nominal price setting rigidities. Furthermore, we include real frictions in the form of convex investment adjustment costs, which impedes adjustments of the stock of physical capital. Frictions on the labor market are introduced through a union framework, where similar to the model of Erceg et al. (2000), nominal wage rigidities lead to wage stickiness. In order to capture the effects of household heterogeneity and inequality, we follow Gali et al. (2004) and distinguish between two types of households that differ in their ability to smooth consumption via savings. The public sector, in terms of monetary and fiscal policy, follows the conventions in the literature. Within this framework we assess the implications of different climate policy instruments in the light of aggregate uncertainty.

2.2 Households

The economy is populated by a continuum of households $I \in [0, 1]$. In particular, we assume that two types of households exist as in Gali et al. (2004). A fraction of households $1 - \lambda$ have access to capital markets where they can accumulate physical capital $k_t$ and rent it to firms. These households will be referred to as Ricardian households, indicated by the subscript $R$. The remaining fraction of households $\lambda$, who have no access to capital markets and consequently own no assets, will be referred to as non-Ricardian households with subscript $N$.

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1As pointed out in Fischer and Heutel (2013), different ways to incorporate pollution into Dynamic Stochastic General Equilibrium (DSGE) models exist. For example Heutel (2012), uses a damage function approach, treating the stock of carbon dioxide as a state variable, comparable to the integration of pollution in most Integrated Assessment Models (IAM).

2The fact that a large fraction of households have no, or even negative net worth and thus behaves as rule-of-thumb consumers has already been stressed by Campbell and Mankiw (1989). Recent figures presented by Kuhn et al. (2018) show that in the United States between 1950 and 2016, the bottom 25% of the wealth distribution had almost zero net worth.
Ricardian Households

The representative Ricardian household chooses consumption \( c_{R,t} \), investment \( x_t \) and labor supply \( h_{R,t} \) in order to maximize expected life-time utility:

\[
E_0 \sum_{t=0}^{\infty} d_t \beta^t \left[ \frac{c_{R,t}^{1-\rho} - \psi h_{R,t}^{1+\chi}}{1-\rho} \right], \quad \beta \in (0,1), \chi > 0,
\]

where \( \beta \) denotes the discount factor of households, \( \psi \) denotes the disutility from labor, \( \chi \) denotes the inverse Frisch elasticity and \( \rho \) denotes the inverse of the intertemporal elasticity of substitution. The terms \( d_t \) and \( \nu_t \) represent stochastic processes, i.e. a preference shock and a labor supply shock respectively. The preference shock affects the time preferences of Ricardian households and evolves according to

\[
d_t = \rho d_t - 1 + \varepsilon_d, \quad \rho \text{ captures the persistence and } \varepsilon_d \text{ the stochastic innovations of the process.}
\]

The labor supply shock evolves as

\[
\nu_t = \rho \nu_{t-1} + \varepsilon_{\nu,t}, \quad \text{with persistence } \rho \nu \text{ and stochastic innovations in labor disutility } \varepsilon_{\nu,t}.
\]

The optimization of Ricardian households is subject to a flow budget constraint. In real terms, the constraint reads:

\[
c_{R,t} + x_t + b_t = \mathcal{W}_t h_{R,t} + R_{t-1} \frac{b_{t-1}}{\Pi_t} + F_{F,t} + F_{F,t} + R_{k,t} k_t - T_t,
\]

where \( \mathcal{W}_t \) denotes the real remuneration of labor supplied to a union. The real return of capital is denoted by \( R_{k,t} \), inflation is denoted by \( \Pi_t = \frac{p_t}{p_{t-1}} \), \( b_t \) denotes the stock of risk-free one period government bonds, \( R_t \) denotes the nominal interest rate and \( T_t \) denotes a lump-sum tax levied by the government. Since firms are owned by Ricardian households, they receive firm profits \( F_{F,t} \) and a share of union profits \( F_{U,t} \).

In order to capture the fact that capital cannot be adjusted instantaneously at business cycle frequencies, convex investment adjustment costs, similar to Christiano et al. (2005), are introduced as follows:

\[
k_{t+1} = \left[ 1 - \frac{\kappa}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right) \right]^2 z_t x_t + (1 - \delta) k_t.
\]

Here, \( \kappa \) captures the degree of adjustment costs and \( \delta \) denotes the depreciation rate. With this formulation, the costs of investment increase in the growth rate of investment. Thus, large increases in investment in a single period are particularly expensive, which implies that households will spread investment over several periods. Therefore, adjustments of the stock of physical capital require time. The term \( z_t \) denotes investment efficiency, i.e. the efficiency to transform investments into physical capital. We assume that investment efficiency is subject to stochastic fluctuations, which are given by \( z_t = \rho z_{t-1} + \varepsilon_{z,t} \), where \( \rho_z \) captures the persistence and \( \varepsilon_{z,t} \) the stochastic innovations of the process. The inclusion of investment shocks is motivated by the findings of Khan et al. (2019), who show that investment shocks appear to be an important determinant for the fluctuations in emissions.

The solution of the household problem yields the following first-order conditions for consumption, labor supply, bond holdings, capital and investment:

\[
\lambda_{R,t} = d_t c_{R,t}^{-\rho}
\]

\[
\lambda_{R,t} = \nu_t \psi h_{R,t}^{\frac{\chi}{1+\chi}} \mathcal{W}_t^{-1}
\]
\[ \lambda_{R,t} = \beta R_t E_t \lambda_{R,t+1} \Pi_{t+1} \]  
\[ 1 = q_t \left( 1 - \frac{\kappa}{2} \left( \frac{x_t}{x_{t+1}} - 1 \right) \right)^2 - \kappa \left( \frac{x_t}{x_{t-1}} - 1 \right) \] 
\[ + \beta E_t \frac{\lambda_{R,t+1}}{\lambda_{R,t}} q_{t+1} k \left( \frac{x_{t+1}}{x_t} - 1 \right) \left( \frac{x_{t+1}}{x_t} \right)^2 \]  
\[ q_t = \beta E_t \frac{\lambda_{R,t+1}}{\lambda_{R,t}} z_t \left( 1 - \delta \right) \left( q_{t+1} + z_{t+1} R_{K,t+1} \right) \]  

Here \( \lambda_{R,t} \) denotes the marginal utility of an additional unit of consumption and \( q_t \) denotes Tobin's Q, which captures the value of installed capital relative to new capital.\(^3\) Therefore, the relative price of installed capital can deviate from the price of newly built capital.

**Non-Ricardian Households**

Non-Ricardian households are not able to smooth consumption through saving and thus seek to optimize their utility period-by-period. In the following, we assume that the utility function is additive-separable and has the functional form: \( u_N = \frac{c_{N,t}^\gamma}{1 - \gamma} - \Psi \frac{h_{N,t}^{\beta \frac{1}{\gamma}}}{(1 + \kappa)} \).\(^4\) In the absence of access to capital markets, non-Ricardian households face a constraint, which restricts their consumption \( c_{N,t} \) to their current income, i.e. \( c_{N,t} = \Psi_t h_{N,t} - T_t + F_{u,t} \). The labor supply of non-Ricardian households \( h_{N,t} \) is given by:

\[ \Psi_t \frac{h_{N,t}^\beta}{(1 + \kappa)} = c_{N,t}^{\gamma - \beta} \Psi_t \]  

which in this case is not constant and depends on the real labor remuneration and the marginal utility from consumption.\(^5\) Aggregate household consumption \( c_t \) and aggregate labor supply \( h_t \) correspond to the weighted averages of the individual variables and are defined as:

\[ c_t \equiv \lambda c_{N,t} + (1 - \lambda) c_{R,t} \]  
\[ h_t \equiv \lambda h_{N,t} + (1 - \lambda) h_{R,t} \]

**Union Wage Setting**

In order to introduce inertia in wage adjustments, we follow the approach of Sims and Wu (2019) and assume that households supply differentiated labor inputs to a continuum of unions \( u \in [0, 1] \).\(^6\) Unions remunerate households and sell labor inputs \( h_{u,t} \) to a competitive labor packer at the price \( w_{u,t} \). The labor packer uses a constant elasticity of substitution (CES) aggregator, given by \( h_{d,t} = \left( \int_0^1 h_{u,t}^{1/(\eta_u-1)} du \right)^{\eta_u}/(\eta_u - 1) \), to transform differentiated union labor into final labor input for the production

\(^3\)More formally, Tobin's Q represents the marginal utility of having an additional future unit of installed capital \( k_{t+1} \) over the marginal utility of having an additional unit of consumption.

\(^4\)While labor supply of non-Ricardian households is subject to the same shock \( \Psi_t \) as the labor supply of Ricardian households, they do not face preference shocks.

\(^5\)If the utility function is non-separable in consumption and leisure, the labor supply of non-Ricardian households remains constant. As shown by Gali et al. (2004), this also implies that consumption of non-Ricardian households will always be proportional to their wage income. This would imply that variations in the income of non-Ricardian households are solely driven by fluctuations in wages. Under the present assumptions, variations in working hours constitute an additional source of variation for the income of non-Ricardian households.

\(^6\)As in Gali et al. (2007), we assume that the distribution of differentiated labor types is identical across both groups of households.
where $\eta_w > 1$ denotes the elasticity of substitution between differentiated labor inputs and $w_j$ denotes the aggregate real wage, which evolves according to $w_t^{1-\eta_w} = \int_0^1 w_{u,t}^{1-\eta_w} \, du$.

Each union transforms the labor supply of households one-for-one and seeks to set its wage in order to maximize the income of union members by maximizing the transfer $F_u$, which union members receive. However, wage setting of unions is subject to a nominal rigidity ala Calvo (1983). Every period unions can adjust the wage with probability $1 - \theta_w$. This implies that a wage set in period $t$ can persist for several periods $k$. Unions take this possibility into account and apply the stochastic discount factor of Ricardian households $A_{t+k} = \beta^{\frac{\pi_{t+k}}{\pi_t}}$ in order to maximize the expected present discounted value of their members’ labor income.\footnote{This assumption seems warranted since the parameters of the utility function are similar across both types of households. Alternatively, as in Colciago (2011), one could assume that unions maximize the weighted sum of household utility across Ricardian and non-Ricardian households.

\footnote{As is common in the literature on energy and business cycles, we assume that $p_{m,t}$ is exogenously given, cf. Kim and Loungani (1992). Note that we also assess the implications of exogenous fluctuations in $p_{m,t}$ and the resulting dynamics are largely comparable to those of uncertainty about emissions prices and are available upon request from the author.}

Under these assumptions, the optimal reset wage $w^*_t$ is common across unions and reads:

$$w^*_t = \frac{\eta_w}{\eta_w - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k A_{t+k}}{E_t \sum_{k=0}^{\infty} \theta^k A_{t+k} + w_t^{\eta_w} \Pi_{t+k} h_{d,t+k}}.$$ \hspace{1cm} (11)

It can be inferred, that in absence of wage adjustment rigidities, unions will set wages as a markup $\eta_w \frac{w_t}{\eta_{w-1}}$ above the level of competitive wages. In addition, the presence of the nominal rigidity induces inertia in wages and alters labor market dynamics. This is also reflected in the dynamic evolution of the aggregate real wage, which can be derived as a weighted average of currently adjusted and past wages:

$$w_t^{1-\eta_w} = (1 - \theta_w) w_{t-1}^{1-\eta_w} + \theta_w \Pi_{t-1}^{\eta_w-1} w_{t-1}^{1-\eta_w}.$$ \hspace{1cm} (12)

### 2.3 Firms

The production sector of the economy can be divided into two layers. Final goods producers operate under perfect competition and aggregate intermediate goods into final output $y_t$. The intermediate goods $y_{j,t}$ are produced by a continuum of intermediate firms $j \in [0,1]$ under monopolistic competition. Intermediate firms face nominal price rigidities as in Calvo (1983). Furthermore, as in Fischer and Springborn (2011), in order to produce output, intermediate firms rely on a polluting intermediate input factor $m_{j,t}$. For simplicity, we assume that they buy the polluting intermediate factor in exchange for final goods at the price $p_{m,t}$.

Final goods producers use a CES aggregator of the form $y_t = \left( \frac{1}{y_{j,t}}^{(1-\varepsilon)/\varepsilon} \right)^{\varepsilon} d j^{\varepsilon/(\varepsilon-1)}$ to combine intermediate goods into final goods. Here $\varepsilon > 1$ denotes the elasticity of substitution between different varieties of intermediate goods. Profit maximization of final goods producers yields the usual downward sloping demand for intermediate goods $y_{j,t} = (\frac{p_j}{p_0})^{\varepsilon} y_t$. The demand for the intermediate good $j$ is a decreasing function of the individual price of the intermediate good $p_{j,t}$ relative to the overall price level
of intermediate goods $p_t$. Using the demand for individual goods, the price level of the economy, defined as the sum over intermediate prices times quantities, is $p_t = \left( \int_0^1 p^1_{j,t} d \mathcal{J} \right)^{1/(1-\varepsilon)}$.

Regarding the production technology, we adopt the formulation of Bosetti and Maffezzoli (2014). Intermediate firms produce according to the following constant returns to scale technology:

$$y_{j,t} = A_t(k_{j,t}^{\alpha} h_{d,j,t}^{1-\alpha})^{1-\gamma} m_j^{\gamma}, \quad 0 < \alpha < 1, \quad 0 < \gamma < 1,$$

(13)

where $A_t$ represents total factor productivity (TFP) which evolves as $A_t = \rho_t A_{t-1} + \epsilon_{a,t}$. Here $\rho_t$ denotes the autocorrelation of the AR(1) process and $\epsilon_{a,t}$ denotes the innovations in productivity that are assumed to be i.i.d. normally distributed. The output elasticity of the polluting intermediate input is denoted by $\gamma$ and $\alpha$ denotes the output elasticity of physical capital. As explained, we assume that emissions $e_{j,t}$ are proportional to the utilization of the polluting intermediate input, i.e. $e_{j,t} = \phi_{e,t} m_{j,t}$. The degree of proportionality thus depends on $\phi_{e,t}$. Now, in order to introduce uncertainty regarding the emission intensity of production, we assume that $\phi_{e,t}$ evolves as:

$$\phi_{e,t} = (1 - \rho_{\phi}) \bar{\phi} + \rho_{\phi} \phi_{e,t-1} + \epsilon_{\phi,t}.$$

(14)

Here $\bar{\phi}$ denotes the steady state emission intensity, which we normalize to unity as in Fischer and Springborn (2011), $\rho_{\phi}$ denotes the persistence of fluctuations in emission intensity and $\epsilon_{\phi,t}$ are independent innovations in emission intensity. This formulation can be regarded as a reduced-form approach that reflects a general uncertainty about technological abilities to absorb or abate emissions in production.

