Sustainable Finance with Green Bonds for a Low Carbon Economy

Joao Braga*, Andreas Lichtenberger†, and Willi Semmler‡

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Abstract

Climate change imposes big challenges and demands an active fiscal and financial policy response. Climate disasters and global warming can move economies onto a lower-growth path with a rise of 'stranded assets' and financial instability. Concerning fiscal and financial policy, we build on a recent World Bank report "Fiscal policies for a low-carbon economy" (see Semmler et al., 2021) and focus on the role of green bonds as an essential pillar for sustainable finance. Though fiscal policy stressing carbon pricing can incentivize a transition to low-carbon economy, green bonds play a complementary role by providing bridge finance and supporting the transition to a zero net carbon emission. However, the financial policy depends on attracting investors and removing financial market roadblocks, such as financial market short-termism. Many recent studies have focused on yield differential between green and conventional bonds. We focus on both yields and volatility and thus on the risk-return performance of the two types of bonds. Using cross-sectional methods, harmonic estimations, bond pairing estimations as well as regression tree methodology, we find that reasonable returns and lower volatility of green bonds deliver superior Sharpe Ratios (risk adjusted returns), protect investors from oil price and business cycle fluctuations, and stabilize portfolio returns and volatility. We demonstrate that, in the long-run, the positive externalities of green bonds benefit the economy and the investors even if these assets have currently lower yields. In contrast, conventional bonds and fossil fuel based assets exhibit lower risk adjusted returns because of higher volatility and the (though slow) perception that they entail long-run negative externalities. We use a dynamic portfolio approach to obtain model-driven results and evaluate those through our empirical evidence.

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*The New School for Social Research
†The New School for Social Research
‡The New School for Social Research, University of Bielefeld, and IIASA

1 Introduction

Sustainable economic growth entails changes in production, consumption and therefore also in the form of financing "green investment". Since financial markets can appear as a roadblock[1] but can also be used as a bridge for the transition to a low carbon economy it is important to understand investment decisions and the financial performance regarding green assets as compared to conventional assets. We want to study the portfolio decisions with respect to green and non-green (brown, i.e. fossil fuel based, or conventional, i.e. plain vanilla) bonds in the context of a dynamic portfolio model. Focusing on its role of a bridge we explore whether the gradual replacement of fossil fuel investments with green investments can reduce negative externalities, create positive externalities, and improve wealth accumulation. The theoretical part of this paper provides a generic model of asset pricing and dynamic portfolio decisions concerning the shift from brown to green investments by including positive and negative economic externalities. In the empirical part, we focus on the performance of bond financed green investments to analyze the differential bond performance of green and non-green bonds.

Though market decisions and market mechanisms are known to have positive or negative external effects since long, which have been demonstrated to exist in microeconomics, but macroeconomic versions of this became more popular with the endogenous growth theory. Economic investments can have positive feedback effects impacting output through some scale effects, for example through some increasing returns to scale or Romer type of inventive investments, see Greiner et al. (2005). Negative externality effects have also been studied in macroeconomic growth theory.[2].

As to our knowledge, what has been however not studied sufficiently are the positive and negative feedback effects, arising on the real side of the economy, impacting asset pricing, financial returns and portfolio decisions of investors. There is now some recent work on how negative externalities arising from GHG emission and subsequent damage and disaster risks impact asset value and returns. Engle et al. (2020) study the impact of climate disaster news on asset risks and portfolio decisions concerning equity portfolios of green and fossil fuel based equity, in an extended Markowitz portfolio model. There are also studies on the green and conventional (and fossil fuel based) bonds and how those asset returns and volatility are impacted by disaster or fuel price shocks.[3]

In the current paper we first present a dynamic asset price model studying a generic link between the real economy with externalities, asset pricing and dynamic portfolio decisions, resembling the Merton (1974) work on dynamic portfolio decision.[4] We then more specifically introduce green and fossil fuel based bonds into the dynamic portfolio framework, using some stylized movements of returns obtained from Fast fourier transform (FFT) and harmonic estimations of actual data from the US economy.[5] Hereby the positive and negative externalities are introduced and their asset price and portfolio allocation effects are studied and the fate of the evolution of wealth explored.

In the empirical section of the paper, we study in more detail, using econometric methods, the drivers for green and conventional bonds performance. We, in particular, use multi variate regressions, regression tree models, and pairing algorithm for studying the differential performance of green and conventional bonds. We also explore subsets of the energy sector such as green renewable energy and fossil fuel bonds. We analyze primary and secondary market returns (using yield at issue and yield to maturity respectively), various volatility measures (over ranges of 30 days, 90 days and 260 days) and risk-adjusted returns (using the Sharpe ratio measure).

The remainder of the paper is structured as follows. Section 2 lists our theoretical hypothesis and discusses the empirical literature. In section 3 the background of our model is described and the dynamic portfolio model is presented. In section 4 we report the results from our numerical analysis, using the NMPC algorithm.[6] Section 5 presents our empirical methodology, data background and our empirical results. Section 6 concludes the paper.

[1]For example short-termism in the financial market has been shown to be a roadblock, see Davies et al. (2014), Semmler et al. (2000).

[2]See Barro and Sala-I- Martin (2004)

[3]For a survey, see Semmler et al. (2021).

[4]See also Chiarella et al. (2016).

[5]As Chiarella et al. (2016) and Semmler & Hsiao (2011).

[6]For details of our solution algorithm used, see Gruene et al. (2015).

2 Theoretical hypotheses and empirical literature

2.1 Theoretical hypotheses

By integrating positive and negative externalities, that one knows from growth theory, into dynamic asset price and portfolio theory, one would expect higher returns on green bonds, since some positive long run externalities are to be expected. In contrast, one would expect lower returns in the long run for conventional (or fossil fuel based) bonds, since, following the Pigou argument that the negative externalities have to be paid in some way, we would expect a lower return.

Yet, what we find is that the empirical results are mixed, partly indicating that green bonds show a better performance, in terms of volatility and the Sharpe ratio, but not necessarily in terms of returns. With respect to returns there is often found a (negative) risk premium. The issue is then where do the extra positive effects (increase of yields) for green bonds come in? On the other hand, one would expect a lower return for conventional (or fossil fuel bonds) if the externalities are properly built into bond pricing – through a carbon tax, anticipation of disaster risks, and disclosure requirements through investments undertaken by well informed agents in the financial market.

Given those model driven predictions, the major question we want to pursue in our empirical study is whether the predictions from theory hold in a visible way in the empirics. We argue that the theoretical predictions may be clarified better by studying not only the returns, but the return-risk ratio, the Sharpe ratio. As to the returns, there are puzzling results that are presumably arising from the fact that the positive and negative externality effects are currently rarely considered in the actual trading and financial decisions in the financial markets.

We also add some consideration on convertible green bonds, an innovative financial instrument that seems to help to achieve sustainable debt. Convertible bonds have recently been issued with green labels as an alternative to conventional fixed income bonds, given the surge of convertible instruments after the COVID-19 crisis (Gregory, 2020).

2.2 Review of empirical literature

As to the return differentials, a number of recent studies on the performance of green bonds have been published, with mixed results and analysis techniques. Most studies find a negative green premium based on bond indices (Ehlers & Packer, 2017) and primary market yields (Kapraun & Scheins, 2019; Immel et al., 2020; Löffler et al., 2021). For secondary market yields, studies find mixed results for green and conventional bond yield differentials (Kapraun & Scheins, 2019; Bachelet et al., 2019), which means that a premium is found only for specific cases (e.g.: institutional and certified green issuers). It is argued that lower yields for green bonds compared to conventional bonds are due pro-environmental attitude of investors (Löffler et al., 2021) and higher ESG credibility of certain issuers, which impacts the demand preferences (Kapraun & Scheins, 2019). As laid out in this paper, the risk structure of financial assets is also impacted by environmental factors: fossil fuel bonds evoke negative environmental externalities while green bonds are environmentally friendly, i.e. should show positive externalities.

In addition to using multivariate regression for primary and secondary market yields of green and conventional bonds and like many other papers (Ehlers & Packer, 2017; Kapraun & Scheins, 2019; Löffler et al., 2021; Bachelet et al., 2019) we analyze bond performance also with regard to risk-adjusted returns (Sharpe ratio) and several bond volatility measures (30d, 90d, 260d). We also deploy a bond pairing algorithm to create a sample of matched green and conventional bonds.

Kapraun & Scheins (2019) analyze the yield performance (primary and secondary market yields) of green and conventional bonds in various regression designs. For the primary market, they evaluate the "yield at issue" in: (a) a fixed effects regression where they use the whole data sample and control for issuer specific effects, year-month fixed effects, currency fixed effects, seniority, maturity, issue size, issue country, yield curve and different interest rate environments; and (b) a fixed effects regression setup for subset of data (e.g.: looking for currency specific effects). For the secondary market, they analyze the "yield to maturity" in: (a) a similar fixed effects regression setup as in the case of the primary market yields, without the rating fixed effect, but adding a control for bond liquidity (using the bid-ask spread), and (b) a regression analysis of matched bonds where one green bond is paired with up to 10 comparable conventional bonds. In the paired bond analysis they control for coupon rate, maturity, issue size, green bonds traded at a green exchange and ESG rating.

Kapraun & Scheins (2019) find that the green yield premia is: in the primary market, on average, 18 bps lower than a conventional bond premia; and, in the secondary market, negative only for green bonds supplied by issuers with a better sustainability reputation, such as multilateral organizations, governments, or other bonds traded at green exchange markets as well as in countries with established environmental policies. They also report a high variation of premia across currencies and issuer types. Kapraun & Scheins (2019) do control for public vs corporate investors, however they do not control for different corporate bond issuing sectors (e.g. energy, finance, utilities) and their analysis does not deal with bond price volatility.

Löffler et al. (2021) analyze the conventional and green bond performance and find that the primary as well as the secondary market yield for green bonds is, on average, 15–20 bps lower than for conventional bonds and that the ask yield volatility of matched green bonds is higher than that of matched conventional bonds. The latter finding motivates their claim that the negative green premium results in a preference for buying green labeled assets. Their study uses two different bond pairing approaches, propensity score matching (PSM) and coarsened exact matching (CEM) methodology, to determine a sample of conventional bonds that is most similar to the sample of green bonds. However, compared to our analysis they do not control for bond rating and amount issued and their conclusions on bond volatility relies on a single volatility measure, whereas our analysis includes three different volatility measures.

Bachelet et al. (2019) analyze a set of 89 paired green and conventional bonds from 2013 until 2017. Their matching criteria are based on issuer, currency, rating, amount issued, coupon rate, maturity date, and coupon type. They analyze green versus conventional bonds with regard to differences in the bond premium (only for secondary markets), the liquidity, and the bond price volatility (also based on secondary bond market prices). Similar to Kapraun & Scheins (2019), they differentiate between private and institutional bond issuers and find that the yield differential of green minus conventional bonds is positive and about 2-3 bps for private issuers and negative, between -1.9 bps and -9.6 bps, for institutional issuers. For private issuers without a green label, the green premium is even bigger, in the range of 3.2 bps and 11.2 bps. This difference also holds for the volatility analysis: green bonds are significantly and slightly less volatile than conventional bonds in the case of established issuers, i.e. institutional and green certified private issuers. Bachelet et al. (2019) does not bring a sector-specific analysis and provides a single measure only for the volatility analysis (ex-post standard deviation of bond yields which considers a spanning period of 20 days). In contrast, our empirical analysis looks into different sectors and uses three different volatility measures with spanning periods of 30, 90, and 260 days.

Additionally, none of those papers that use a bond pairing algorithm investigate extensively the financial performance of green and brown bonds based on a risk-reward performance measure, like the Sharpe ratio (SR). In fact, Ehlers & Packer (2017) uses the Sharpe ratio to investigate the green bond performance, however they only use a cross-sectional sample of 21 green bonds issued between 2014 and 2017. Our sample includes 1529 green bonds that were issued between 2017 and 2020 and, in contrast to their bond analysis, our algorithm also controls for rating. Their study finds that the risk-adjusted performance (i.e., the Sharpe ratio) was, in some cases, slightly higher for green bond indices than for global bond indices, though that difference was not statistically significant to evaluate the bond performance of green bonds. The paper, with a CoVaR model[7], finds that green bonds improve the SR of a stock-bond portfolio in different market environments and can provide downside risk protection during the COVID-19 pandemic.

The novelty of our paper is to combine an established bond pairing procedure with new bond performance measures (the Sharpe ratio and several volatility measures in addition to primary and secondary market yield measures) to analyze a recent set of green and conventional bonds (from 2017 until 2020). We also apply a categorization and regression tree analysis which has not been applied so far in the analysis of bond performance. Our dataset is obtained through the Bloomberg terminal and considers the sectors in which most green bonds were issued globally: finance, utilities, energy, and government (i.e. we consider the following Bloomberg Industry Classification Sectors (BICS 2): banks, real estate, power generation, utilities, sovereigns, renewable energy, and development

[7]The CoVaR model looks at the impact of financial distress in one market (using the value at risk, VaR, indicator) on the VaR of another market.

banks).

Though, overall the econometric results on the differential performance of green and conventional, or fossil fuel bonds respectively, appear to be mixed (see Section 5). Next, we want to explore, by employing recent macroeconomic and dynamic portfolio theory, why the empirics is likely to show those features.

3 Background and description of the model

In terms of a macro model – as model background for the portfolio framework – we can consider some endogenous growth type macro model with externalities[8], and a Romer type of growth model with expenditure for innovations impacting economic growth rates.[9]

In terms of modeling[10] one can consider the possibility of having a fraction of accumulated wealth wealth going into consumption, and addition a fraction going into the spending of resources for new technologies. As in Romer (1986) we can assume this latter fraction being spent for human capital creating innovations. Yet, we want to let it spent for innovations either for green or fossil fuel based innovations. So this spending is again subdivided into two purposes which will be taken for reason of simplicity as fixed.

The investment of the fraction of wealth into innovation generates a time-varying return $r^e(t)$ which can generate positive or negative effects on growth in the long run. The value of an investment which does not create long-term negative externalities (such as renewable energy) can thus have positive effects on growth but also increase portfolio returns; whereas the value of investment which creates long run adverse effects to the economy through negative externalities (such as CO2 emission) will negatively affect the portfolio return and growth.

Building on dynamic portfolio models of the Merton type (Merton, 1974), the allocation paths can be assumed to evolve driven by a finite decision horizon. Further we can assume that portfolio decisions that promote green innovations are likely to yield higher asset returns than investment decisions that fund non-renewable technology. The latter induces negative environmental externalities which is usually not reflected in the asset return but maybe in the longer run having destructive effects and embody greater risk – climate risks– usually not immediately taking into account while investing. Thus, neglecting to invest in green technologies can lower returns for the portfolio in the long run due to negative externality effects and realized risks resulting from CO_2 emissions (for example, disrupted production processes due to environmental damages). Negative externalities not internalized in asset price formation will prevent green investments from easily taking off and accumulation of wealth will be lower. This is often the case as argued by Davies et al. (2016) due to short-termism by cash flow oriented investors.

Though assets from climate projects often do not promise such high returns as from fossil fuel assets, since they are still in the stage of learning and with high risk of failure, but they are usually less vulnerable to exogenous shocks and should be of interest for private long term investors. This has been shown in empirical studies comparing bonds of the same issuer and similar maturity, see Kapraun and Scheins (2019) and Section 5. A comparison of the financial performance of fossil fuel bonds and green bonds often show lower yields of the latter. Yet, the puzzle is why it can be beneficial for long run asset accumulation? Is it the case that low yield real investors (for example, in the renewable energy sector) can issue bonds purchased by financial investors that pursue the social and environment good, and thus those would be expected to add in the long run social returns? Is there a long run service that a green bond provides for the environment not accounted for in the market yield?