Intermediate firms take factor prices as given, so that in the absence of emission reduction policies, their static cost minimization problem yields the following optimality conditions for factor inputs:

$$R_{k,j} = \lambda_{j,t} \left( 1 - \gamma \right) \alpha A_t (k_{j,t}^{\alpha} h_{d,j,t}^{1-\alpha})^{1-\gamma} m_j^{\gamma} k_j^{1-1}\gamma$$

(15)

$$w_t = \lambda_{j,t} \left( 1 - \gamma \right) (1 - \alpha) A_t (k_{j,t}^{\alpha} h_{d,j,t}^{1-\alpha})^{1-\gamma} m_j^{\gamma} h_{d,j,t}^{1-1}\gamma$$

(16)

$$p_{m,j} = \lambda_{j,t} \gamma A_t (k_{j,t}^{\alpha} h_{d,j,t}^{1-\alpha})^{1-\gamma} m_j^{\gamma-1}.$$

(17)

Here, the Lagrange multiplier $\lambda_{j,t} = mc_{j,t}$ can be interpreted as marginal costs of the firm, i.e. the cost of producing an additional unit of output. From (17) we can infer that at the optimum, firms choose the amount of the polluting intermediate input, such that marginal revenues equate marginal costs. This implies that regulatory measures that increase the cost of employing intermediate inputs, such as permit requirements or a tax, will distort the choice of input factors and incentivize firms to reduce emissions. Furthermore, conditions (15) - (17) imply that all firms will choose the same capital-labor and intermediate inputs-labor ratios so that marginal costs are common to all firms, i.e. $mc_{j,t} = mc_t$, where:

$$mc_t = \left( \frac{1}{(1-\alpha)(1-\gamma)} \right)^{1-\gamma} \left( \frac{(1-\alpha)}{\alpha} \right)^{\alpha(1-\gamma)} \left( \frac{1}{\gamma} \right) \gamma w_t^{1-\gamma} \frac{R_{k,j}^{(1-\gamma)} p_{m,j} \gamma}{A_t}.$$

(18)

Intermediate goods producers use their market power and choose the price of intermediate goods $p_{j,t}$ that maximizes discounted real profits. To this end, they apply the stochastic discount factor of Ricardian households defined as $\Lambda_{t+i} = \beta \frac{\lambda_{t+i}}{\lambda_t}$. Price stickiness is introduced according to Calvo (1983), every period only a fraction $1 - \theta_p$ of firms can adjust their prices. The firms that cannot adjust their prices
remain at their previously chosen prices. The solution to this dynamic price-setting problem implies that all firms that can reset prices will choose the same optimal reset price \( p^*_t \), given by:

\[
p^*_t = p_{j,t} = \frac{\varepsilon}{(\varepsilon - 1)} \frac{E_t \sum_{i=0}^{\infty} \theta^i p^{E}_{j,t+i} + p^{E}_{j,t} y_{t+i} + \gamma^i mc_{j,t+i}}{E_t \sum_{i=0}^{\infty} \theta^i y_{j,t+i} + p^{E}_{j,t} y_{t+i}} .
\]

With \( \theta_p = 0 \), all firms can freely adjust their prices and the price of intermediate goods will be a markup \( \frac{\varepsilon}{(\varepsilon - 1)} > 1 \) over marginal costs. With \( \theta_p > 0 \), the evolution of the aggregate price level is given by

\[
p_t = \left[ (1 - \theta_p) p_t^{1-\varepsilon} + \theta_p p_{t-1}^{1-\varepsilon} \right]^{1/(1-\varepsilon)},
\]

which implies that the current aggregate price level corresponds to the weighted average of recently adjusted and previous prices. For later reference, we rewrite this in terms of inflation as

\[
1 = (1 - \theta_p) \Pi_t^{1-\varepsilon} + \theta_p \Pi_{t-1}^{1-\varepsilon},
\]

where \( \Pi_t = \frac{p_t}{p_0} \) and \( \Pi_{t-1} = \frac{p_{t-1}}{p_0} \). Finally, using the Calvo assumption, the price dispersion in equilibrium \( \nu^p_t \) can be written as

\[
\nu^p_t = (1 - \theta_p) \Pi_t^{1-\varepsilon} + \theta_p \Pi_{t-1}^{1-\varepsilon} \nu^p_{t-1}.
\]

The profits of firms \( F_{F,j} \) are distributed lump-sum to Ricardian households.

### 2.4 Environmental Policies

In the present model, environmental regulation is conducted through an independent institution such that the revenues of the carbon reduction scheme in place, denoted as \( T_{e,t} \), are not necessarily part of the government budget. In modeling the environmental regulator, we follow the general principals as laid out in the proposal of Delpla and Gollier (2019). This entails that the policies under consideration follow the polluter pays principle, i.e. the costs of emissions have to be borne by the entity that emits carbon dioxide. In the model, emissions result from the utilization of a polluting intermediate input by intermediate goods producers. Hence, climate policies directly affect the production side of the economy and alter the optimization problem of intermediate firms.

In the following, we consider four different policy instruments. A constant price instrument (comparable to a tax), a flexible price instrument where the price of emissions adjusts dynamically, a cap-and-trade system where the price of permits is formed in competitive permit markets, and an intensity target. This set of instruments spans the space between fully flexible emissions at a constant price under the price scenario and fixed emissions at fully flexible prices under the cap-and-trade scenario. In contrast, the flexible price and the intensity target allow for fluctuations of both prices and quantities. The main difference between the later instruments is that the flexible price scheme dampens the procyclical reaction of emissions, whereas the intensity target allows firms to adjust emissions procyclically. As documented by Heutel (2012), this distinction is important. According to his analysis, policies that allow for procyclical emissions adjustments in response to fluctuations tend to be preferable from a welfare perspective.

To illustrate the different policy instruments, we use the fact that from the perspective of intermediate goods producers, the unit costs associated with intermediate inputs \( \hat{p}_{m,t} \) can be broken into two components. Formally, \( \hat{p}_{m,t} = p_{m,t} + \phi_{e,t} p_{e,t} \), where \( p_{e,t} \) denotes the price of emissions under the different policy instruments \( g(\cdot) \), such that \( p_{e,t}(g(\cdot)) \). Thus, emissions policies alter the optimization problem of intermediate goods producers by increasing the costs of intermediate inputs. Taking this into account, firms’ marginal costs under emission policies are given by:

\[
mc_t = \left( \frac{1}{(1-\alpha)(1-\gamma)} \right)^{1-\gamma} \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha(1-\gamma)} \frac{1}{(1-\gamma)} \frac{w^t_{\gamma}(1-\alpha)(1-\gamma) \hat{p}_{m,t}^{1-\gamma}}{A_t} \hat{p}_{e,t}^{\gamma}. \tag{20}
\]
We assume that under the price instrument, the unit price of emissions is fixed at the exogenous level \( p_e = \mu \). In case of the cap-and-trade scenario, we assume that firms face a constantly binding emissions constraint \( e_t = \bar{e} \) and that one unit of emissions requires firms to provide one unit of permits. The number of permits issued by the regulator is kept constant over time. Under the flexible price scheme, the price of emissions takes the form of a reaction function \( p_{e,t} = \mu + \eta_e (e_t - \bar{e}) \), and depends on the level of emissions relative to steady state emissions. The parameter \( \eta_e \) captures the intensity in which the emission price reacts to deviations of emissions from the target. Finally, the intensity target requires \( e_t = \xi \gamma_t \), i.e. the emissions to output ratio is fixed at the exogenous value \( \xi \). Note that we assume that intermediate goods producers trade permits in a perfectly competitive market. Thus, in the case of quantity instruments, the price of emissions corresponds to \( \omega_h \), i.e. the shadow value of an additional permit resulting from the optimization of firms under the emission constraint.  

Table 1 summarizes the different instruments and the corresponding price of the intermediate input.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Functional form</th>
<th>Price of intermediate inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>( g(e_t) = \mu )</td>
<td>( \hat{p}<em>{m,t} = p</em>{m,t} + \phi_{e,t} \mu )</td>
</tr>
<tr>
<td>Flex Price</td>
<td>( g(e_t) = \mu + \eta_e (e_t - \bar{e}) )</td>
<td>( \hat{p}<em>{m,t} = p</em>{m,t} + \phi_{e,t}(\mu + \eta_e (e_t - \bar{e})) )</td>
</tr>
<tr>
<td>Cap-and-Trade</td>
<td>( g(e_t) = e_t \leq \bar{e} )</td>
<td>( \hat{p}<em>{m,t} = p</em>{m,t} + \phi_{e,t} \omega_h )</td>
</tr>
<tr>
<td>Intensity Target</td>
<td>( g(y_t, e_t) = e_t \leq \xi \gamma_t )</td>
<td>( \hat{p}<em>{m,t} = p</em>{m,t} + \phi_{e,t} \omega_h )</td>
</tr>
</tbody>
</table>

Table 1: Climate policy instruments and intermediate input prices.

As can be inferred, all policy instruments attach a price to emissions that translates into a price increase of intermediate inputs. This induces two types of adjustments on the supply side of the economy. First, in response to an increase in \( \hat{p}_{m,t} \), firms adjust the proportion of input factors and reduce the amount of the polluting intermediate goods used in production. To see this, combine (16) and (17) under an active climate policy, to obtain the optimal factor input ratio of the polluting intermediate good vis-a-vis labor:

\[
\frac{m_t}{h_{d,t}} = \frac{\gamma}{(1-\alpha)(1-\gamma)} \frac{w_t}{\hat{p}_{m,t}}.
\]

Apparently, an exogenous increase in \( \hat{p}_{m,t} \) requires ceteris paribus a decline in \( m_t/h_{d,t} \).  

Given the underlying production structure, firms react to the change in relative factor prices by substituting intermediate inputs through additional capital and labor. This substitution effect has implications for the deterministic steady state of the economy and also determines the short-run dynamics in response to shocks. In general, the deterministic long-run adjustments of factor inputs depend on the relative productivity of input factors. In the present model, at the same emissions reduction goal, all policy instruments lead to identical long-run adjustments, i.e. the same deterministic steady state. However, in the short-run with frictional adjustment, by design, all four instruments differ in terms of price and emissions dynamics.

Furthermore, as depicted in (19), the dynamic price setting decision of firms is based on current and expected future production costs. Hence, an increase in the user costs of polluting intermediate goods will lead to price adjustments on the firm level, followed by a corresponding increase in the aggregate price.

---

9 Formally, at the optimum, firms equate the marginal gains from using an additional unit of emissions to its costs, thus \( \omega_h \) represents the Lagrange multiplier associated with the emissions constraint.

10 This follows from: \( \frac{\partial(m_t/h_{d,t})}{\partial \hat{p}_{m,t}} < 0 \).
level. On the one hand, as implied by (6), this affects the consumption decisions of Ricardian households. On the other hand, the inflationary impulse leads to interactions between climate, monetary and fiscal policies. To the extent that the effect on the emission price differs between the instruments, this will alter macroeconomic dynamics in response to shocks.

Finally, as laid out in the introduction, the actual size of the global emissions budget is subject to uncertainty. To capture this, we assume that environmental policies are fully committed to the desired emissions reduction goal and therefore adjust the policy instruments accordingly. This implies that an unexpected adjustment in available emissions translates directly into adjustments of the price of emissions or the number of emissions permits issued under price or quantity regulations, respectively. Formally, we will introduce this uncertainty about the emission budget as follows:

\[ \tilde{e}_t = (1 - \rho_e)\tilde{e} + \rho_e \tilde{e}_{t-1} + \epsilon_{e,t}, \]  

(21)

where \( \rho_e \) denotes the persistence of changes in the carbon budget and \( \epsilon_{e,t} \) denotes the stochastic innovations in the emission cap.\(^{11}\)

2.5 Public Sector and Market Clearing

Central Bank

The short-term gross nominal interest rate \( R_t \) is under the control of the central bank, which has the objective to maintain price stability and reacts to deviations of inflation from inflation target \( \hat{\Pi} \). Monetary policy is conducted according to the monetary policy rule:

\[ \frac{R_t}{R} = \left( \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left( \frac{\Pi_t}{\hat{\Pi}} \right)^{\gamma_\Pi} \right)^{1-\gamma_R} \exp(\epsilon_{R,t}) \]  

(22)

Here, \( \gamma_\Pi \) denotes the coefficient that captures the reaction of the central bank to deviations of inflation from the target and \( \gamma_R \) captures the persistence in nominal interest rates and ensures empirically plausible smooth adjustments in nominal rates. The steady state nominal interest rate is denoted by \( \bar{R} \), and \( \epsilon_{R,t} \) denotes stochastic innovations in the nominal rate.

Government

Since the main focus of the present analysis is on environmental policies, the government sector is kept rather simple. In particular, we abstract from distortionary taxation apart from climate policies. In real terms, the government flow budget constraint is given by:

\[ g_t + R_{t-1}b_{t-1}/\Pi_t = b_t + T_t + T_{E,t} \]  

(23)

i.e. real government expenditures \( g_t \) are financed via issuing risk-free bonds \( b_t \) through lump-sum taxes \( T_t \) levied upon households and through the revenues of the emission reduction scheme in place \( T_{E,t} = p_{e,t}e_t \).

\(^{11}\)In case of a price instrument, innovations in the carbon budget induce adjustments of the carbon price. Formally \( p_{e,t} = (1 - \rho_e)\mu + \rho_e p_{e,t-1} + \epsilon_{e,t} \), where the mapping between \( \mu \) and \( \tilde{e} \) can be used to generate equivalent fluctuations in prices.
To ensure the long-run sustainability of government debt, the government follows the fiscal rule:

$$T_t = T + \phi_T (b_t - \bar{b})$$  \hspace{1cm} (24)

where $T$ denotes steady state taxes and $\phi_T$ captures the intensity of adjustments in taxes in response to deviations of the stock of government debt from an exogenously defined target $\bar{b}$. Government expenditures are stochastic and follow the AR(1) process $g_t = (1 - \rho_g) g + \rho_g g_{t-1} + \epsilon_{g,t}$. Here, $\bar{g}$ denotes an exogenous target of government consumption and $\rho_g$ captures the persistence of i.i.d. innovations in government consumption denoted by $\epsilon_{g,t}$.