If this is so then we have to take account the immediate private yields and the additional social returns which would then provide a higher return for green energy than for fossil fuel energy. Yet this evaluation effect is undertaken presumable very imperfectly in the market and thus the evidence on performance green investments might be mixed. And of course the better overall performance of the green investments would come out more distinctively if at the same time fossil fuel energy is facing a carbon tax or is forced to disclose CO2 emission – which is presumably also imperfectly

[8]See Greiner et al (2005, ch. 4)

[9]See Greiner et al (2005, ch. 4)

[10]See Semmler et al. (2020)

done. Thus in theory we would expect clear results though the empirics could look mixed, see Section 5

Yet pursuing the theory nexus here first, in a model portfolio model of wealth build up we consider a fixed decision horizon and compare the possibility of the different externality impacts associated with portfolio decisions on wealth accumulation (either a positive impact due to environmental investments, or a negative impact due to fossil fuel investments and negative externalities). Though the model will first be written in terms of regular fixed-income fossil fuel and green bonds, and later we will introduce convertible green bonds, that might also be a good transitional instrument to avoid excess debt accumulation and ensure varying returns linked to green innovation success.

The model distinguishes two risky assets with fluctuating returns: a green and a fossil fuel asset. For the long-term oriented investor, these returns can be represented by low frequency movements, as in harmonic estimations.[11] By definition, savings and portfolio decisions represent low-frequency movements (Chiarella et al., 2016). This is also true for green investments, that require overcoming short-termism in financial markets (Semmler et al., 2020). We use a Fast Fourier Transform (FFT) - as in Chiarella et al. (2016, ch. 2), and Semmler & Hsiao (2011) - to empirically estimate the harmonic oscillations of the monthly annual total returns for green and fossil fuel bonds, based on two market indices provided by Bloomberg Barclays MSCI[12] and available from December/2015 to December/2020. The FFT is smoothing asset returns to obtain low-frequency movements, filtering out short-term shocks that dot not impact long-term investors' preferences (See Appendix A).

In our model, the change in prices of the bond i in the economy is $\frac{dP_{i,t}}{dt} = r^e_{i,t} \cdot P_{i,t}$. Thereby, $P_{i,t}$ depends on the time-varying return of the asset i which is $r^e_{i,t,}$ for the green or the fossil fuel asset. This is given by sine-cosine functions obtained by harmonic estimations (see eqs. 1 and 2) from US data.[13] Using such harmonic estimations we presume that financial market practitioners dynamically re-balance portfolios by looking at low frequency movements in the financial data. We get the following results for the returns of green and fossil fuel bonds:

$$\begin{aligned} r^e_{green}(t) = (&0.029\ sin(2\pi/60)t + 0.0273\ cos(2\pi/60)t - 0.0119\ sin(2\pi/20)t - 0.007\ cos(2\pi/20)t) \\ &+ 0.004\ sin(2\pi/30)t + 0.0217\ cos(2\pi/30)t - 0.0037\ sin(2\pi/15)t - 0.0004\ cos(2\pi/15)t) \end{aligned} \tag{1}$$

$$r^e_{fossilfuel}(t) = (-0.0361\ sin(2\pi/30)t + 0.0113\ cos(2\pi/30)t). \tag{2}$$

Next we presume that in a stylized baseline portfolio model an investor invests a fraction of his/her wealth to each risky asset, defining the share of green assets given by the decision variable π_t, which may also allow divestment in fossil fuels as well. Thus we can have π_t>1. The decision on the share of investment for innovations and consumption is a maximization problem with a budget constraint. The maximization of the value function is expressed using the usual dynamic discounting cash flow model:

$$V(W, x, t) \equiv max_{\{c_s, \pi_s\}} \mathbb{E}\{\int_t^T e^{-\delta_0(s-t)} F(c_s W_s, u_s W_s) ds\} \tag{3}$$

$$s.t. \dot{W}(t) = \pi_t r^e_{t_{green}} W_t + (1 - \pi_t) r^e_{t_{fossilfuel}} W_t - c_t W_t - X(\Pi_t, W_t) \tag{4}$$

The budget constraint of equation 4 is a sum of wealth gains from the different types of investment ($\pi_t r^e_{t_{green}} W_t + (1 - \pi_t) r^e_{t_{fossilfuel}} W_t$) minus a fraction of the assets going to consumption

[11] In economics, harmonic estimations can capture the business cycles in prices, industrial production, employment, and asset returns. For the potential application of harmonic estimations, see Artis et al. (2007).

[12] Respectively, the Bloomberg Barclays MSCI US Global Green Bond in USD (GBUSTRUU) and the Bloomberg Barclays US Corporate Energy in USD (I00388US).

[13] The harmonic estimations are obtained using a FFT. As described in Appendix A, we first de-trend the time series for the real asset returns and we are able to apply the FFT in order to filter short-term movements by estimating the coefficients for a linear combination of a sine-cosin function, based on the original data. The harmonic regression model is estimated for six different frequencies. We select the estimation with the lowest sum of squared errors.

$(c_t W_t)$ and minus additional adjustment costs that the investor incurs by obtaining the equity assets $(X(\Pi_t, W_t))$. The preferences in eq. (3) are defined by a log-utility function over both objectives.

This baseline model is extended by including a fraction $u_t = \frac{U_t}{W_t}$ of the investor's wealth going into innovation efforts. Such efforts can be tailored to the development of clean technology, as in the spirit of directed technical change in Acemoglu et al. (2012). Additionally to consumption, efforts to develop clean technology is added to the log-utility function, each of the spending types weighted with w_1 and $(1 - w_1)$. This change adds another decision variable to the maximization problem of the baseline model which therefore yields. Now we have 3 decision variables, the share of investment for innovations, consumption and the allocation decisions on asset holdings:

$$V(W, x, t) \equiv max_{\{u_s, c_s, \pi_s\}} \mathbb{E}\{\int_t^T e^{-\delta_0(s-t)} F(c_s W_s, u_s W_s) ds\} \tag{5}$$

$$s.t. \dot{W}(t) = \pi_t r^{e^{\text{`}}}_{t_{green}} W_t + (1 - \pi_t) r^{e^{\text{`}}}_{t_{fossilfuel}} W_t - (u_t + c_t) W_t - X(\Pi_t, W_t) \tag{6}$$

The investment of a fraction of wealth into innovation efforts, which could be investment into renewable and/or fossil fuel energy innovations, generates a time-varying return $r^{e^{\text{`}}}(t)$ which can be positively or negatively affected by the investment decision (depending if there are climate positive or negative externalities). The renewable energy oriented innovation investments additionally take into account the long-term return benefits of green investment (lower volatility of returns and higher social returns). We thus can define the two types of returns

$$r^{e^{\text{`}}}_{green}(t) = (1 + \mu(\nu u_t W_t))(\alpha + \gamma * r^{e}_{green}(t)). \tag{7}$$

$$r^{e^{\text{`}}}_{fossilfuel}(t) = (1 - \mu((1 - \nu) u_t W_t)) r^{e}_{fossilfuel}(t). \tag{8}$$

Hereby the fractions of wealth used for renewable energy innovations, ν,and the remaining part used for fossil fuel innovations will be taken as fixed with equal shares.[14]

As defined by $r^{e^{\text{`}}}_i(t)$ in eq. (7) an investment which does not create long-term negative externalities (such as renewable energy) and has positive externality effects will be positive $(+\mu(u_t W_t))$ and additional will have lower volatility of returns, and possibly higher returns in the long-run (α is a mean-adjustment term that represents the positive impact of externalities which will be varied in our simulations). In eq (8) an asset return is represented which creates long-run adverse effects on the economy caused by the creation of negative externalities (such as CO_2 emission, affecting temperature and creating damages in the long run) which will be negatively affecting returns, thus $(-\mu(u_t W_t))$.

Thus the two above return equations (7) and (8) indicate if we have the sign "+" there is an asset such as renewable energy impacted by an innovation, not creating long-run negative externalities but rather having a beneficial effect for the economy. On the other hand, if we have the sign " − " one can think of a fraction of wealth invested in fossil fuel innovations (or as fraction of population operating as fossil fuel engineers) that are creating long-run adverse effects on the economy through the creation of negative externalities, such as CO_2 emission, affecting temperature and creating damages in the long run.[15]

As mentioned, the term $\mu(\nu u_t W_t)$ and $\mu((1 - \nu) u_t W_t)$,could depend for example (as in the Romer model) on the engineers' innovation effort and the therefrom resulting returns. But in order to smooth out the transition we use in the simulation a logistic function for u_t so that we have for example for renewable energy innovations: $\mu(\nu u_t) = \mu\nu L(\beta u_t)$,

$$\mu(\nu u_t) = \mu\nu L(\beta u_t) \tag{9}$$

[14]Note that in order to avoid additional state variables defining stocks of innovation capacity we undertaken a short cut and let the respective innovations being driven by the respective fraction of wealth. Simulations could be undertaken with various fractions of v_t.

[15]We could also think of the sign " − " as indicating that the fossil fuel subsidies are reduced and thus the return on fossil fuel asset would fall.

in which μ is the externality scale factor (for example set as 0.2); βu_t is the fraction of wealth now in the logistic function, allocated to green or fossil fuel innovations. The logistic function L is here introduced to capture the possibly increasing social returns of innovation effort (Hall et al., 2010; Leibowicz, 2018 and Jones & Summers, 2020). The logistic function is given by

$$L(\beta u_t) = 130/(1 + e^{-30u_t}). \tag{10}$$

The upper bound in the logistic function is based on estimations by Jones & Summers (2020). They estimate that the social returns of an innovation expense in the health sector is from 79% to 159%.

Furthermore, the adjusted-returns for green and fossil fuel assets for the cases of α in eq. (7) equals to 0.034, 0.024 and 0.014 which are shown by Figure 1.

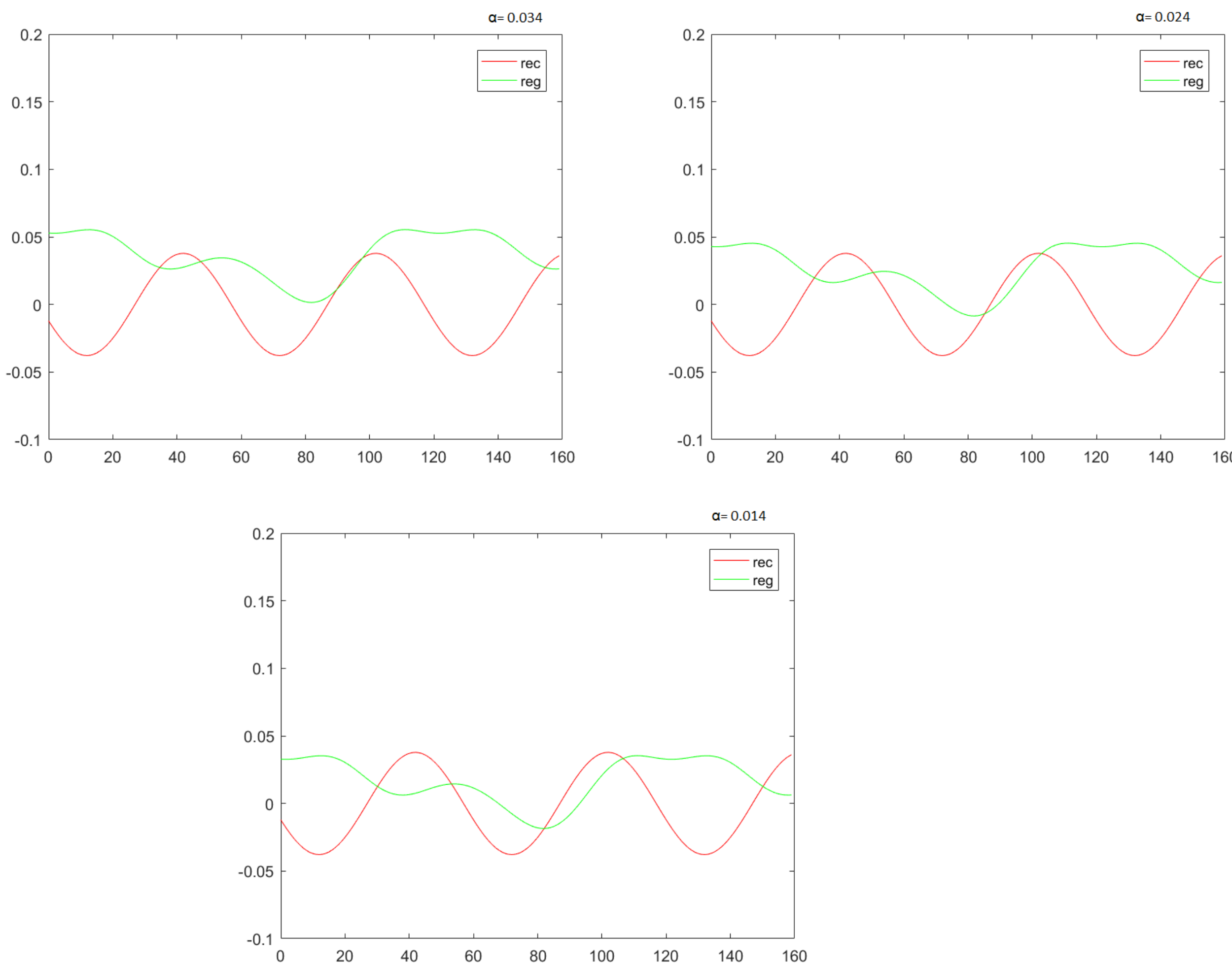

Figure 1: Returns for green (reg) and fossil fuel bonds (rec): adjusted harmonic estimations as in eqs. 7 and 8.

Using our harmonic estimations of eqs (1) and (2) the Figure 1 shows the paths of two returns of green energy bonds, *reg*, and carbon energy bonds, *rec*, for the different scaling factors. Since the actual empirical effects are not known with some certainty we explore the paths of the harmonic estimates for different size of the α.

Though assets from climate projects often do not promise such high returns as from fossil fuel assets, but they are less vulnerable to exogenous shocks and should be of interest for investors. This has been shown in empirical studies comparing bonds of the same issuer and similar maturity, see Kapraun and Scheins (2019) and Semmler et al. (2021).

Overall, we might observe the case that in fact a carbon tax could be very complementary to a green bond strategy since a higher carbon tax (or a disclosure requirements) can make the risky return turn sour. This can visibly reduce the fossil fuel returns, and can make green bonds even with negative bond premia, as reported in Kapraun and Scheins (2019), profitable in dynamic portfolios and successfully help the transition to a low carbon economy. Moreover, to indicate this possible positive externality effect, the "+" we have introduced a positive drift in the returns arising from energy innovation, the α. The negative externality effect is represented by the negative sign, the "-".[16]

The possibility to have (negative) premia for green bonds – possibly reflecting the individual preferences of green bond holders (as compared to fossil fuel bonds) – is also indicated, since the short and medium run effects could just be driven by the preferences of the asset buyers. Thus, there can be (direct) negative premia for green bonds, see Kapraun and Scheins (2019). Yet, in fact, through green investments (green energy, conservation of energy etc.) there is through the externality for the society some "extra productivity" or "service" achieved (through scale effects with freely available energy and avoidance of destruction through CO_2) which should show up in extra long-run returns in portfolio holdings, indicated by the the size of the α.

Next, numerical solutions are obtained by applying to our model the method Nonlinear Model Predictive Control (NMPC) as a solution procedure, see Gruene et al. (2015).

4 Modelling results

As noted much recent literature has pointed out that large scale fossil fuel energy firms tend to short-termism, see Davies et al. (2014). In Semmler et al (2020), it is shown that portfolio decisions arising from short-termism in terms of higher discount rates or hyperbolic discounting may inhibit the accumulation of low carbon based assets. It has been also explored the role of the decision horizon as another manifestation of short-termism.[17]

On the other hand, as climate research showed, fossil fuel bonds and fossil fuel equity are linked to negative externalities through CO_2 emission, temperature rise, weather extremes and climate disasters, requiring a carbon tax to internalize the externality cost. Fossil fuel assets might also be quite volatile in value, in particular in contractions and recessionary periods, triggering financial instability. Some countries have introduced a mandatory disclosure on traded assets. An introduced carbon tax as well the threat of financial instability and the disclosure requirements will lead to either lower net cash flows of fossil fuel firms and/or their assets will face a devaluation in the market, triggered by higher discount rates capturing the long run environmental risk involved.[18]

The above mentioned two opposite effects on the return and firm value can be illustrated by the following simulations for the dynamic system eqs. 5 - 8. The dynamic saving and asset allocation choices are modeled in continuous time. In the objective function we have included, in addition to spending a fraction c_t of wealth on consumption, a policy maker's decision to spend also a fraction u_t of the wealth on innovation efforts; efforts aimed for instance for developing new energy technologies, each of the spending types weighted with w_1 and $(1 - w_1)$ in the utility function. As noted, this is in line with the previous work of Acemoglu et al. (2012) on directed technical change. We have have further split up the innovation effort u in different fractions, one fraction for clean energy and possible one fraction for fossil fuel energy. We can then explore those modeling procedure on the time consumption wealth ratio, the time varying returns and the fate of wealth. This will allow us to observe whether wealth is increasing or decreasing over time for our two types of assets.