**Aggregation and Market Clearing**

In equilibrium factor and goods markets clear. Labor market clearing requires that the labor supply of unions equates the labor demand of the labor packer, which under perfect competition corresponds to labor demand of intermediate good producers. Aggregate labor supply is given by $\int_0^1 h_u, t \, du = h_t$ taking the demand function for differentiated labor (10) into account, we have $h_t = h_d, v_t^w$, where $v_t^w$ denotes wage dispersion, defined as $v_t^w = \int_0^1 (w_u/w_t) - \eta \, du$. Given the wage adjustment frictions, the dynamic evolution of wage dispersion is given by:

$$v_t^w = (1 - \theta_w) \left( \frac{w^*_t}{w_t} \right)^{-\eta_w} + \theta_w \Pi_t^{\eta_w} \left( \frac{w_t}{w_{t-1}} \right)^{\eta_w} v_{t-1}^w. \hspace{1cm} (25)$$

Market clearing for capital and intermediate inputs implies: $k_t = \int_0^1 k_j, d j$ and $m_t = \int_0^1 m_j, d j$. Taking the demand for intermediate goods into account, integration yields $\int_0^1 y_j, d j = \int_0^1 (P_j/p_t)^{-\epsilon} y_t, d j = y_t v_t^p$, where $v_t^p$ evolves dynamically as defined above. Aggregate final output can thus be written as:

$$y_t = A_t(k_t^\alpha \eta^1-h_d,^1-\alpha_j)1^{-\gamma} m_t^\gamma / v_t^p. \hspace{1cm} (26)$$

Since $v_t^p > 1$, if $\theta_p > 0$, price dispersion reflects the inefficiency of aggregate output associated with price rigidity. The resource constraint of the economy requires:

$$y_t = x_t + c_t + g_t + \tilde{p}_{m,s} m_t, \hspace{1cm} (27)$$

where $\tilde{p}_{m,s} m_t$ includes the revenues of the environmental policy regime whenever they are not part of the government budget. A full summary of the equilibrium conditions for the baseline scenario, as well as details regarding the solution procedure, are provided in 5.2.

**3 Quantitative Analysis**

This section presents the quantitative evaluation of the theoretical model. Since no solution to the full non-linear model exists, the model is solved numerically using a second-order Taylor approximation around the deterministic steady state of the model. The first part of this section compares the four policy instruments, and assesses the resulting implications for macroeconomic dynamics in response to different
sources of business cycle fluctuations. This exercise includes a comparison of macroeconomic stability and welfare across instruments. Subsequently, we evaluate the distributive implications of the policy instruments and examine the role of uncertainty and frictions in this context. In the second part of the analysis we focus on uncertainty regarding the carbon budget. Here, we incorporate carbon budget shocks into the model. We begin with an examination of the macroeconomic dynamics in response to an unexpected change in the carbon budget, which leads to adjustments of climate policy instruments and compare the implications of a price instrument to those of a cap-and-trade scheme. Afterwards, we use the stochastic model with fluctuations in the emission cap and the emission price to examine the general role of uncertainties associated with the carbon budget.

3.1 Calibration

The numerical simulations require defining specific parameter values. As is common in the literature, the model is calibrated to capture some empirically observed moments of the economy. In particular, we try to match the dynamics of output and consumption of the German economy. To this end we specify the parameters of the production sector based on empirical data to match the average ratios of private consumption to GDP and private investment to GDP. The share of polluting intermediate inputs in production is set to match the share of energy in production. Parameters which reflect monetary and fiscal policy are set to match the average inflation rate, the ratio of government consumption to GDP and the debt to GDP target. Regarding the structural parameters of the model, which capture household preferences and frictions, we largely follow the existing literature on German business cycles. Table 2 summarizes the parameters used in the baseline specification.

The parameters that capture household preferences correspond to the values used by Hristov (2016). The subjective discount factor of households is set to \( \rho = 0.998 \), the inverse of the intertemporal elasticity of substitution is set to \( \rho = 2 \) and the inverse of the Frisch elasticity of labor supply \( \chi \) is set to 1.5. These values are broadly in line with values used in most studies of the German economy. Based on the results reported by Grabka and Halbmeier (2019), the share of non-Ricardian households \( \lambda \) is set to 0.28. The wage-setting frequency of unions \( \theta_w \) is taken from Gadatsch et al. (2016) and set to 0.83. The elasticity of substitution between labor types is set to \( \eta_w = 4 \). This implies a wage markup in the deterministic steady state of 1.33. Finally, the labor disutility parameter \( \psi \) is set in order to reach an average working time of \( h_t = 0.33 \) in the deterministic steady state.

We set the capital share to \( \alpha = 0.3 \), which corresponds to the average capital share in Germany between 1990 and 2015 as reported by the Federal Statistical Office (Destatis). Fischer and Springborn (2011) calibrate the production elasticity of polluting intermediate inputs \( \gamma \) to match the average energy expenditures relative to GDP in the United States. For Germany, we set \( \gamma = 0.1 \), which corresponds to the average total energy supply relative to GDP as reported by the International Energy Agency (IEA) for the period from 1990 – 2015. The quarterly depreciation rate of physical capital \( \delta \) is set to 0.025. In line with the estimation results of Drygalla et al. (2020), we set the investment adjustment cost parameter \( \kappa \) to 3.9. According to the estimation results of Jondeau and Sahuc (2008) for the Germany economy, we set the

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\[ ^{12} \text{A well documented phenomenon is the occurrence of indeterminacy of equilibrium, conditional on the share of non-Ricardian households and the degree of price adjustment frictions. In the given model, this issue arises for a share of non-Ricardian households of 70\% and at values of } \theta_p > 0.7. \text{ The corresponding plot illustrating parameter combinations associated with indeterminacy of the model is provided in appendix 5.3.} \]
Calvo parameter to $\theta_p = 0.86$ and the elasticity of substitution between intermediate goods $\varepsilon$ to 6, which corresponds to a markup of 1.2.\textsuperscript{13}

Regarding the choice of the parameters that capture monetary and fiscal policy, we again consider the estimation results of Drygalla et al. (2020). The stance on inflation $\gamma_\Pi$ is set to 1.47 and the degree of interest rate smoothing $\gamma_R$ is set to 0.91. The target rate of inflation is set to 1%.\textsuperscript{14} The parameter $\phi_T$ that captures the strength of the reaction of lump-sum taxes to deviations of government debt from target is set to 0.38. As explained, the steady state levels of government debt $\bar{b}$ and consumption $\bar{g}$ are set to match a debt-to-GDP ratio of 0.6 and a government consumption to GDP ratio of 0.19.\textsuperscript{15}

In the first exercise, we test the implications of climate policies for short-run macroeconomic dynamics. Since the studies on German business cycles cited above differ in terms of models and sample periods, we cannot adopt all their estimates of the shock processes directly. Therefore, we use the reported results and set the autocorrelations and standard deviations of shock processes within the reported range.\textsuperscript{16} The uncertainty of emission intensity, formally described by (14), is specified in line with the properties of quarterly emission intensity data for Germany. The corresponding time series ranging from 1991Q3 to 2012Q4, is constructed using data on carbon dioxide emissions from Crippa et al. (2020) and data on real GDP obtained from Destatis. The cyclical component of emission intensity is found to be relatively persistent with a statistically significant autocorrelation of $\rho_e = 0.78$ and $\sigma_e = 0.023$.\textsuperscript{17} Overall, the employed parameter values are comparable to the literature and are summarized at the bottom of table 2.

In the second exercise, in order to assess the role of carbon budget uncertainty, we extended the baseline model by equation (21). Uncertainty about the remaining carbon budget evolves for several reasons. First of all, carbon budgets differ with respect to the specific climate goal. Clearly, keeping global warming below 2°C is associated with a larger remaining carbon budget compared to keeping the temperature increase below 1.5°C. But even for a specific climate goal, carbon budgets differ due to scientific uncertainty about atmospheric dynamics. Furthermore, some decarbonization pathways allow for overshooting, which keeps the carbon budget more flexible. To specify the uncertainty in the carbon budget, we restrict the space of carbon budgets to the 1.5°C scenarios as reported by IPCC (2018). Then we proceed in two ways. First, we extract all scenarios from Huppmann et al. (2019) and follow Fujimori et al. (2019), who compute the remaining cumulative CO$_2$ emissions for each study. Based on the reported carbon budgets, we obtain a relative standard deviation in the estimates of the remaining carbon budget of $\sigma_e = 0.060$. In addition, we also compute the standard deviation in the reported budgets, including studies which allow for low-overshooting. Based on 51 studies, we obtain a relative standard deviation in the carbon budgets of $\sigma_e = 0.126$. However, these studies can differ with respect to the assumptions regarding emission absorption technologies and therefore partially capture technological uncertainty as well. A second approach is based on the information provided by Rogelj (2018), where based on the transient climate response (TCR) to cumulative emissions of carbon within the 1.5°C scenario we obtain a relative

\textsuperscript{13}As documented by Annicchiarico and Dio (2015), the degree of nominal frictions has strong implications for the relative performance of climate policy instruments. We therefore assess this issue explicitly in a following analysis.

\textsuperscript{14}These parameter values fulfill the Taylor principle and ensure that the stationary equilibrium is uniquely determined.

\textsuperscript{15}The debt-to-GDP ratio is chosen in accordance with the Maastricht criteria and the ratio of government consumption corresponds roughly to the empirically observed share of government consumption over GDP in Germany between 1991 – 2016, as reported by Destatis.

\textsuperscript{16}In addition, we also take the estimation results of Pytlarczyk (2005) and Gadatsch et al. (2015) into account.

\textsuperscript{17}As is common in the literature, the cyclical component is obtained applying the HP-Filter with $\lambda = 1600$. More details on the data used in the calibration and additional information are presented in 5.1.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.998</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.5</td>
<td>Inverse Frisch elasticity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>2</td>
<td>Inverse elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\psi$</td>
<td>45</td>
<td>Labor disutility</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.28</td>
<td>Share of non-Ricardian households</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>0.83</td>
<td>Wage adjustment frictions (unions)</td>
</tr>
<tr>
<td>$\eta_w$</td>
<td>4</td>
<td>Elasticity of substitution labor types</td>
</tr>
<tr>
<td>Firms:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.1</td>
<td>Output elasticity polluting goods</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.30</td>
<td>Output elasticity capital</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>3.9</td>
<td>Investment adjustment costs</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>0.86</td>
<td>Price stickiness</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>6</td>
<td>Elasticity of substitution intermediate goods</td>
</tr>
<tr>
<td>Polices:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\Pi}$</td>
<td>1.47</td>
<td>Interest rate rule inflation coefficient</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>0.91</td>
<td>Interest rate rule smoothing coefficient</td>
</tr>
<tr>
<td>$\bar{\Pi}$</td>
<td>1.01</td>
<td>Target inflation</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>0.38</td>
<td>Reaction of taxation</td>
</tr>
<tr>
<td>$b_y$</td>
<td>0.6</td>
<td>Debt-GDP-ratio</td>
</tr>
<tr>
<td>$g_y$</td>
<td>0.19</td>
<td>Government consumption to GDP ratio</td>
</tr>
<tr>
<td>Stochastic processes:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.95</td>
<td>Persistence TFP shock</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>0.86</td>
<td>Persistence government spending shock</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>0.82</td>
<td>Persistence preference shock</td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>0.88</td>
<td>Persistence labor supply shock</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.77</td>
<td>Persistence investment shock</td>
</tr>
<tr>
<td>$\rho_{\phi}$</td>
<td>0.78</td>
<td>Persistence emission intensity shock</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.0049</td>
<td>S.D. TFP shock</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.0004</td>
<td>S.D. Monetary shock</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>0.0039</td>
<td>S.D. Government spending shock</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.0118</td>
<td>S.D. Labor supply shock</td>
</tr>
<tr>
<td>$\sigma_{d}$</td>
<td>0.0044</td>
<td>S.D. Preference shock</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0183</td>
<td>S.D. Investment shock</td>
</tr>
<tr>
<td>$\sigma_{\phi}$</td>
<td>0.023</td>
<td>S.D. Emission intensity shock</td>
</tr>
</tbody>
</table>

Table 2: Calibrated Parameters – Baseline Scenario

standard deviation of $\sigma_e = 0.167$.\(^{18}\) Thus we see that overall variations in carbon budget estimates fall into a broad range between $0.060 < \sigma_e < 0.167$.

### 3.2 Business Cycle Shocks – Volatility and Welfare

First, to compare the implications of the present model formulation to previous studies and to understand the role of heterogeneity in income and wealth in light of emissions reduction policies, we examine the response of the economy to six frequently examined business cycle shocks. In particular, we focus on supply shocks, which are modeled as fluctuations in total factor productivity, demand shocks, which take the form of preference shocks, shocks to the nominal interest rate, i.e. monetary policy shocks, shocks to

\(^{18}\)We consider all scenarios within the 67% percentile and compute emission budgets from 2011 onward.
investment efficiency, shocks to government consumption and labor supply shocks. This wide range of shocks captures several relevant drivers of business cycles. In addition to the common sources of business cycle fluctuations, we allow for stochastic innovations in emission intensity, based on the estimated shock processes for the German economy. In the absence of climate policy, emission intensity shocks only affect emissions but do not alter aggregate dynamics. However, under an active climate policy, fluctuations in emission intensity affect the optimal utilization of input factors and can exert effects on macroeconomic aggregates.

In the baseline scenario, we assess the effects of a 10% reduction in emissions for the deterministic steady state of the model relative to a no-policy scenario. This reflects a situation in which climate policies require emissions adjustments on the firm level. The policy parameters $\mu$, $\bar{e}$ and $\xi$, which capture the price of emissions, the emissions cap and the intensity target, respectively, are calibrated to yield the same level of steady state emissions. The parameter $\eta_e$ that captures the reaction of emission prices to deviations from the target is set to 5.\textsuperscript{19} Overall, the reduction target is set to ensure a constantly binding emissions constraint when considering quantity instruments. The revenues generated through emissions policies are completely absorbed.\textsuperscript{20} Table 3 provides a comparison of key macroeconomic variables and welfare between the no-policy case vis-a-vis the 10% reduction scenario in the deterministic steady state.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$y_t$</th>
<th>$k_t$</th>
<th>$h_t$</th>
<th>$e_t$</th>
<th>$c_t$</th>
<th>$x_t$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-Policy</td>
<td>0.534</td>
<td>4.462</td>
<td>0.333</td>
<td>0.044</td>
<td>0.274</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>(% change)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Policy</td>
<td>0.529</td>
<td>4.414</td>
<td>0.336</td>
<td>0.040</td>
<td>0.271</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>(% change)</td>
<td>-1.1%</td>
<td>-1.1%</td>
<td>0.4%</td>
<td>-10.0%</td>
<td>-1.1%</td>
<td>-1.1%</td>
<td>-1.05%</td>
</tr>
</tbody>
</table>

Table 3: Comparison of macroeconomic aggregates in the deterministic steady state under climate policy and percentage changes relative to the no-policy scenario. Welfare effects are expressed as consumption equivalent variations relative to the no-policy scenario.