We want to note in the model above so far we referred to those two generic risky assets. But we can allow the two risky assets, to be a green and a fossil fuel bond that display time varying returns impacting wealth accumulation. These bonds could be long-term bonds, generating returns from some coupon payments. These bonds can also be long-term bonds, generating returns from some coupon payments. Hereby the bond prices for long term bonds can be made dependent on

[16]We also want to note that the weights ω_1 and $(1 - \omega_1)$ of the two parts of the objective functions (3) and (5) can be varied and some Pareto frontier can be computed to explore which weights are the realistic ones, see Kaya & Maurer (2014).

[17]Risk aversion and discount rates have been explored in its relevance for portfolio dynamics dynamics in (5)-(6).

[18]See Davies et al (2014) and there the discussion on stranded assets.

the expected return of the short term bonds.[19]

In this section we solve the model using NMPC for a decision horizon $N = 6$ fixed and 160 iteration time periods T for the different returns showed in Figure 1. We solve the model where the term $(1 \pm \mu(u_t W_t))$ using the logistic function, L, and where it holds for "+" , which implies that $\mu(\cdot) > 0$,for example for the new innovations in renewable energy firms. Note that there might also be some temporary risk premium harvested by fossil fuel assets so that we have $\mu(\cdot) > 0$. On the other hand, for the fossil fuel asset we might have $(1 \pm \mu(u_t W_t)) < 1$), with the minus sign " $-$ ", holding when $\mu(\cdot) < 0$.

In Figure 2 we have depicted only the case of positive externalities. For this purpose we use for example a computational parameterization of $\mu(\cdot) = 0.2$, running this with different parameterization of α. It is obvious that a $\mu(\cdot) < 0$ will always generate lower returns. with mostly fossil fuel bonds held in the portfolio facing the prospects of a carbon tax, stranded assets and downgrading through requirements of CO_2 disclosure, and higher liquidity and default risks and lower accumulation of wealth we do not need to solve for this.

Our depicted cases present only results for $\mu(\cdot) > 0$.. We use for this case, for example a computational parameterization of $\mu(\cdot) = 0.2$. In Figure 2, the lowest graph represents the effect with a parameter $\alpha = 0.014$, .the middle graph is for $\alpha = 0.024$ and the upper graph for $\alpha = 0.034$. As can be observed the greater the positive externality effects, the more wealth is accumulated over time. And even without simulations we can conclude that having negative externalities these are are likely to lead to less asset built up, but more specifically to dissipating asset value in the long run.

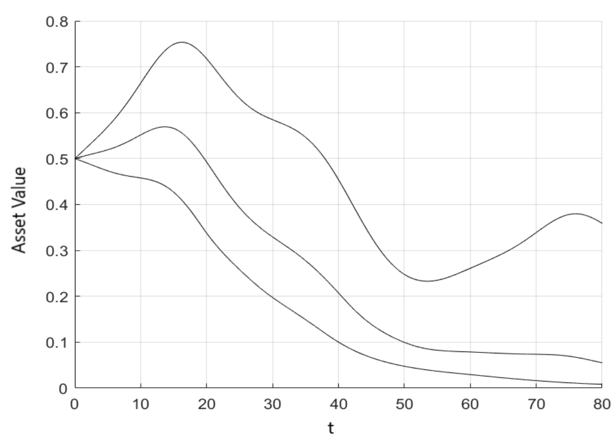

Figure 2: Solutions path of wealth for different asset returns, the upper graph with $\alpha = 0.034$, middle graphs with $\alpha = 0.024$ and lowest graph $\alpha = 0.014$ ($N = 6$, $T = 160$)

We observe that the referred effects on wealth and investor's asset value are as well associated with their portfolio composition choice. If investors hold a larger share of green bonds, with long-term positive externality effects, the negative effects of climate transition tend to be mitigated. Figure 3 shows the share of green bonds hold by investors for three cases simulated in Figure 2. We observe that, in the case with $\alpha = 0.034$, upper panel of Figure 3, which is associated with the upper solution path of wealth, investors invest only in green assets and divest fossil fuel assets, since the asset share held in green assets is $\pi = 1.2$,meaning that there is short selling for the fossil fuel asset. On the other hand, in the other two cases, investors tend to diversify and still chose (partially or totally) carbon-intensive securities in some periods.

[19]For details see Cochrane (2001, ch.19) where then the bond price is solved for solving then the appropriate discount rate forward. In the portfolio context, see Semmler (2011, ch. 17.5).

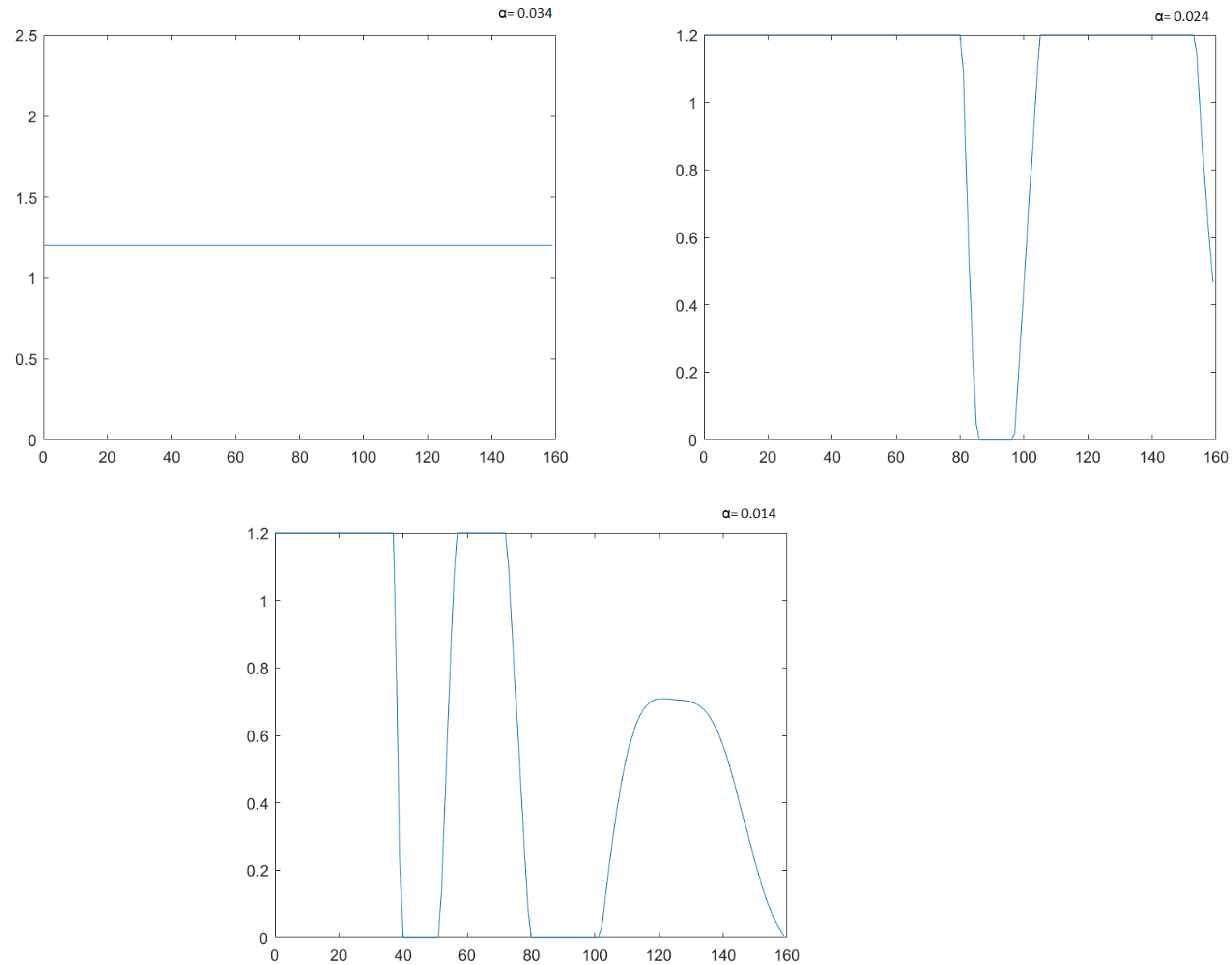

Figure 3: Portfolio decision - share of green assets for the cases with $\alpha = 0.034$, $\alpha = 0.024$, and $\alpha = 0.014$.

As mentioned in the model above when referring to green and fossil fuel assets as the two risky assets, they could also be green bonds and fossil fuel based bonds. We could also assume that we have green convertible bonds, instead of fixed-income securities. But these are bonds that can be converted into equity. The condition of convertibility to equity might be tight to some equity price per share through some strike price as the Merton model for debt suggests and Black-Scholes for derivatives in general entails. A green bond is convertible to equity if the asset value goes up, which might be the case of a successful green start-up firm. This is likely to be accelerated if the distance to default also decreases, given the firm's debt issuance, and the equity value of the firm increases.[20] Moreover if these are sovereign bonds, the conversion of sovereign bonds into firms' equity could allow the sovereign to reduce debt that it had increased first by selling bonds and increasing debt. This way debt sustainability could be achieved.

Convertible bonds have recently been issued with green labels as an alternative to conventional fixed income bonds, but still represents a small share of the market (Gregory, 2020). A convertible green bond is an innovative instrument that can address climate challenges and benefit long-term issuers and investors, with higher future returns linked to the success of green innovations. A surge in the convertible bond market was observed in 2020 following the COVID-19 crisis, with creates opportunities also for convertible green bonds (Semmler et al., 2021). In the United States, new convertible bonds totaled \$ 77 billion as of September 2020, an increase of 45 percent over

[20] It might help to support green start up firms, ensuring a minimum return if the business does not succeed but high returns if it succeeds. If we think in terms of green treasury bonds, it can help to reduce sovereign debt due to debt to equity swaps.

2019 and 200 percent over 2015. The convertible bond market index (ICE BofA US Convertible Index—VXA0) outperformed the S&P 500 and S&P 500 bond index.

5 Data, empirical analysis, and results

Next we want to explore to what extent our model-driven hypotheses can be supported by the data. Our analysis is based on green and conventional bond data downloaded from the Bloomberg terminal. Bloomberg provides a "green instrument indicator" which was our criterion for green bonds. Conventional, or plain vanilla, bonds are simply non-green bonds; in other words, bonds for which the "green instrument indicator" does not hold. The last complete version of bond data was downloaded from the Bloomberg Terminal on October 1, 2020. Due to the higher availability of conventional bond data and the restrictions on Bloomberg download limits, we restricted our analysis to a set of sectors with the highest amount of green bonds and to the period of time in which the most green bonds were issued. As we are mostly interested in comparing the performance of green and conventional bonds and since green bonds became more popular in 2015 and their issuance kicked off in 2017, our period of selection includes data from January 1, 2017, to October, 1 2020. Also, we select conventional bonds only from the sectors that showed the highest amount of green bonds: (i) the financial sector with the banking and real estate sub-sector, (ii) the utilities sector with the utilities and power generation sub sector, (iii) the government sector with the government development bank, supranational and sovereign sub-sectors, and (iv) the energy sector with the renewable energy sub-sector.

In our analysis, we measure bond performance by looking at (i) the expected return of bonds (yield at issue for the primary bond market yields and the yield to maturity for the secondary bond market), (ii) the volatility measure of bonds, which is represented by the Bloomberg volatility measure (the day to day logarithmic historical price changes, for the 30, 90 and 260 most recent trading days closing prices[21], and (iii) the bond specific Sharpe ratio (SR_b).

The bond specific Sharpe ratio is similar to the original Sharpe ratio (which we call "portfolio Sharpe ratio" SR_p) which is an information criterion of the risk-to-return measure of a portfolio, whereby the portfolio standard deviation describes the risk of a portfolio (see Sharpe, 1994). The combination of both the yield level and the variation in yields (portfolio volatility and risk) is integrated in the portfolio Sharpe ratio, which is defined as $SR_p = \frac{\bar{R}_p - R_f}{\sigma_p}$,where $\bar{R}_p$ is the average portfolio return, R_f the risk free rate, and $\sigma_p$the portfolio standard deviation. The bond specific Sharpe ratio SR_b(or SRb) is inspired by the original SR_p(or SRp), carries individual excess bond returns in the numerator and a measurement for a bond return volatility over time in the denominator and is defined as $SR_b = \frac{R_b - R_f}{v_b}$,where R_b is the individual asset return, R_f is the risk free rate, and v_b is the individual asset volatility measure. The advantage of using the bond specific over the portfolio Sharpe ratio is that we can obtain a reward risk measurement for each separate bond which can then be used in a regression analysis. For computing the SRb we use the yield to maturity rate in the numerator and different volatility measures (the 30d, 90d or 260d) in the denominator.[22]

We use four different types of empirical analysis: a base regression (analysis 1), that only uses key explanatory variables; a first extension, that adds sector controls (analysis 2); an extension to the second model, by controlling for USD as a currency (analysis 3); and an extension to the second model that controls for EUR as a currency (analysis 4). In this section, each analysis is presented in a separate subsection.

5.1 Multivariate regression

As a first step we determine the bond performance of green and conventional bonds by running multivariate regressions on three different dependent variables: yield at issue (YAI), yield to ma-

[21]We abbreviate volatility measures for these 30, 90, and 260 day ranges by writing 30d, 90d, and 260d

[22]In our empirical analysis, we compared the bond specific Sharpe ratio results with and without the risk-free rate for the two most frequent currencies (USD and EUR) and did not find a relevant change (differences in the size of the second or the third digit after the decimal). Hence, for simplicity, we do not consider the risk-free rate in our analysis when reporting results on the bond specific Sharpe ratio SRb.

turity (YTM), bond specific Sharpe ratio (SRb)[23]. Model 1 is defined in in equation 11 where X_1 is a green dummy variable (when equal to one, the bond is green), X_2 is the S&P rating (integer variable which where AAA is 1, AA+ 2 and so on), X_3 is the maturity structure (this is a dummy variable which is one for long-term bonds and zero for short-term bonds)[24], X_4 is the coupon rate[25], X_5 is the liquidity (computed as the ask minus the bid price)[26], X_6 is the amount of bonds issued in US$ divided by 10^9 (dividing by a billion gives us more similar numbers in comparison to the yield values)[27], X_7 is the debt-to-assets ratio, and X_8 is the 90-day bond price volatility rate[28].

$$Y_i = \beta_0 + \sum_{k=1}^{8} \beta_k X_{k,i} + \epsilon_i \tag{11}$$

The extension for model 2 are defined by equation 12 and for model 3 and model 4 are defined by equation 13 where $c = \{USD, EUR\}$.

$$Y_i = \beta_0 + \sum_{k=1}^{8} \beta_k X_{k,i} + \sum_{l=1}^{4} \gamma_l X_{l,i} + \epsilon_i \tag{12}$$

$$Y_{i,c} = \beta_{0,c} + \sum_{k=1}^{8} \beta_{k,c} X_{k,i,c} + \sum_{l=1}^{4} \gamma_{l,c} X_{l,i,c} + \epsilon_{i,c} \tag{13}$$

The multivariate regression of the primary market yields shows negative yields for green bonds (see Table 1). This holds for the simple model (model 1) as well as for the case that controls for sectors (model 2) including a control for USD (model 3). Only in the EUR specific model we don't find a significant effect for green bonds.

[23] YAI values showed few outliers and only observations above the 99th percentile were truncated. In terms of YTM observations values below the 2.5th percentile and above the 97.5th percentile were truncated. The SRb was calculated based on the filtered YTM and volatility values and outliers below the 2.5th percentile and above the 97.5th percentile were truncated.