As can be inferred, compared to the no-policy case, a 10% emissions reduction leads to a decline in output of roughly 1.1%. The relative decline in the physical capital stock, aggregate consumption and investment are of the same magnitude. Since firms substitute the polluting intermediate good partially by employing additional labor, we see a small increase in aggregate working hours of about 0.4%. In terms of consumption equivalent variations, aggregate welfare declines by about 1.05% relative to the no-policy scenario. Quantitatively, the results are by and large comparable to the results presented by Annicchiarico and Dio (2015) and Fischer and Springborn (2011), and the implied steady state ratios for the no-policy scenario are close to empirically observed statistics for Germany.\textsuperscript{21}

\textsuperscript{19}Appendix 5.4 illustrates the effects of the choice of $\eta_e$ on model dynamics for a TFP shock. In general, a larger reaction parameter tends to dampen the observed dynamics.

\textsuperscript{20}This assumption corresponds to a scenario in which the revenues of climate policies are treated as an increase in the import price of natural resources and therefore flow out of the small open economy. While this assumption is clearly arbitrary, it provides an unbiased reference point to compare policy instruments in the absence of changes to fiscal policies.

\textsuperscript{21}While the model matches the target value for government consumption over GDP, the implied ratio of private consumption to GDP of $c_t/y_t \approx 0.51$ and the ratio of private investment to GDP of $x_t/y_t \approx 0.21$ in the no-policy scenario are also comparable to the values observed for the German economy between 1991 and 2015.
**Impulse Responses**

We begin with a graphical examination of the model dynamics under the four different policy regimes in response to aggregate shocks. All results are based on the baseline parameter values and refer to a 10% emission reduction relative to the no-policy scenario. If not indicated differently, the dynamics are illustrated in terms of percentage deviations from the deterministic steady state over a horizon of 20 quarters.

Figure 1: Impulse responses of macroeconomic aggregates to TFP shock, in percentage deviations from steady state. The underlying parameter values correspond to table 2.

Figure 1 depicts the dynamics in response to a one standard deviation increase in TFP. First, we observe that emissions policies do not qualitatively alter the reaction of macroeconomic aggregates in response to an increase in TFP. The innovation in productivity leads to a persistent increase in output, consumption and investment. Due to the increased productivity, firms’ marginal costs fall. In the model, because of price rigidities, firms cannot adjust their prices in accordance with the decline in marginal costs. Therefore, the responses of aggregate demand for intermediate goods and consequently aggregate output are dampened. In this situation firms adjust factor inputs in order to utilize the increased productivity. Here, despite the increase in wages, which due to the presence of nominal wage rigidities displays a hump-shaped pattern, aggregate working hours decline. This decline in response to the shock is essentially
a feature of empirical data as emphasized by Gali (1999). Since investment is subject to convex adjustment costs the increase in productivity causes a decline in the real interest rate on impact. In addition, investment also displays a hump-shaped pattern.

Overall, there appear to be no qualitative difference in the reaction of macroeconomic aggregates across policy instruments, but we observe quantitative differences in the model dynamics. In general, under the price instruments, aggregate dynamics are most pronounced, whereas the dynamic is least pronounced with a cap-and-trade system. The dynamics of emissions and emission prices display notable differences between all instruments. While under the cap-and-trade scheme emissions do not react, they increase under the intensity target and decrease in the case of price instruments. With an intensity target, the amount of emissions permits increases with output, which leads to a decline in the permit price and induces firms to increase the utilization of polluting inputs. In contrast with a constant price of emissions, firms have no incentive to increase the amount of polluting inputs on impact. After a while, good prices can adjust and emissions start to increase in proportion to the increase in the other input factors. Under a flexible price regime, the same effects occur, but emission dynamics are dampened due to adjustments in the price of emissions. The largest increase in the emissions price arises with a cap-and-trade regime. In response to the shock, firms demand for the polluting input factor declines, which is reflected in a decline in the market price of permits. In later periods, price adjustments take place and increase the demand for intermediate goods, which induces firms to further expand production and spurs firms’ demand for input factors. Now, with the strict cap on emissions, firms cannot increase intermediate inputs due to a lack of permits. Here, the binding of the emissions constraint is associated with the largest price increase in emissions. Under the other instruments, the amount of emissions is, at least to some extent flexible, which allows quantity adjustments.

Figure 2 depicts the aggregate dynamics in response to a one standard deviation increase in government expenditures. Under the given parameter values for the government sector, the additional government expenditures are mostly financed through an increase in government debt. The increase of aggregate demand, associated with the increase in government consumption, then leads to a jump of output and aggregate consumption, and at the same time crowds out private investment. The crowding out of investment is caused by Ricardian households, who increase their holdings of government bonds. Many studies report a decline in aggregate consumption in response to government spending shocks. Here, as explained by Gali et al. (2007), the initial increase in aggregate consumption results from the presence of non-Ricadian households who have a marginal propensity to consume equal to one. Due to the increase in hours and wages, their income increases, which directly translates into an increase in aggregate consumption. Again, we observe that the policy instruments have quantitative implications for aggregate dynamics and generally alter the responses. We observe that under a cap-and-trade system the dynamics are stronger compared to the other policy regimes. This is particularly evident in the case of marginal costs, which react much stronger under the cap-and-trade scheme. The constant price instrument results in the smallest increase in consumption, marginal costs, hours and wages. In terms of magnitude, the effects under the intensity target are somewhere between the price instruments and the cap-and-trade scheme.

The different effects of the instruments can be explained by their effects on the utilization of polluting intermediate inputs through the emissions price. While under the constant price regime, emissions show the strongest increase, and the increase is dampened under the flexible price regime and the intensity target. As can be inferred from the dynamics of the emissions price, the government spending shock causes the
strongest increase, under the cap-and-trade scheme, followed by the intensity target and the flexible price instrument. This increase in the unit cost of emissions translates into the strongest increase in marginal costs under the cap-and-trade scheme. Here, firms cannot increase emissions and in turn utilize additional labor to satisfy the increase in aggregate demand. This leads to higher wages, which also contributes to the stark rise in marginal costs under the cap-and-trade policy. This adjustment through the labor market can also explain why consumption reacts most pronounced under the cap-and-trade system. Here, the effects on the consumption of non-Ricardian households through increased labor input and wages are stronger compared to the other regimes.

Figure 3 depicts the reaction of the economy to an increase in the emission intensity of polluting intermediate goods under the different policy regimes. The impulse responses of a one standard deviation increase in emission intensity are depicted as percentage deviations from steady state over a 20 quarter horizon. In the absence of climate policies, this shock only affects emissions, but does not induce fluctuations in macroeconomic aggregates. It is therefore a good example for additional uncertainties that are directly associated with emission reduction policies.

The innovation in emission intensity increases the amount of emissions per employed unit of the intermediate input and therefore causes a change in relative factor prices under all emissions reduction policies.
regimes. As can be inferred, the reaction of output, aggregate consumption and investment differs substantially across policy regimes. Under the price instruments, we observe almost negligible reactions, but under the cap-and-trade and the intensity target, the observed effects are sizeable. Here, output and consumption increase on impact, accompanied by a decline in investment. In later periods, the initial increase reverses and both output and aggregate consumption persistently fall below their steady state levels. To understand the underlying mechanism, we compare the responses of the amount of intermediate inputs used in production, emissions and emission prices across instruments. As one would expect, the utilization of intermediate inputs declines under all regimes. However, the decline under the price instruments is comparably small relative to the other instruments, such that overall emissions increase in response to the shock. Under cap-and-trade, emissions remain constant and increase slightly under the intensity target. However, under the later policy regimes, we see a marked rise in emission prices, which translates into an increase in marginal costs. In this situation, firms substitute polluting inputs with additional labor, which leads to a (sluggish) increase in wages. This finally explains the marked differences in the reaction of output and aggregate consumption. Due to the increase in the real wage, non-Ricardian households increase consumption and stimulate aggregate demand in the first periods. For the underlying parameter values, this increase in the consumption of non-Ricardian households overcompensates for
the decline in the consumption of Ricardian households. After around six to eight quarters, the effect diminishes, causing a decline in aggregate consumption and output.\footnote{The impulse responses of the consumption of non-Ricardian and Ricardian households is depicted in figure 14.}

Since the general effects of climate policies on macroeconomic aggregates are qualitatively similar across the remaining shocks, we summarize the impulse responses and put a particular focus on the implied dynamics of output, consumption, emissions and emissions prices. Figure 4 depicts the responses of these variables to one standard deviation shocks in monetary policy, time preferences of Ricardian households, labor supply and investment efficiency. The time horizon is again 20 quarters and the results are based on the baseline parameter values.

The responses of output, consumption, emissions and the emissions price, as depicted in the first row of figure 4 are as expected. An exogenous increase in the nominal interest rate is followed by a contraction in output and consumption. This leads to a reduction in emissions and the emissions price. This reveals an important channel through which monetary policy interacts with climate policies. As explained earlier, adjustments in emissions prices affect the aggregate price level, but as we observe here, changes in aggregate activity in response to innovations in interest rates also alter emissions dynamics. After around 15 quarters, all variables stabilize and converge back to their steady state. In the second...
row, we see the reaction of output, consumption, emissions and emission prices to a preference shock, which increases aggregate demand. Here, output and consumption increase on impact and converge back over time. This increase in output, spurred by a reduction in savings, is accompanied by corresponding increases in emissions under the flexible emissions schemes. The tighter the constraint on emissions, the more pronounced are the resulting dynamics in emissions prices. In this case, the quantitative differences between the instruments on the reaction of output and consumption are of negligible magnitude.

The decline in labor supply depicted in the third row of figure 4 results in a drop in output and consumption on impact. Under schemes with flexible emissions, firms initially substitute labor through the additional utilization of intermediate inputs to stabilize output. But eventually, we see a drop in emissions and emission prices under all schemes. Again, the inability to adjust emissions freely translates into stronger dynamics of the emission price under a cap-and-trade and under the intensity target. Finally, in the bottom row we see the effects of an investment efficiency shock. Here, turning output into investment becomes cheaper, leaving more resources for consumption. The contemporaneous increase in investment and consumption induces an increase in output, which comes again with increasing emissions and emissions prices. While the observed differences in aggregate dynamics are again comparably small, the constant price allows for the largest expansion of output and consumption by putting the fewest restrictions on emissions. Overall, we find that regardless of the shocks, a cap-and-trade system is associated with the largest fluctuations in emissions prices, whereas the largest fluctuations in emissions occur under a constant price regime. The intensity target and the flexible price instrument are somewhat in between those two extremes, allowing for quantity and price adjustments.

From the inspection of the model dynamics, we can draw some initial insights regarding the implied effects of emissions reduction schemes for business cycle dynamics. Under uncertainty and with the presence of rigidities, these instruments affect the way in which firms can respond to shocks. Technically, the increase in the unit costs of emissions raises the marginal costs of production. Furthermore, whenever the price of emissions adjusts, the marginal costs of firms change too. Therefore, depending on the situation, schemes with dynamic prices either dampen or amplify aggregate fluctuations, but always imply procyclical dynamics of emission prices. In the extreme case under a fixed cap-and-trade scheme, the missing flexibility of emissions can induce stronger adjustments of capital and labor. This has direct implications for labor market dynamics, which are generally most pronounced under the cap-and-trade system.

Volatility and Welfare

Gauging the model dynamics in response to shocks has revealed quantitative and, for some variables, even qualitative differences in their dynamic behavior, conditional on the environmental policy regime in place. Among others, Dissou and Karnizova (2016), have shown that emissions reduction policies generally differ with respect to their effects on aggregate volatility. To quantify these differences, we use a second order approximation of the theoretical moments of the model and compare the standard deviations of key variables. Table 4 shows the empirically observed and the model-generated statistics.

Before comparing the implications of the different policy instruments in terms of macroeconomic volatility, we compare the model generated standard deviations in the no-policy scenario to the empirically
Table 4: Standard deviations and relative standard deviations of macroeconomic variables, based on a second-order approximation of the HP-filtered theoretical moments of the model. Parameter values correspond to table 2.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\sigma_y$</th>
<th>$\sigma_c$</th>
<th>$\frac{\sigma_y}{\sigma_c}$</th>
<th>$\sigma_e$</th>
<th>$\frac{\sigma_e}{\sigma_y}$</th>
<th>$\sigma_c$</th>
<th>$\frac{\sigma_c}{\sigma_y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.0160</td>
<td>0.0075</td>
<td>0.47</td>
<td>0.0367</td>
<td>2.29</td>
<td>0.0151</td>
<td>0.94</td>
</tr>
<tr>
<td>No-Policy</td>
<td>0.0162</td>
<td>0.0095</td>
<td>0.58</td>
<td>0.0414</td>
<td>2.55</td>
<td>0.0463</td>
<td>2.86</td>
</tr>
<tr>
<td>Price</td>
<td>0.0162</td>
<td>0.0095</td>
<td>0.58</td>
<td>0.0414</td>
<td>2.55</td>
<td>0.0452</td>
<td>2.79</td>
</tr>
<tr>
<td>Price (flex)</td>
<td>0.0163</td>
<td>0.0096</td>
<td>0.59</td>
<td>0.0410</td>
<td>2.52</td>
<td>0.0395</td>
<td>2.43</td>
</tr>
<tr>
<td>Cap-and-Trade</td>
<td>0.0170</td>
<td>0.0106</td>
<td>0.62</td>
<td>0.0387</td>
<td>2.28</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Intensity Target</td>
<td>0.0166</td>
<td>0.0100</td>
<td>0.60</td>
<td>0.0401</td>
<td>2.41</td>
<td>0.0166</td>
<td>1</td>
</tr>
</tbody>
</table>

observed moments of the German economy.\textsuperscript{23} We find that the model provides a good fit for the standard deviation of the cyclical fluctuations in output. The implied standard deviations of the model in aggregate private consumption and investment are relatively large compared to the data, but still within a reasonable range. Finally, with respect to emissions, the model generates a relative volatility of $\frac{\sigma_e}{\sigma_y} = 2.86$. This value is close to the average relative volatility between carbon dioxide emissions and GDP of 3.08 documented by Doda (2014) for a set of 122 countries. However, he reports an average relative volatility of 1.8 for Germany and also the value reported here based on data between 1991 and 2012 of 0.94 is significantly smaller.\textsuperscript{24} Overall, the predicted fluctuations in macroeconomic aggregates appear reasonably close to the data, while the fit of emission dynamics within a DSGE model remains challenging.\textsuperscript{25}

As can be inferred from table 4, climate policy instruments differ in terms of their effects for macroeconomic fluctuations. We find that price instruments have only minor (statistically insignificant) effects on the volatility of output, while the cap-and-trade scheme and the intensity target tend to amplify output dynamics. The volatility in aggregate consumption also remains largely unaffected under the price instruments and increases for the other instruments. Under the cap-and-trade and the intensity target, we observe a decline in the volatility of investment which is again absent under price instruments. Finally, as expected, we see that all instruments reduce emissions volatility. This dampening effect is stronger under a flexible price instrument compared to a constant price instrument. The cap fully eliminates emissions dynamics, and the intensity target results in a perfect correlation between output and emission dynamics. Therefore, both instruments appear to be more suitable to control emissions dynamics at the cost of higher volatility in other aggregates.