[24] We define long-term bonds similar to Kapraun & Scheins (2019) who categorize them as bonds with a maturity structure of more than ten years (or see Kenny, 2021: https://www.thebalance.com/choosing-bond-fund-term-416948). Our non-long-term bonds include bonds with a maturity structure of 10 years or less and subsume short term and intermediate-term bonds. In order to reduce the influence of outliers we excluded observations with a maturity structure of more than 100 years.

[25] In order to reduce the impact of outliers coupon rate observations below the 1st and above the 99th percentile were truncated.

[26] In order to reduce the impact of outliers bid-ask spread observations below the 1st and above the 99th percentile were truncated.

[27] In order to reduce the impact of outliers amount issued observations above the 97.5th percentile were truncated

[28] We carried out all analyses with all three different volatility measures but for the matter of simplicity (and unless specified, as in the case of analysis 4 on the energy sector) we only report regressions with the 90 day volatility measure. Results for the different volatility measures are similar and are available upon request. In order to reduce the impact of outliers the 90d volatility observations above the 95th percentile were truncated.

	Dependent variable: Yield at issue			
	(1) Base model	(2) Sector model	(3) USD model	(4) EUR model
(Intercept)	0.283***	0.262***	0.572***	−0.102
	(0.047)	(0.045)	(0.088)	(0.072)
typegreen	−0.119***	−0.106***	−0.133***	0.024
	(0.024)	(0.026)	(0.034)	(0.066)
duration.2catlong	0.252***	0.297***	0.377***	0.257*
	(0.043)	(0.048)	(0.055)	(0.099)
snp_rating_num	0.001	0.016***	0.051***	0.028*
	(0.002)	(0.003)	(0.009)	(0.012)
cpn	0.930***	0.920***	0.805***	0.890***
	(0.011)	(0.013)	(0.031)	(0.058)
amount_issued.G	−0.049**	−0.043*	−0.159***	−0.046
	(0.017)	(0.017)	(0.035)	(0.061)
debt_to_assets	0.001	−0.002***	−0.001**	0.001
	(0.001)	(0.000)	(0.000)	(0.001)
bics_level_2Energy		0.031	0.003	0.154
		(0.020)	(0.021)	(0.134)
bics_level_2Utilities		−0.106***	−0.220***	−0.096
		(0.021)	(0.035)	(0.067)
bics_level_2Government		0.308***	0.304**	0.160
		(0.078)	(0.094)	(0.112)
R^2	0.882	0.884	0.900	0.943
Adj. R^2	0.882	0.884	0.900	0.939
Num. obs.	2969	2969	1794	144

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 1: Analysis 1a - Multivariate regression yield at issue

The results for the secondary yield regressions are similar: there is no evidence for positive yield premia of green bonds (see Table 2). There is a strong significance of negative yields in the EUR model case and weak evidence in the case of the second model that controls for different sectors but not for currencies.

	Dependent variable: Yield to maturity			
	(1) Base model	(2) Sector model	(3) USD model	(4) EUR model
(Intercept)	-1.236^{***}	-1.342^{***}	-1.496^{***}	-0.829^{***}
	(0.086)	(0.087)	(0.128)	(0.091)
typegreen	-0.119^{*}	-0.160^{**}	-0.133	-0.168^{**}
	(0.054)	(0.052)	(0.097)	(0.052)
liquidity.o	-0.530^{***}	-0.366^{***}	-0.549^{***}	-0.275^{**}
	(0.084)	(0.080)	(0.096)	(0.099)
snp_rating_num	0.154^{***}	0.191^{***}	0.258^{***}	0.108^{***}
	(0.008)	(0.009)	(0.016)	(0.007)
duration.2catlong	0.013	0.046	0.235	-0.022
	(0.069)	(0.067)	(0.123)	(0.088)
cpn	0.596^{***}	0.566^{***}	0.447^{***}	0.771^{***}
	(0.017)	(0.018)	(0.034)	(0.061)
amount_issued.G	-0.524^{***}	-0.572^{***}	-0.682^{***}	-0.361^{***}
	(0.035)	(0.036)	(0.056)	(0.031)
debt_to_assets	0.007^{***}	0.004^{***}	0.004^{**}	0.003^{**}
	(0.001)	(0.001)	(0.001)	(0.001)
volat_90d	0.126^{***}	0.121^{***}	0.153^{***}	0.016^{*}
	(0.010)	(0.010)	(0.013)	(0.008)
bics_level_2Energy		0.250^{***}	0.204^{*}	0.415^{*}
		(0.068)	(0.085)	(0.168)
bics_level_2Utilities		-0.470^{***}	-0.472^{***}	-0.443^{***}
		(0.041)	(0.058)	(0.049)
bics_level_2Government		0.692^{***}	1.111^{***}	0.184^{*}
		(0.075)	(0.122)	(0.075)
factor(start_quart)2017 Q2	-0.011	0.060	0.106	-0.050
	(0.089)	(0.087)	(0.133)	(0.069)
factor(start_quart)2017 Q3	0.127	0.135	0.151	-0.023
	(0.082)	(0.080)	(0.122)	(0.068)
factor(start_quart)2017 Q4	0.140	0.110	0.053	0.061
	(0.083)	(0.080)	(0.122)	(0.096)
factor(start_quart)2018 Q1	-0.024	-0.019	-0.093	0.005
	(0.071)	(0.069)	(0.110)	(0.062)
factor(start_quart)2018 Q2	-0.057	-0.018	-0.128	0.074
	(0.074)	(0.071)	(0.111)	(0.070)
factor(start_quart)2018 Q3	-0.158^{*}	-0.148^{*}	-0.281^{**}	0.005
	(0.069)	(0.066)	(0.101)	(0.065)
factor(start_quart)2018 Q4	-0.264^{***}	-0.296^{***}	-0.309^{*}	-0.074
	(0.080)	(0.079)	(0.121)	(0.073)
factor(start_quart)2019 Q1	-0.186^{**}	-0.163^{*}	-0.157	-0.094
	(0.070)	(0.068)	(0.101)	(0.067)
factor(start_quart)2019 Q2	0.135	0.115	0.179	0.142
	(0.074)	(0.072)	(0.111)	(0.074)
factor(start_quart)2019 Q3	0.054	0.061	0.031	0.415^{***}
	(0.094)	(0.091)	(0.138)	(0.091)
factor(start_quart)2019 Q4	0.139	0.127	0.134	0.372^{***}
	(0.089)	(0.086)	(0.133)	(0.087)
factor(start_quart)2020 Q1	0.928^{***}	0.873^{***}	0.264	0.295
	(0.261)	(0.246)	(0.245)	(0.210)
factor(start_quart)2020 Q2	0.477^{***}	0.427^{***}	-0.257	0.173
	(0.118)	(0.111)	(0.290)	(0.112)
R^2	0.755	0.772	0.771	0.690
Adj. R^2	0.753	0.771	0.769	0.685
Num. obs.	4454	4454	2433	1739

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 2: Analysis 1b - Multivariate regression for yield to maturity

And the regression results for the SRb regression in table 3 show a positive impact green bonds the bond specific Sharpe ratios.