Compared to the results of Fischer and Springborn (2011), who integrate pollution in a similar way into a frictionless real business cycle model, the implications regarding macroeconomic volatility differ substantially. According to their simulations, a cap-and-trade policy stabilizes output and consumption, and is generally associated with a decline in aggregate volatility. The intensity target does not affect aggregate volatility, and they observe a slight increase in volatility under a price instrument. To rationalize

\textsuperscript{23}This comparison ignores that during the observational period, German firms were at least partially subject to environmental policies. Since 2005, the German energy sector and selected other industries have been part of the European Union Emissions Trading System (EU ETS). Furthermore, the German government imposes energy taxes on primary energy usage.

\textsuperscript{24}The difference between the estimates likely results from different observation periods. Furthermore, the estimates of Doda (2014) are based on yearly data, while the empirical standard deviations reported here are based on quarterly data.

\textsuperscript{25}For comparison, the reported relative volatility between emissions and output of Annicchiarico and Dio (2015) is about 30 times larger than in the data. If we abstract from shocks to emission intensity, the present model predicts a relative volatility between emissions and output of 2.29, which is a bit closer to the data.
the differences compared to the present results, it is important to emphasize that in their model, output prices are formed on frictionless competitive markets, and input factors can adjust directly in response to shocks. In the present framework, prices and wages, as well as the capital stock, cannot adjust directly in response to shocks but are subject to adjustment frictions. In particular, the presence of price rigidities requires additional adjustments from firms on several margins. But since capital and labor markets are also subject to frictions, these adjustments are impeded. The implementation of climate policies now introduces additional constraints on the ability of firms to adjust input factors in response to shocks. While emissions can still fluctuate under the price instruments, and to a smaller extent under the intensity target, the cap-and-trade completely eliminates emissions dynamics. The absence of emissions dynamics under the cap-and-trade scheme also helps to explain the differences in terms of macroeconomic stabilization. In the present framework, the price instruments and the intensity target allow for adjustments in labor, capital and emissions in response to shocks. In contrast, a binding emissions cap restricts the adjustment possibilities of firms to capital and labor. Since factor prices are subject to adjustment frictions under the cap-and-trade scheme, the reaction of firms to shocks is inhibited, which results in larger output volatility and less volatility in investment. Similar reasoning applies to the intensity target here emissions can still fluctuate but are directly linked to output dynamics, which dampens emissions dynamics compared to the price instruments. Hence, in the present context, the degree to which firms can vary emissions determines their ability to smooth overall production and explains the differences in macroeconomic volatility.

The role of adjustment frictions for the effects of climate policies on macroeconomic volatility has also been stressed by Annicchiarico and Dio (2015). In line with the present study, they report moderate effects of price instruments on output volatility and an increased volatility in output under an intensity target. However, they also report a decline in the volatility of output and consumption under a cap-and-trade scheme. Since their model also includes nominal price-setting frictions and capital adjustment costs, these differences are related to the way they integrate pollution into the model. They follow the approach of Heutel (2012) and incorporate a damage function into production and emissions abatement possibilities for firms. The possibility to adjust the abatement effort provides firms with an additional margin of adjustment to comply with climate policies. In this framework, under the cap-and-trade scheme, firms are able to expand emissions at the cost of additional abatement efforts. Thus, with a binding emissions cap, firms vary abatement efforts to stabilize output, which leads to lower volatility in macroeconomic aggregates, while the volatility in abatement increases substantially. This confirms the previous argument, according to which the effects of climate policy on macroeconomic stability depend largely on how strongly the adaptation options of firms are influenced.

As discussed, from a long-run perspective, all instruments are able to achieve an equivalent reduction in emissions and are associated with the same deterministic steady state. Consequently, in the deterministic long-run, all instruments have identical implications for aggregate welfare. However, besides abstracting from aggregate uncertainty, this static comparison also neglects the effects of adjustment frictions for welfare. As observed, economic dynamics differ across policy instruments, which also alters the respective welfare implications. In order to quantify the differences between the instruments under frictions and aggregate uncertainty, we compare the unconditional welfare effects of a 10% emissions reduction relative to the no-policy scenario in the stochastic model. As is common, welfare results are expressed in terms of consumption equivalent variations, i.e. as the fraction of consumption that households would require to be indifferent between the no-policy and the respective climate policy regime. To give a first idea
of the distributional implications of climate policies in this context, we compute the welfare effects for Ricardian and non-Ricardian households separately. Overall welfare is reported as the weighted average of Ricardian and non-Ricardian welfare. The effects reported in table 5 are based on the parameter values of the baseline specification (table 2) and computed from a second-order approximation of the theoretical moments of the model.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Overall Welfare</th>
<th>Ricardian</th>
<th>Non-Ricardian</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>-1.06%</td>
<td>-1.06%</td>
<td>-1.05%</td>
</tr>
<tr>
<td>Price (flex)</td>
<td>-1.09%</td>
<td>-1.09%</td>
<td>-1.09%</td>
</tr>
<tr>
<td>Cap-and-Trade</td>
<td>-1.62%</td>
<td>-1.57%</td>
<td>-1.73%</td>
</tr>
<tr>
<td>Intensity Target</td>
<td>-1.55%</td>
<td>-1.52%</td>
<td>-1.64%</td>
</tr>
</tbody>
</table>

Table 5: Welfare changes of a 10% emissions reduction, reported in terms of consumption equivalent compensations (in %) relative to the no-policy scenario, based on a second-order approximation of the theoretical moments of the model.

Comparing the instruments with respect to their implications for aggregate welfare, we see that the welfare losses under all instruments are generally larger than in the deterministic long-run scenario. While the welfare losses under a constant and a flexible price instrument amount to about 1.06% and 1.09%, the welfare loss under the cap-and-trade scheme is about 1.62%, and the welfare loss under the intensity target is 1.55%. In terms of magnitude, the reported effects are somewhat larger than the welfare effects found by Fischer and Springborn (2011), who evaluate climate policies within a real business cycle model without frictions. Since the present model features different types of frictions and several sources of uncertainty, this difference in terms of magnitude is not surprising. As laid out by Annicchiarico and Dio (2015), nominal frictions can amplify the welfare effects. Quantitatively, the present results are comparable to the effects documented in their study, and under the baseline specification, the ranking of policy instruments in terms of welfare effects is similar.

In order to provide a general idea of the magnitude of the welfare effects computed under the baseline specification, we do a back on the envelope calculation and translate these consumption equivalent welfare effects into monetary units. According to data from Destatis, real aggregate private consumption for Germany in 2015 amounts to roughly 1.6 trillion euros. Thus, in absolute terms, the implied welfare losses associated with climate policies range from 16.9 billion euros under a constant price regime to about 25.7 billion euros under a cap-and-trade regime. In per-capita terms this corresponds to 208–317 euros, respectively.26 These numbers indicate that the choice of policy instrument has relevant implications for aggregate and per-capita consumption. However, it is important to emphasize that the effects obtained do not reflect all of the costs potentially associated with climate policy instruments, and crucially depend on the underlying parameter values and the assumptions of the model.27

While the model abstracts from the costs of climate change, it is still informative to compare these welfare effects of mitigating climate change with the estimated costs associated with global temperature increases. For example, Hübler et al. (2008) estimate the climate induced health risks for Germany over the period 2071–2100 for different climate scenarios. They focus on the costs associated with increases in

26 All figures are expressed in per capita terms and based on the German population of 81.2 million in 2015, as reported by Eurostat.

27 We assess the robustness of the baseline results to variations of parameter values in the following section.
mortality rates, heat induced hospitalization and reduced productivity. According to their estimates, the expected annual costs range between 2.2 and 8.2 billion euros in absolute terms. In per capita terms, this corresponds to costs between 27 and 101 euros, conditional on the specific climate scenario. Kemfert (2007) provides estimates of the potential costs for different economic sectors based on an IAM. According to her results, the average annual costs from 2015 to 2100 associated with an temperature increase of 4.5°C amount to roughly 24 billion euros per year, which corresponds to 295 euro per capita.28 Another example of costs associated with climate change is the increasing number of flood events. According to Rojas et al. (2013), the potential annual costs of this increase for Germany will amount to about 1.8 billion euros between 2020 and 2089. In per capita terms, this corresponds to about 22 euros per year. This small overview shows that it is hard to pin down the costs of climate change exactly, conditional on the effects included in the respective studies, as the estimated costs vary substantially. Compared to the predicted costs of climate policies stemming from uncertainty and frictional adjustments presented here, we see that the expected costs under a no-policy scenario can easily exceed the costs of mitigation. Moreover, the reported baseline effects rely on the assumption that revenues from climate policies are completely absorbed, an aspect that we will explore in more detail in the following section.

Besides the comparison of the aggregate welfare implications, the present model also allows us to assess the distributional effects of climate policies. Comparing the welfare effects of the policy instruments between the two groups of households reveals that the costs associated with climate policies are not evenly distributed. For price instruments, we observe almost identical welfare effects across households, in this case climate policy exerts neutral or even slightly progressive effects. However, under a cap-and-trade policy, the difference in welfare losses between Ricardian and non-Ricardian amounts to about 0.16 percentage points. Under an intensity target, we find a difference of 0.13 percentage points between the welfare losses between households. Thus, under the baseline specification, in the context of short-run dynamics under uncertainty, both instruments are regressive, posing a larger relative burden in terms of welfare losses on poor households.

3.3 Inequality

The results of the welfare comparison for the baseline specification presented in table 5 have revealed differences between climate policy instruments regarding their distributional implications. While the price instruments are neutral or even slightly progressive, the cap-and-trade and the intensity target exert regressive effects. From a policy perspective, the direction and magnitude of the distributional effects of climate policies is important, since uneven effects can erode broad support for climate policies. Therefore, the distributive effects of climate policies have received considerable attention. As summarized by Ohlendorf et al. (2020), the existing literature provides ambiguous results on the distributional consequences of climate policies. They highlight that the different findings are context specific and depend on the type of effects captured in the respective studies. Furthermore, at least partially, methodological differences help to explain the variation in distributional outcomes across studies. As discussed by Wang et al. (2016) in the analysis of the distributional effects of climate policies, different dimensions of inequality, such as location or demographic aspects, are also relevant. Against this backdrop, it is

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28We exclude the predicted costs of structural adjustments to increasing temperatures from this comparison.
important to emphasize that the focus of the present study is on wealth inequality, which translates into income inequality.

In order to set the present results into context and to identify the relevant drivers, we assess the different channels through which climate policies exert distributional effects in the present model. It is important to emphasize that the present framework abstracts from the effects of long-run adjustments, but intends to capture the effects of climate policies that result from interactions with frictional short-run dynamics in the presence of aggregate uncertainty. In the following, we adopt the classification of Fullerton (2011), who distinguishes between six different types of distributional effects. In particular, the present framework directly captures the distributional effects of changes in relative factor incomes and the transitory costs of labor and goods market frictions. Both channels are intimately linked, since frictions substantially alter the response of factor incomes to fluctuations. In contrast, the restriction on a single final good in combination with homothetic preferences rules out distributional effects on consumers driven by adjustments in consumption.

Furthermore, since we do not model pollution externalities explicitly, we cannot address distributional differences in the benefits from alleviating climate change. While we have largely ignored the revenues from climate policy instruments so far, we can use the model to assess some aspects of the distributional implications of different revenue allocation schemes. In addition to these channels, the present analysis points to the importance of macroeconomic uncertainty for the distributive implications of climate policies. While the effects of uncertainty clearly relate to the transitional effects associated with climate policies, this mechanism has not yet been discussed in the literature.

In the present model, the major channel through which climate policies exert distributional effects lies in its effects on factor prices. Since firms rely on the polluting intermediate input factor, a price increase here translates into adjustments of the combination of inputs used in production. In a static general equilibrium analysis, Fullerton and Heutel (2011) term these effects the "sources-side" effects, i.e. effects that occur through factor price changes at the source of factor incomes. According to their results, the sources side effects can differ substantially and are particularly sensitive to assumptions regarding parameter values. Whether a policy exerts regressive or progressive sources-side effects depends on the elasticity of substitution between the polluting input and other input factors. Within a static general equilibrium model, Dissou and Siddiqui (2014) report progressive effects of a carbon tax resulting from adjustments in factor prices. In their model, this result emerges because the income losses in the capital intensive energy sector are relatively larger than the reduction in wage incomes. Thus, the factor substitution effects turn out relatively favorable for wealth poor households. In contrast to these studies, which capture sectoral aspects of the production side in detail, the present model builds on a simple aggregate production function. In particular, the assumption of a Cobb-Douglas production function for intermediate goods producers implies an elasticity of substitution between input factors of unity. Thus,
under complete markets with perfect competition, the implied relative change in the employed quantities is exactly proportional to the relative change in factor prices. This structure ignores the long-run adjustments of the production side and also rules out situations where the substitution effect does not eliminate the price effect. However, as is well-known with market imperfections and frictions, this is not necessarily the case. Thus, while a comparison between deterministic steady states of the model does not reveal the distributive consequences of factor substitution effects, within the dynamic framework, those effects play an important role. On the one hand, as shown by Gali (1995), factor shares will depend on the dynamics of price markups. On the other hand, the sluggish adjustment of wages and the capital stock crucially affect the factor substitution decisions of optimizing firms.

Figure 5: Cumulative deviations of the capital share and the labor share from steady state in percentage points in response to different shocks over a 20 quarter horizon. The underlying parameter values correspond to table 2.

To see how factor income shares behave under the different instruments, figure 5 depicts the cumulative deviations of the capital share and the labor share in response to different shocks. The effects are computed over a 20 quarter horizon and based on the baseline parameter values. Apparently, there are no qualitative differences in the cumulative changes of income shares between the instruments. Nevertheless, for some shocks, e.g. the TFP shock or the government spending shock, we observe marked differences in the cumulative development of income shares between the instruments. These differences in the dynamics
of factor incomes show that price and quantity adjustments on capital and labor markets are directly affected through climate policy instruments. In general, we observe the largest cumulative deviation in factor shares under a cap-and-trade scheme. The constant price instrument is associated with the smallest movements in factor shares. The fluctuations under the flexible price instrument and the intensity target lie between these two boundaries. Across all shocks, we also observe a tendency that movements in the capital share are stronger under the cap-and-trade and the intensity target regime compared to the price instruments. However, while this comparison illustrates the channel through which instruments exert distributional effects, it cannot reveal the direction of these effects. Both types of households are directly affected by movements in the labor share, but movements in the capital share affect only Ricardian households. Furthermore, firms and unions use their price-setting power to earn profits that are not reflected in the development of factor shares as depicted here.

To further illustrate the factor substitution channel and to show that the distributional effects of the policy instruments are driven by their implications for input factor dynamics, we compare the unconditional volatility in factor shares and household consumption. Table 6 depicts the standard deviations of the capital share, the labor share, aggregate consumption, consumption of Ricardian households and consumption of non-Ricardian households. The figures are again based on the baseline specification and correspond to the implied theoretical moments of the model.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Capital share</th>
<th>Labor share</th>
<th>$c_t$</th>
<th>$c_{R,t}$</th>
<th>$c_{N,t}$</th>
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</thead>
<tbody>
<tr>
<td>No-Policy</td>
<td>0.0051</td>
<td>0.0140</td>
<td>0.0095</td>
<td>0.0051</td>
<td>0.0367</td>
</tr>
<tr>
<td>(% change)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Price</td>
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<td>0.0141</td>
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<td>0.0368</td>
</tr>
<tr>
<td>(% change)</td>
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<td>0.2%</td>
<td>0.1%</td>
<td>0.1%</td>
<td>0</td>
</tr>
<tr>
<td>Price (flex)</td>
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<td>0.0148</td>
<td>0.0097</td>
<td>0.0050</td>
<td>0.0381</td>
</tr>
<tr>
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<td>5.9%</td>
<td>5.6%</td>
<td>1.1%</td>
<td>0.1%</td>
<td>3.7%</td>
</tr>
<tr>
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<td>0.0106</td>
<td>0.0049</td>
<td>0.0469</td>
</tr>
<tr>
<td>(% change)</td>
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<td>41.9%</td>
<td>10.6%</td>
<td>-3.9%</td>
<td>27.6%</td>
</tr>
<tr>
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<td>0.0175</td>
<td>0.0100</td>
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<td>0.0424</td>
</tr>
<tr>
<td>(% change)</td>
<td>26.7%</td>
<td>24.9%</td>
<td>4.7%</td>
<td>0.4%</td>
<td>15.5%</td>
</tr>
</tbody>
</table>

Table 6: Standard deviations of the capital share, the labor share, aggregate consumption, consumption of Ricardian households and consumption of non-Ricardian households. Results are based on a second-order approximation of the HP-filtered theoretical moments of the model. Parameter values correspond to table 2.

As we can see, in accordance with the impression from the cumulative responses to shocks, the volatility in the capital share and the labor share is significantly larger under the cap-and-trade system and the intensity target compared to the no-policy scenario. In contrast, under the price instrument, we only see a moderate increase in volatility in income shares. Furthermore, we also see that the increased volatility in income shares accompanies an increased volatility in aggregate consumption under the cap-and-trade scheme and the intensity target. In particular, we find that under the cap-and-trade system, consumption of Ricardian households becomes less volatile, while the volatility in non-Ricardian consumption increases

31Since the figure depicts cumulative deviations, negative and positive innovations in income shares cancel out, and we cannot infer the overall variation in factor shares from here.
by roughly 27%. A similar pattern can be observed for the intensity target. Compared to the price instrument, where we observe a more or less proportional increase in the volatility of consumption across households, this provides a clear indication for the source of the distributive effects. Under the cap-and-trade scheme and the intensity target, as volatility in income shares increases substantially since non-Ricardian households cannot smooth consumption through savings, this increased volatility in labor income translates directly into an increased volatility in their consumption. Overall, these results confirm that income dynamics under adjustment frictions are an important determinant for the distributional effects observed here.

Therefore, in order to better understand the differences in the distributional effects of policy instruments, we need to assess the role of market imperfections and frictions in the present context. As discussed by Fullerton and Heutel (2011), the transitory costs of climate policies stemming from adjustment frictions exert heterogeneous effects on economic agents and constitute an important channel for distributive effects. The importance of frictions for the comparison of policy instruments in terms of welfare has already been emphasized by Annicchiarico and Dio (2015). Their results show that the degree of price setting rigidities can significantly alter the welfare effects associated with different policy instruments. In the present model, two types of nominal rigidities, namely price and wage setting frictions, alter the behavior of firms. To understand how these frictions affect the implications of the present study and to assess their role for the distributional effects, we conduct a sensitivity analysis.

The degree of price rigidities in the economy determines the duration of firms price setting decisions. At high values of $\theta_p$, intermediate goods prices can only adjust slowly in response to shocks, which creates an inefficiency and contracts output dynamics. The reduction in the ability of firms to adjust output

![Figure 6: Welfare changes relative to the no-policy scenario in % at different values of $\theta_p$. The underlying parameter values correspond to table 2.](image-url)
prices requires larger variations of input factors and generally tends to increase aggregate volatility. The distributive implications associated with a higher degree of nominal price setting rigidities are a priori unclear. To assess them, we compute the welfare effects of the policy instruments for a 10% emissions reduction relative to a no-policy scenario at different values of \( \theta_p \). The other parameter values correspond to the baseline scenario. Again, we express welfare effects in terms of consumption equivalent variations. Figure 6 depicts the results for aggregate and household specific welfare.

In the absence of climate policies compared to the situation with flexible prices, aggregate and group specific welfare decline with increasing price setting frictions. By and large, we see that the pattern of welfare effects as reported in table 5 does not change with the degree of price stickiness. The welfare losses associated with the price instruments have a similar magnitude and are generally lower than under the cap-and-trade scheme or under an intensity target. Furthermore, we observe that the welfare effect of the fixed price instrument remains almost constant over the whole range of parameter values. For the other instruments, we observe stable welfare effects for \( 0 < \theta_p < 0.6 \). At higher values of \( \theta_p \), the welfare losses associated with the instruments, which generate flexible emission prices, start to increase. Given that the baseline specification uses a value of \( \theta_p = 0.86 \), the increasing welfare effects in this parameter range show that the quantitative results are sensitive to the degree of price rigidities. Qualitatively, the reported effects remain robust over the whole range of parameter values.

Figure 7: Difference between the welfare changes of non-Ricardian and Ricardian households at different values of \( \theta_p \). The underlying parameter values correspond to table 2.

Figure 7 depicts the percentage point difference between the absolute welfare change of non-Ricardian and Ricardian households at the same values of \( \theta_p \) for the four policy instruments. We see that the price instruments have a small progressive effect, i.e. welfare losses of non-Ricardian households are a magnitude smaller than those of Ricardian households. As before, the cap-and-trade and the intensity
target exert regressive effects with larger welfare losses for non-Ricardian households. Apparently, the
distributional effects of all instruments are stable for $0 < \theta_p < 0.7$. For higher degrees of price rigidity, we
see a declining progressivity of the flexible price instrument and increasing regressivity for the cap-and-
trade and the intensity target. This indicates that price rigidities are not only relevant for overall welfare
effects, but also affect the magnitude of the distributional effects. As we can infer, the size of the reported
regressive effects of the cap-and-trade and the intensity target sharply increase at high values of $\theta_p$. This
suggests that wealth-poor households are over-proportionally affected by the implementation of these
instruments when goods markets are particularly inflexible.

![Figure 8: Welfare changes relative to the no-policy scenario in % at different values of $\theta_w$. The underlying parameter values correspond to table 2.](image)

While the general welfare losses associated with price rigidities are qualitatively similar across agents,
the welfare losses associated with wage rigidities differ across agents. In the model, the consumption
of non-Ricardian households depends crucially on the real wage. As explained by Colciago (2011), the
degree of wage rigidities determines the volatility of real wages, i.e. higher degrees of wage stickiness
imply smaller fluctuations in wages. Under the present assumptions regarding household preferences, non-
Ricardian and Ricardian households are both risk-averse and therefore adversely affected by fluctuations
in consumption. However, Ricardian households are able to smooth consumption through savings, which
is not possible for non-Ricardian households. Therefore, lower variations in the real wage tend to stabilize
consumption and welfare of non-Ricardian households. Analogous to before, figure 8 illustrates the
aggregate and household specific welfare effects of climate policy instruments at different values of $\theta_w$.

As we can see, at low-degrees of wage setting frictions, households experience substantial welfare
losses associated with climate policies compared to the no-policy scenario. Again, we find that a constant
price of emissions has monotonous welfare implications which remain comparably stable across all values
Figure 9: Difference between the welfare changes of non-Ricardian and Ricardian households at different values of $\theta_w$. The underlying parameter values correspond to table 2.

of $\theta_w$. In contrast, for the remaining instruments, we observe a decline in welfare losses as $\theta_w$ increases. It can be inferred that these welfare losses tend to converge at values of $\theta_w > 0.7$. This implies that the results under the baseline specification are stable in the relevant range of parameters. It also highlights that the additional variation in income, induced through climate policies, is an important driver of welfare effects.

Figure 9 depicts the percentage point difference between the absolute welfare change of non-Ricardian and Ricardian households at the same values of $\theta_w$ for the four policy instruments. We see that regardless of the degree of wage stickiness, the distributional implications of the price instrument remain constant. Regarding the flexible price instrument, we observe a regressive effect for values of $\theta_w$ between 0 and 0.6. This effect diminishes at higher degrees of wage rigidities. Similarly, we observe a high degree of regressivity associated with the cap-and-trade and the intensity target regime for values of $\theta_w$ between 0 and 0.4. When wage rigidities increase further, the difference between non-Ricadian and Ricadian welfare losses gets smaller and converges to the values reported earlier. Overall, we see that wage rigidities affect the distributional implications of the present model, in particular, climate policies tend to be more regressive when wage rigidities are moderate. This observation supports the argument that the overall welfare effects and the distributional effects of climate policy instruments are driven by market-imperfections. It also shows that in the present model, these effects emerge as a consequence of increased fluctuations in household income.

The preceding analysis has illustrated that climate policy instruments are not only associated with aggregate welfare implications, but also exert distributional effects. In the present model, these effects result from frictional supply side adjustments under uncertainty. Yet so far, we have assumed that all
revenues generated through the policy instruments are completely absorbed. While this allows one to assess the direct implications of climate policies for polluting firms and provides an unbiased comparison of the instruments, it generally amplifies the welfare effects and rules out potential interactions with fiscal policies. Therefore, to provide less biased welfare effects and to illustrate possible measures that specifically aim to alleviate the distributional effects arising in the present context, we compare different revenue recycling schemes.

There is a large body of literature that examines the role of revenue recycling from environmental policies for the public budget. A general theme in this discussion is the question of whether it is possible to earn a “double dividend” using the revenues of environmental policies to finance adjustments of the tax system. An early assessment of this aspect is given by Goulder (1995). The idea of a double dividend has often been addressed through Computational General Equilibrium (CGE) models, a recent meta analysis of these studies is provided by Freire-González (2018). With regard to distributional effects, Klenert et al. (2018) show that the availability of a double dividend from environmental policies extends the scope for re-distributive policies. In the present model, government expenditures are completely financed via lump sum taxes that exclude the occurrence of a double dividend. However, since the focus is on short-run fluctuations, the integration of revenues into the government budget creates cyclical interactions between the two. Thus, to some extent, the following exercise relates to the longstanding debate about tax smoothing as described by Barro (1979).

In the following, we compare the welfare effects associated with the policy instruments under different revenue recycling schemes. Figure 7 presents the unconditional welfare effects of a 10% reduction in emissions relative to the no-policy scenario (without revenues) associated with the different instruments. We study three different scenarios. As a reference scenario, we assume that the government uses the entire income to increase government spending, i.e. \( T_{E,I} = \Delta \bar{g} \). This constitutes a less restrictive form of absorption compared to the scenario reported above. The corresponding welfare effects are shown in the upper panel of the table. The panel in the middle shows the results under the assumption that the government increases spending and uses half of the revenues from environmental policy to reduce lump sum taxes. Finally, the bottom panel refers to a scenario where the government fully redistributes the revenues via lump sum transfers.

Taking the revenues of climate policies into account reveals several insights. We see, that when the government absorbs the entire revenue, welfare losses are smaller compared to the earlier scenario. This can be explained by the fact that in this situation, government spending remains in the economy and dynamically adjusts in response to increases in climate policy revenues. By and large, we see only a small effect for the price instruments, but for the cap-and-trade scheme and the intensity target, the welfare losses are reduced by 0.29 and 0.23 percentage points, respectively. Under both regimes, the reduction is roughly proportional across households. For the second scenario, we observe an even more pronounced reduction in welfare losses for all agents. The tax cut does not affect the savings decisions of Ricardian households, but generally provides households with additional resources for consumption. This also mitigates the distributional effects, but the cap-and-trade system and the intensity target are still associated with a relatively larger decline in welfare of non-Ricardian households.

Finally, in the full transfer scenario, aggregate welfare losses reduce substantially across all instruments. Here, the implementation of the price instruments leads to a 0.22% decline in welfare relative to the no-policy scenario. The welfare losses associated with a cap amount to roughly 0.51%, almost identical to
the effects observed for the intensity target. A full transfer scheme significantly alters the distributional implications of the model. While the welfare losses of Ricardian households under the price instrument are roughly 0.33% of consumption equivalent variations, non-Ricardian households see a negligible increase in welfare, or at least do not experience any welfare losses. With the transfer scheme, we also observe a reversal of the distributional effects of the cap-and-trade system and the intensity target. In both cases, the relative welfare loss of non-Ricardian households is smaller than the loss experienced by Ricardian households. This result is not surprising. Firms’ factor input decisions are still distorted toward an emissions reduction, which also implies lower income from labor and capital. However, since the entire revenue is transferred to households, aggregate private consumption is larger than in the other scenarios. Therefore, Ricardian households experience a decline in capital and labor income, but are partially compensated by the transfer. Non-Ricardian households are not directly affected by the decline in capital income but receive the same transfer. Put differently, they also receive a share of the revenue that is paid by capital.

As we can see, the welfare implications have changed substantially compared to the scenario with complete absorption. In the government spending scenario, the implied average annual costs per capita now vary from 205 euros under the price instrument, to 260 euros under the cap-and-trade system. For the full transfer scenario under the price instrument the average annual costs per capita associated with climate policies correspond to 43 euros and to 100 euros under the cap-and-trade system. Thus in monetary units, the aggregate welfare loss associated with the cap-and-trade system is still more than two times larger than the loss under a price instrument. Against the backdrop of the costs associated with climate change reported earlier, these figures reveal that even under a full redistribution of revenues, the costs associated with climate policy instruments that evolve through production side effects under frictions and uncertainty are sizeable and should be taken into account when designing policy instruments. Furthermore, we can

<table>
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<tr>
<th>Scenario</th>
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<th>Non-Ricardian</th>
</tr>
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<td>-1.04%</td>
<td>-1.04%</td>
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<tr>
<td><strong>Tax cut and Spending</strong></td>
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<td><strong>Full Transfer</strong></td>
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<td>-0.43%</td>
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<tr>
<td>Intensity Target</td>
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<td>-0.55%</td>
<td>-0.39%</td>
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Table 7: Welfare changes of a 10% emission reduction, reported in terms of consumption equivalent compensations (in %) relative to the no-policy scenario for different revenue recycling schemes. Based on a second-order approximation of the theoretical moments of the model.
conclude here that the distributive effects exerted through climate policies can be mitigated or even fully eliminated through an appropriate revenue recycling scheme.

3.4 Uncertainty of Climate Policies

So far, we have assessed the different instruments in the context of common business cycle shocks. As we have seen, climate policies interact with short-run macroeconomic dynamics and can alter the response of an economy to aggregate shocks. Moreover, as laid out above, the efforts to limit global warming through emissions reductions can itself introduce additional sources of uncertainty. Fluctuations in emission intensity are one potential aspect which we have already examined. Another type of uncertainty directly results from limitations in the precision of carbon budget estimates. As pointed out by Fujimori et al. (2019), the available estimates of the carbon budget remaining to limit increases of global temperature vary substantially. Limitations in the understanding of atmospheric dynamics and potential non-linearities in the effect of GHG on global temperature, such as trigger points through the thawing of the permafrost, imply that current estimates of the carbon budget may need to be adjusted in the future. This manifests in a general uncertainty regarding the precise size of the overall carbon budget. Hence, climate policies based on specific budgets might also undergo adjustments in response to new scientific information. Therefore, as a last step, we explicitly examine the effects of carbon budget uncertainty.

In order to assess the implications of uncertainties regarding the carbon budget, we proceed in two steps. First, we assess the consequences of a permanent adjustment of policy tools in response to a 10% decline in the carbon budget. As explained in the calibration section, this value fits roughly into the range of standard deviations in carbon budget estimates under the 1.5°C goal. This setup allows us to assess the model dynamics under different implementation schemes and provides a general idea about the implications of an unexpected permanent adjustment in the carbon budget. Here, we compare the performance of the constant price instrument to the cap-and-trade system in terms of aggregate dynamics. Second, we incorporate the carbon budget shock, as laid out in (21) into the baseline model, and assess the implications of carbon budget uncertainty under the cap-and-trade system and the price instrument. To this end, we compare the implied volatility in macroeconomic aggregates and unconditional welfare relative to a scenario without carbon budget uncertainty. Throughout this exercise, we do not include the flexible price instrument and the intensity target since both instruments introduce substantial feedback dynamics in response to changes in the carbon budget.

To analyze the consequences of a permanent decrease in the carbon budget, we focus on the transitional dynamics of the model economy, which starting in the deterministic steady state under a 10% emissions reduction policy learns in period \( t = 1 \) that an additional 10% reduction is necessary. In particular, we follow Diluviso et al. (2020) and model three different implementation scenarios. In the first scenario (direct implementation), the policy tools are adjusted directly, i.e. the number of emission permits is instantly reduced by 10% and the emission price increases accordingly. In the second scenario (linear implementation), we assume that policymakers linearly phase in the adjustments over a 20 quarter period.

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32 Under an active climate policy, fluctuations in emission intensities induce changes in firms demand for polluting goods, and consequently alter the demand for permits and the returns from price instruments. In contrast, shocks to the carbon budget alter emissions prices or the number of permits, which reflects a supply-side effect in the context of climate policy instruments.

33 This approach resembles a perfect foresight transition commonly called MIT shock.
such that the final reduction goal is reached after five years through linear increases in the price, and decreases in the number of permits. In the third scenario (exponential implementation), the instruments are adjusted at an exponential rate over a five year period, which corresponds to a Hotelling scheme. In this analysis, after the initial unexpected innovation in the carbon budget, economic agents are directly informed about the planned adjustments in policy instruments. Given rational expectations, this implies that agents fully anticipate the future time path of policy variables. The model solution is computed from the full set of non-linear equations. Since we cannot solve the full model for the baseline parameter values of the Calvo rigidities, we set \( \theta_w = \theta_p = 0.75 \) and the other parameter values correspond to the baseline scenario.\(^{34}\) Figure 10 depicts the adjustments of output, consumption, investment, wages, emissions and the emissions price to the permanent change in the carbon budget for a constant price instrument (upper panel) and a cap-and-trade policy (lower panel).

As can be inferred from the figure, the adjustment trajectories differ conditionally on the implementation scheme, but appear relatively similar across instruments. Over the entire period, all schemes exhibit recessionary tendencies. In case of a direct implementation, we observe an immediate drop in output, consumption, investment, wages and emissions, accompanied by an increase in the price of emissions. The most notable difference between the policy instruments manifests in the dynamics of emissions and the emissions price. Under the emissions cap, emissions decline directly, whereas under the price instrument, emissions drop on impact and then decline slowly until the same emissions reduction is reached after around 15 quarters. The opposite picture emerges with respect to the price of emissions. The direct adjustment of the price instrument leads to a constant price of emissions over the entire period, but under the cap-and-trade, we observe an initial overshooting of the carbon price of about 1.5% relative to the constant price, followed by a subsequent convergence. This overshooting appears due to the unexpected decrease in the number of permits, which interferes with the production plans of firms and drives up the permit price in the initial periods before firms can fully adjust to the new policies.

When the policy adjustments are phased in over time, we observe a different pattern. Under both the linear and the exponential implementation scheme, output and consumption increase on impact and decline subsequently. The observed initial increase in both variables is a magnitude larger under the exponential scheme, where we also observe a small increase in investment during the first quarters. The difference between the direct implementation and the gradual schemes emerges for two reasons. On the production side, firms anticipate the gradual increase in future production costs driven by the planned adjustments in policy instruments. Therefore, in contrast to the direct implementation scenario, they do not need to curb production immediately and can smoothly decrease output over a longer horizon. On the demand side, households anticipate the future decline in output and the gradual implementation provides them with additional flexibility to smooth their consumption over time. The initial increase in output and consumption results from the assumption that households are impatient and prefer present consumption over future consumption. Therefore, Ricardian households intertemporally substitute future consumption for present consumption. Non-Ricardian households cannot smooth consumption, but given the slower decline in wages compared to the direct implementation, the gradual implementation schemes also stabilize their consumption, which in turn tends to stabilize aggregate demand. In general, the trajectories under both schemes mimic the implementation scheme, i.e. under the linear scheme,

\(^{34}\)We can numerically solve for the transition path with \( \theta_p = 0.86 \) and \( \theta_w = 0.83 \) if we consider a 1% decline in the carbon budget. The resulting transition dynamics are qualitatively comparable to the ones depicted in figure 10.
Figure 10: Simulated time path of macroeconomic aggregates in response to an unexpected 10% decline in the carbon budget over a 20 quarter horizon. All variables are expressed as percentage deviations from the initial steady state. The upper panel (A) shows the transition under a price instrument. The lower panel (B) shows the transition under a cap-and-trade system.

the adjustments evolve almost linearly, and under the exponential scheme, we observe trajectories with additional curvature. This features prominently in the dynamics of aggregate consumption. While both schemes lead to an initial increase of consumption, under the exponential implementation, aggregate
consumption stabilizes between the fifth and the sixteenth quarter, whereas we observe a steady decline under the linear scheme.

As indicated, over the entire period, all implementation schemes are associated with declining output and consumption and achieve the desired emission reduction after 20 quarters. However, quantitatively, the adjustment paths differ substantially across implementation schemes, but are almost identical for both policy instruments. Under the direct implementation, the cumulative drop in output and consumption correspond to about 25.5% and 14.5% relative to the initial steady state. Consequently, the transition toward the new steady state is associated with a decline in aggregate welfare measured in terms of utility over the transition period. In contrast, under the linear implementation scheme, the cumulative decline of output and consumption over the 20 quarter period are 14.1% and 7.6%, respectively. With an exponential adjustment of policy instruments in response to the carbon budget shock, the cumulative decline in output and consumption amount to 6.7% and 3.7%. Given that households discount future utility exponentially, it is not surprising that the aggregate welfare losses under both schemes are lower than under the direct implementation scheme, with the smallest welfare losses under the exponential implementation scheme. However, the opposite picture emerges in terms of cumulative emissions. While all implementations schemes achieve the same emissions reduction after 20 quarters, cumulative emissions over the transition period differ. Naturally, if the policy instruments are adjusted directly, cumulative emissions over the period are lower compared to the case of an direct implementation, which by construction achieves the desired emission reduction throughout the whole period. Taking this as the benchmark, under the linear implementation, cumulative emissions are 101 percentage points higher, and even 166 percentage points higher if the implementation proceeds exponentially. Thus, while the later schemes allow one to smooth the decline in output and consumption and appear preferable from a welfare perspective, policymakers must also take the effects of an staggered emission reduction into account. Importantly, we do not observe significant differences in terms of aggregate dynamics and welfare between the price instrument and the cap-and-trade system in response to a tightening of the carbon budget.

While the previous exercises showed the dynamics in response to an unexpected change in emissions policies and allowed for a comparison of the welfare effects along such a transition, it neglected the potential effects of this type of uncertainty in the general context of business cycles. To capture the effects of this kind of uncertainty on unconditional welfare and aggregate volatility, we compare the theoretical moments of the stochastic model between the baseline scenario and a scenario with stochastic innovations in the carbon budget or price, conditional on the instrument. Since the model is solved via a second-order approximation around the deterministic steady state, the policy functions are only accurate in the vicinity of the steady state. Therefore, we implement relatively small fluctuations of 1% in the carbon budget, which corresponds roughly to 9% fluctuations in the emissions price. Since we focus on temporary shocks in a stationary environment, we model fluctuations as an AR(1) process. In absence of a clear idea about the persistence of fluctuations in the carbon budget, we compare the effects for different values of \( \rho_e = [0, 0.4, 0.8] \). Again, the other parameter values correspond to the baseline specification.

The results reported in table 8 show that aggregate volatility increases if we take carbon budget uncertainty into account. Generally, we see a larger increase in volatility in the case of fluctuations in the number of permits compared to fluctuations in the emissions price. We also observe that volatility under both schemes increases with the increasing persistence of carbon budget shocks. For uncorrelated shocks in the carbon budget, we see only a negligible increase in volatility under the price instrument.
Table 8: Macroeconomic volatility and welfare effects under a 10% emission reduction relative to the no-policy scenario. Welfare effects are reported in terms of consumption equivalent compensations (in %) relative to the equivalent policy scenario without carbon budget uncertainty. Volatility is reported as % change relative to the equivalent policy scenario without carbon budget uncertainty. All results are based on a second-order approximation of the theoretical moments of the model.

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<th>y_t</th>
<th>e_t</th>
<th>x_t</th>
<th>Aggregate</th>
<th>Ricardian</th>
<th>Non-Ricardian</th>
</tr>
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</tbody>
</table>

In contrast, under the cap-and-trade scheme, volatility in output increases by about 7% compared to a situation without carbon budget shocks. If fluctuations in the carbon budget are assumed to be highly serially correlated, i.e., $\rho_e = 0.8$, we find that the volatility in output increases by about 3% under the price instrument and by about 30% under the cap-and-trade scheme. A similar pattern evolves regarding the volatility in aggregate consumption and investment. Since to some extent, carbon budget shocks resemble exogenous variations in energy prices, the general increase in volatility is not unexpected and broadly fits the results of Kim and Loungani (1992). In the present context, the significantly stronger increase in volatility under the cap-and-trade scheme again results from the binding nature of the emissions constraint. In order to fully comply the with the imposed time-varying emission cap, firms need to adjust input factors more frequently, which translates into higher volatility on all margins of the model.

Furthermore, we find that carbon budget uncertainty is associated with additional welfare losses compared to the baseline scenario. In general, these losses are smaller under the price instrument. In addition, the cap-and-trade is not only associated with a larger reduction in aggregate welfare, but again tends to exert regressive effects. Similar to the pattern observed for aggregate volatility, we find that the welfare effects amplify at higher degrees of serial correlation in the carbon budget shocks. While the welfare losses under the price instrument remain relatively stable between -0.58% and -0.65%, the aggregate welfare losses under the cap-and-trade system amount to -3.2% for $\rho_e = 0.8$. As laid out above, this substantial increase in welfare losses under the cap-and-trade system reflects the strong increase in aggregate volatility under a binding emissions cap.

Taken together, we see that carbon budget uncertainty that requires adjustments of climate policy instruments can exert large effects on the aggregate economy. Not only does a permanent change in the carbon budget induce substantial adjustment dynamics associated with declining output and consumption, as we have seen, relatively small stochastic fluctuations in the carbon budget already result in an increased macroeconomic volatility and welfare losses. In response to an unexpected permanent adjustment in the carbon budget, we do not observe significant differences between the policy instruments. According to the present results, in this situation, the time path of policy adjustments is more important. A direct
adjustment of policy instruments is favorable in terms of cumulative emissions, but exerts the largest economic costs. A staggered adjustment of policy instruments with exponential increments drastically reduces the economic costs, but is also associated with higher emissions over the implementation period. The linear implementation strikes a balance between the other schemes. With respect to the unconditional effects of carbon budget uncertainty, under the present formalization of the policy instruments, we again find that the flexibility in emissions under a price instrument exerts smaller effects on macroeconomic aggregates and welfare compared to a cap-and-trade scheme. However, regardless of the instrument, the last exercises highlighted that climate policies should be implemented in a predictable and transparent way to avoid additional economic consequences stemming from trembling regulations.

4 Conclusion

To conclude, the present study examined the implications of climate policies for short-run dynamics under aggregate uncertainty with a particular focus on the role of inequality and economic frictions. Within a TANK model we assess four different emission reduction schemes in terms of welfare, macroeconomic stability and their implied distributional effects. In addition to common sources of macroeconomic uncertainty, we extended the analysis for uncertainties associated with climate policies, i.e. carbon budget and emission intensity uncertainty. In order to solve the numerical model and to understand the role of uncertainty and frictions for climate policies we calibrate the model to match key properties of the German economy.

In a first quantitative exercise, we compare the model dynamics under the different instruments in the light of business cycle fluctuations. The results of this analysis largely confirm previous studies and highlight that climate policies interact with aggregate short-run dynamics. We find that a price instrument is preferable in terms of aggregate welfare, followed by a flexible price instrument and an intensity target, while a cap-and-trade scheme is associated with the lowest welfare. In terms of macroeconomic stabilization, we find that a cap-and-trade scheme, as depicted here, can fully stabilize emission dynamics. However, this comes at the cost of an increased volatility in output and consumption. The effects of the intensity target are similar but weaker. In general, price instruments are found to exert only minor effects on aggregate volatility. The finding of an increased output volatility under a cap-and-trade scheme results from the inability of firms to adjust the utilization of polluting inputs, which requires larger adjustments on other margins.

The assessment of the distributional implications shows that the costs associated with climate policies and frictional adjustments are not necessarily equal across households. Under the assumption of a full absorption of revenues, price instruments exert neutral effects but the cap-and-trade system, as well, as the intensity target imply larger welfare costs for wealth-poor households. A sensitivity analysis reveals that these effects stem from the interaction between climate policies and firms factor substitution decision under frictions. We see that at higher degrees of market-imperfections not only aggregate welfare effects magnify, but also the regressive effects associated with the policy instruments increase. This result reflects the inability of wealth-poor households to smooth consumption through savings, so that larger variations in wage income directly increase the variability of their consumption. Under different revenue recycling schemes, these regressive effects can be overcome. Under the baseline parameter values we find that the documented annual welfare costs associated with climate policy in Germany vary between 43 Euro per
capita under a price instrument with full redistribution, and 317 Euros per capita under a cap-and-trade scheme with full absorption of revenues. Importantly, under the full absorption scenario, the individual loses of wealth-poor households are about 0.3 percentage points larger. Overall, the present study shows that policymakers should take interactions with macroeconomic dynamics and the potential distributional consequences into account when implementing climate policies.

Finally, as one would expect, uncertainty which is directly associated with climate policies alters macroeconomic dynamics. Changes in emission reduction possibilities, modeled as stochastic fluctuations in emission intensity of production, affect production costs and lead to adjustments in macroeconomic aggregates. Furthermore, we find that the scientific uncertainty regarding the size of the carbon budget also plays a role. An unexpected adjustment of climate policies causes a decline in aggregate output and consumption. Through a staggered adjustment of climate policies the economic costs of the tightening of the carbon budget can be reduced. Nevertheless, within the present framework, we find that already the presence of carbon budget uncertainty affects economic decisions and outcomes.

Given these results, it is important to make some remarks. First, the present analysis of inequality is rather stylized and as emphasized, abstracts from demand side effects which according to the literature can be expected to have regressive tendencies. Second, the effects of the cap-and-trade system obtained here result from the specific assumptions regarding its design. A more flexible approach, where a cap-and-trade system allows for intertemporal banking and borrowing of permits could potentially overcome these problems. Third, while the present model provides a reasonable fit of empirical data, there is still room for improvements, especially with regards to the models ability to match emission dynamics. All these points can provide fruitful starting points for future research.
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5 Appendices

5.1 Data and Calibration

The data used for the calibration of the model stems from various sources:

- Quarterly data on real GDP of Germany 1991–2015 (Destatis: 81000-0020)
- Quarterly data on real private consumption in Germany 1991–2015 (Destatis: 81000-0022)
- Quarterly data on gross fixed capital formation in Germany’s private sector from 1991–2015 (Destatis: 81000-0024)
- Annual data on the labor share in Germany 1991–2015 (Destatis: 81000-0015 and 81000-0003)
- Quarterly CO₂ emissions in Germany 1991Q1–2012Q4 (EDGAR: Crippa et al. (2020))
- Annual data on total energy supply in Germany 1991–2015 (EIA: TES in toe/thousand 2015 US Dollar)
- Data on carbon budgets, consistent with 1.5°C scenarios (including scenarios with low-overshooting) is extracted from Huppmann et al. (2019). Some scenarios are filtered out, following the procedure of the IPCC (https://data.ene.iiasa.ac.at/sr15_scenario_analysis/assessment/sr15_2.5_carbon_price_analysis.html).

All data series obtained from Destatis are seasonally adjusted. The quarterly time series used to compute the statistics reported in table 4 were log-transformed and the cyclical components were extracted using a HP-Filter with a smoothing parameter of 1600.

In order to calibrate the stochastic properties of (14), we construct quarterly data on emission intensity in Germany for the period 1991Q3–2012Q4. Since emission intensity in Germany displays a downward trend over the period, we apply a HP-Filter with \( \lambda = 1600 \) to isolate the cyclical component. Figure 11 depicts the cyclical fluctuations in emission intensity (left panel) and the trend component (right panel). Table 9 shows the respective statistics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>AR(1)</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_t/y_t )</td>
<td>0.36</td>
<td>0.78</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Table 9: Cyclical properties of emission intensity in Germany 1991Q3–2012Q4. The sample mean, measured in kg CO₂ emissions per unit of output, refers to the untransformed data.
5.2 Equilibrium Conditions and Solution

The model is solved numerically in Dynare. For details, see Adjemian et al. (2011). The solution is based on a 2nd order Taylor approximation around the deterministic steady state of the model. Therefore it is necessary to rewrite the full model in terms of stationary variables. Furthermore, when solving the model, we rely on a recursive representation of the dynamic wage setting equation of unions and the dynamic price setting equation of intermediate goods producing firms. Recursively the wage setting equation (11) can be represented as follows:

\[
w^*_t = \frac{\eta_w}{(\eta_w - 1) f_{2,t}} f_{1,t} = \frac{\eta_w}{\eta_w - 1} h_{d,t} + \theta_w \Lambda_{t+1} \Pi_t^{\epsilon} f_{1,t+1}\\
\]

\[
f_{2,t} = \frac{\eta_w}{\eta_w - 1} h_{d,t} + \theta_w \Lambda_{t+1} \Pi_t^{\epsilon-1} f_{2,t+1}.
\]

Analogously, the price setting equation (19) reads:

\[
\Pi_t^\epsilon = \frac{\epsilon}{\epsilon - 1} g_{1,t}\\
\]

\[
g_{1,t} = mc_t y_t + \theta_p \Lambda_{t+1} \Pi_t^\epsilon g_{1,t+1}\\
\]

\[
g_{2,t} = y_t + \theta_p \Lambda_{t+1} \Pi_t^{\epsilon-1} g_{2,t+1}.
\]
The full set of equations that describe the decentralized competitive equilibrium of the economy is given by:

\[ \lambda_{R,j} = d_t c_{R,j}^\rho \]  
\[ \lambda_{R,j} = w_t \Psi_t^{\text{h}_{R,j}} \Psi_t^{-1} \]  
\[ \lambda_{R,j} = \beta_t R_t \lambda_{R,t+1} \Pi_t^{-1} \]  
\[ 1 = q_t \left( \frac{1 - \kappa}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 - \kappa \left( \frac{x_t}{x_{t-1}} - 1 \right) \frac{x_t}{x_{t-1}} \right) + \beta_t E_t \frac{\lambda_{R,t+1}}{\lambda_{R,t}} q_{t+1} \kappa \left( \frac{x_{t+1}}{x_t} - 1 \right) \left( \frac{x_{t+1}}{x_t} \right)^2 \]  
\[ q_t = \beta_t E_t \frac{\lambda_{R,t+1}}{\lambda_{R,t}} \frac{z_t}{z_{t-1}} \left( (1 - \delta)q_{t+1} + z_{t+1} R_{k,j+1} \right) \]  
\[ \Lambda_{t,t+1} = \beta_t \frac{\lambda_{R,t+1}}{\lambda_{R,t}} \]  
\[ k_{t+1} = \left[ 1 - \frac{\kappa}{2} \left( \frac{x_t}{x_{t-1}} - 1 \right)^2 \right] z_t x_t + (1 - \delta) k_t \]  
\[ c_{N,t} = \Psi_t h_{N,t} - T_t + F_{u,t} \]  
\[ h_t = \lambda_t h_{N,t} + (1 - \lambda_t) h_{R,t} \]  
\[ w_t^p = \frac{\eta_t}{(\eta_t - 1) f_{z,t}} f_{1,t} \]  
\[ f_{1,t} = \Psi_t^{h_t} h_{d,t} + \theta_t \Lambda_{t,t+1} \Pi_t^{n_t} f_{1,t+1} \]  
\[ f_{2,t} = w_t^{n_t} h_{d,t} + \theta_t \Lambda_{t,t+1} \Pi_t^{n_t-1} f_{2,t+1} \]  
\[ h_t = h_{d,t} v_t^w \]  
\[ v_t^w = (1 - \theta_t)(w_t^p / w_t)^{-\eta_t} + \theta_t \Pi_t^{b_t} (w_t^p / w_{t-1})^{\eta_t} v_{t-1}^w \]  
\[ w_t^{1-\eta_t} = (1 - \theta_t)w_t^{1-\eta_t} + \theta_t \Pi_t^{b_t} w_t^{1-\eta_t} \]  
\[ y_t = \Lambda_t (k_t^{1-a} h_t^{1-a})^{1-\gamma} / v_t^p \]  
\[ k_t = \frac{\alpha}{(1 - \alpha)(1 - \gamma)} w_t \]  
\[ m_t = \frac{\gamma}{(1 - \alpha)(1 - \gamma)} w_t \]  
\[ m_{n_t} = \frac{\gamma}{(1 - \alpha)(1 - \gamma)} \lambda_t \]  
\[ \gamma \frac{w_t^{1-\alpha}(1-\gamma) R_t^{\alpha(1-\gamma)} \lambda_t^{\alpha(1-\gamma)} p_{m,t} \} \]  
\[ g_{1,t} = m_{c,t} y_t + \theta_t \Lambda_{t,t+1} \Pi_t^{e_t} g_{1,t+1} \]  
\[ g_{2,t} = y_t + \theta_t \Lambda_{t,t+1} \Pi_t^{e_t-1} g_{2,t+1} \]  
\[ \Pi_t^{e_t} = \frac{\varepsilon}{\varepsilon - 1} g_{1,t} \]  
\[ 1 = (1 - \theta_t) \Pi_t^{1-e_t} + \theta_t \Pi_t^{e_t-1} \]  
\[ v_t^p = (1 - \theta_t) \Pi_t^{1-e_t} + \theta_t \Pi_t^{e_t-1} v_{t-1}^p \]  
\[ c_t = \frac{1}{(1 - \alpha)(1 - \gamma)} w_t h_{d,t} \]  
\[ F_{u,t} = (w_t - \Psi_t^2) h_t \]  

\[ 49 \]
\[
\frac{R_t}{R} = \left( \left( \frac{R_{t-1}}{R} \right)^{\gamma} \left( \frac{\Pi_t}{\Pi} \right)^{\gamma_i} \right)^{1-\gamma} \exp(\epsilon_{R,t}) \quad (A.5.30)
\]
\[
g_t = b_t + T_t - R_{t-1} b_{t-1} / \Pi_t \quad (A.5.31)
\]
\[
T_t = \bar{T} + \phi_t (b_t - \bar{b}) \quad (A.5.32)
\]
\[
y_t = c_t + x_t + g_t + p_{m,t} m_i + p_{e,t} \epsilon_t \quad (A.5.33)
\]
\[
p_{m,t} = 1 \quad (A.5.34)
\]
\[
p_{e,t} = g(\cdot) \quad (A.5.35)
\]
\[
\hat{p}_{m,t} = p_{m,t} + p_{e,t} \quad (A.5.36)
\]
\[
\ln(d_t) = \rho_d \ln(d_{t-1}) + \epsilon_{d,t} \quad (A.5.37)
\]
\[
\ln(\nu_t) = \rho_v \ln(\nu_{t-1}) + \epsilon_{v,t} \quad (A.5.38)
\]
\[
\ln(z_t) = \rho_z \ln(z_{t-1}) + \epsilon_{z,t} \quad (A.5.39)
\]
\[
\ln(g_t) = (1 - \rho_g) \ln(\bar{g}) + \rho_g \ln(g_{t-1}) + \epsilon_{g,t} \quad (A.5.40)
\]
\[
\ln(A_t) = \rho_a \ln(A_{t-1}) + \epsilon_{a,t} \quad (A.5.41)
\]
\[
\phi_{e,t} = (1 - \rho_{\phi_e}) \phi_e + \rho_{\phi_e} \phi_{e,t-1} + \epsilon_{\phi_e,t} \quad (A.5.42)
\]

Thus the complete model comprises 42 variables: \{\lambda_{R,t}, d_t, c_{R,t}, v_t, h_{R,t}, \Pi_t, q_t, x_t, R_{k,t}, \Lambda_{t,t+1}, k_t, z_t, c_{N,t}, R_t, h_{N,t}, T_t, f_{a,t}, c_t, h_t, w_t^*, f_1, f_2, j, h_{d,t}, v_t, \nu_t, y_t, A_t, m_t, v_t^p, \hat{p}_{m,t}, m_c, g_{1,t}, g_{2,t}, \Pi^*, e_t, F_{\pi,t}, g_t, b_t, p_{m,t}, \phi_{e,t}, p_{e,t} \}.

In accordance with the environmental policy regime, \( p_{e,t} = g(\cdot) \) differs as laid out in table 1.
5.3 Determinacy of Equilibrium

As discussed by Gali et al. (2004), the presence of non-Ricardian households can cause indeterminacy and instability of equilibrium in the New Keynesian framework. In particular, indeterminacy arises if the degree of price stickiness is high and a large share of non-Ricardian households is present. We test for parameter combinations that lead to indeterminacy or instability of equilibrium by checking the Blanchard-Kahn conditions. For the main specification of the model, the parameter combinations for the share of non-Ricardian households $\lambda$ and the degree of price stickiness $\theta_p$ that lead to indeterminacy are plotted in figure 12.

![Indeterminacy of Equilibrium](image)

Figure 12: Non-Ricardian households, price stickiness, and indeterminacy. The results are based on the parameter values of table 2.

As can be inferred when the share of non-Ricardian households is 1, all equilibria are unstable. At a share of non-Ricardian households below 80%, the equilibrium is uniquely determined for values of $\theta_p$ between 0 and 0.7. At degrees of price stickiness of $\theta_p > 0.8$, indeterminacy arises for a share of non-Ricardian households above 60%. Furthermore, for $0.2 < \theta_p < 0.7$ and $\lambda > 0.8$, instability of equilibrium can also occur. However, for the baseline specification with $\theta_p = 0.86$ and $\lambda = 0.28$, the equilibrium is uniquely determinant.
5.4 Sensitivity Flexible Price

To demonstrate how the choice of the intensity parameter $\eta_e$ in the flexible price regime affects the adjustment dynamics of the model, the following figure shows the impulse responses to a TFP shock for $\eta_e = [0, 1, 50, 100]$. As can be inferred from the figure, qualitatively, the dynamics remain unchanged. At higher values of $\eta_e$, the dynamics of the emissions price get amplified. As a consequence, emissions dynamics are less pronounced. The size of the reaction parameter has similar effects with respect to other shocks.

Figure 13: Impulse responses to a TFP shock with a flexible price instrument at different values of $\eta_e$. The simulations are based on the parameter values of table 2.
5.5 Additional IRFs

Figure 14: Impulse responses of Non-Ricardian and Ricardian consumption to an emission intensity shock. The simulations are based on the parameter values of table 2.