	Dependent variable: Bond specific Sharpe ratio (SRb)			
	(1) Base model	(2) Sector model	(3) USD model	(4) EUR model
(Intercept)	0.727^{***}	0.594^{***}	0.851^{***}	0.135
	(0.063)	(0.055)	(0.078)	(0.070)
typegreen	0.603^{**}	0.535^{**}	1.189^{*}	0.004
	(0.215)	(0.204)	(0.537)	(0.029)
liquidity.o	-0.519^{***}	-0.441^{***}	-0.405^{***}	-0.489^{***}
	(0.050)	(0.043)	(0.058)	(0.053)
duration.2catlong	-0.105^{***}	-0.119^{***}	-0.002	0.078^{*}
	(0.025)	(0.026)	(0.038)	(0.034)
snp_rating_num	-0.033^{***}	-0.004	-0.005	0.040^{***}
	(0.010)	(0.007)	(0.011)	(0.004)
cpn	0.161^{***}	0.152^{***}	0.087^{***}	0.183^{***}
	(0.018)	(0.017)	(0.024)	(0.021)
amount_issued.G	-0.270^{***}	-0.289^{***}	-0.443^{***}	-0.227^{***}
	(0.038)	(0.042)	(0.083)	(0.024)
debt_to_assets	0.000	-0.002^{***}	-0.000	0.001
	(0.000)	(0.000)	(0.001)	(0.000)
factor(start_quart)2017 Q2	-0.011	0.002	0.052	-0.024
	(0.048)	(0.047)	(0.059)	(0.076)
factor(start_quart)2017 Q3	-0.036	-0.028	-0.060	-0.028
	(0.046)	(0.045)	(0.055)	(0.076)
factor(start_quart)2017 Q4	0.016	-0.003	-0.021	-0.019
	(0.051)	(0.050)	(0.065)	(0.069)
factor(start_quart)2018 Q1	-0.085	-0.082	-0.038	-0.055
	(0.044)	(0.043)	(0.056)	(0.059)
factor(start_quart)2018 Q2	-0.118^{**}	-0.103^{*}	-0.099	-0.050
	(0.042)	(0.041)	(0.051)	(0.063)
factor(start_quart)2018 Q3	-0.161^{***}	-0.145^{***}	-0.123^{*}	-0.092
	(0.042)	(0.040)	(0.055)	(0.062)
factor(start_quart)2018 Q4	0.138	0.128	0.294	-0.081
	(0.134)	(0.131)	(0.206)	(0.073)
factor(start_quart)2019 Q1	-0.028	-0.029	0.036	-0.071
	(0.067)	(0.067)	(0.089)	(0.062)
factor(start_quart)2019 Q2	-0.114^{**}	-0.127^{**}	-0.084	-0.137^{*}
	(0.043)	(0.041)	(0.056)	(0.059)
factor(start_quart)2019 Q3	-0.190^{***}	-0.184^{***}	-0.186^{**}	-0.017
	(0.049)	(0.049)	(0.062)	(0.064)
factor(start_quart)2019 Q4	-0.154^{*}	-0.155^{*}	-0.361^{*}	-0.080
	(0.070)	(0.069)	(0.146)	(0.065)
bics_level_2Energy		-0.132^{***}	-0.165^{***}	0.160^{*}
		(0.026)	(0.035)	(0.067)
bics_level_2Utilities		-0.100^{***}	-0.202^{***}	-0.116^{***}
		(0.022)	(0.031)	(0.029)
bics_level_2Government		0.512^{***}	0.949^{***}	0.198^{***}
		(0.089)	(0.213)	(0.059)
R^2	0.115	0.133	0.163	0.255
Adj. R^2	0.111	0.129	0.155	0.245
Num. obs.	4028	4028	2357	1532

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 3: Analysis 1c - Multivariate regression for bond specific Sharpe ratio (SRb)

The results of tables 1, 2, and 3 are also visualized by coefplots in Figures 4, 5, and 6, respectively.

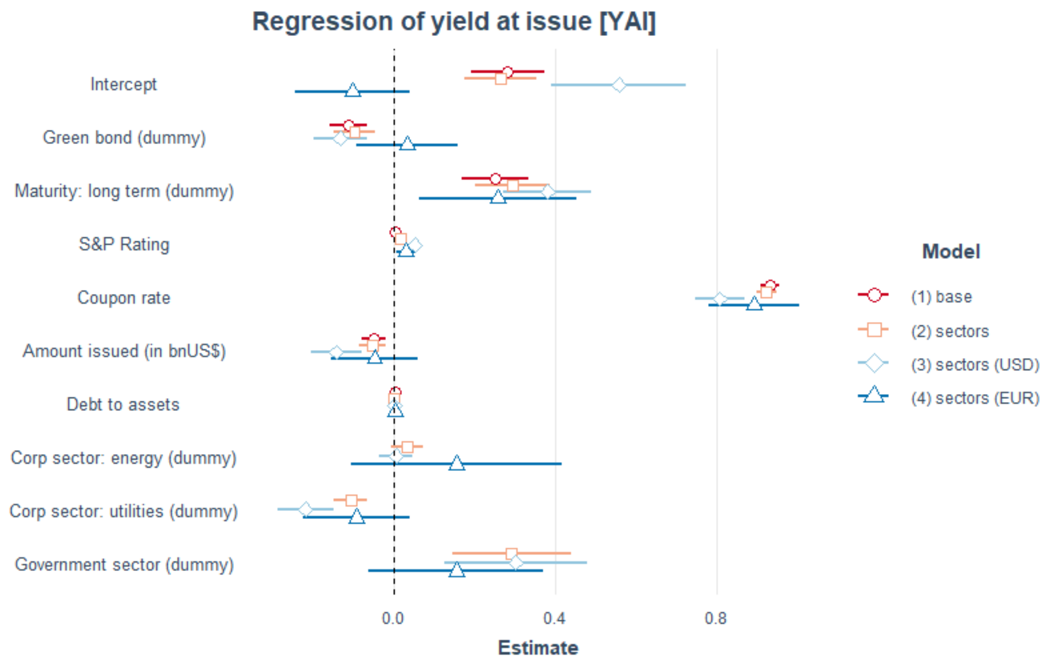

Figure 4: Analysis 1a - Coefplot yield at issue

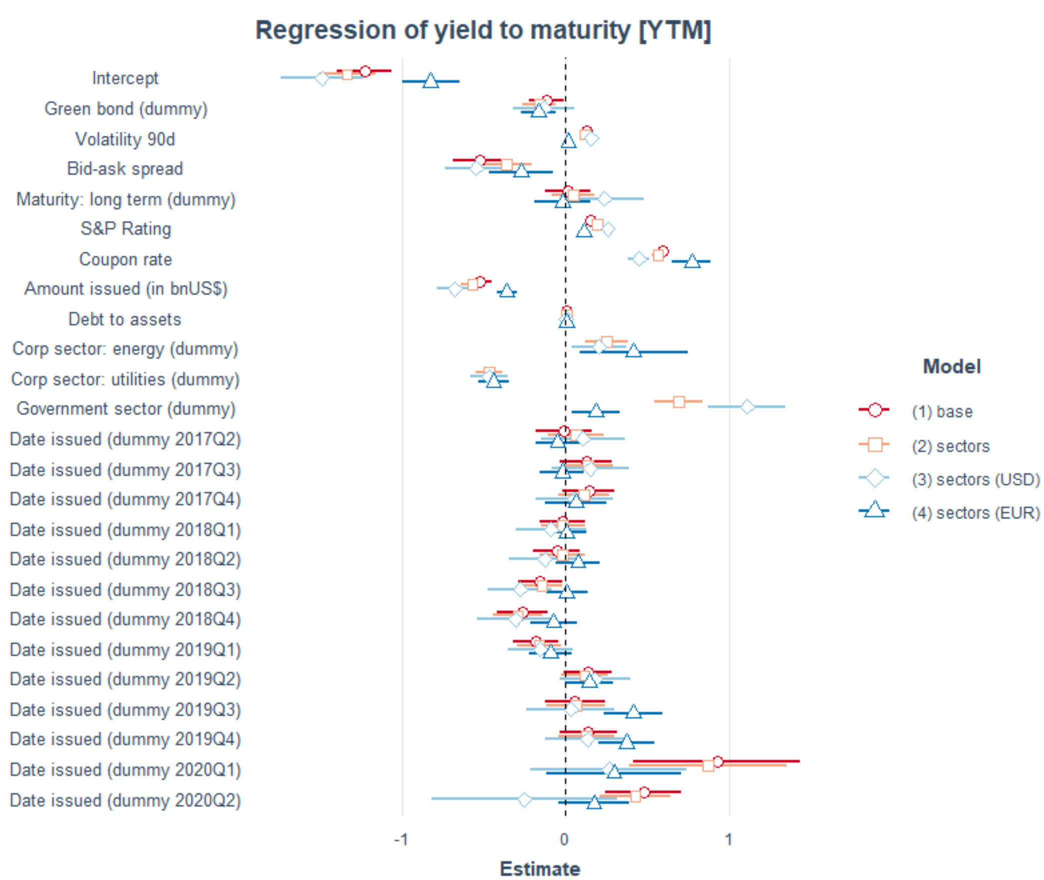

Figure 5: Analysis 1b - Coefplot yield to maturity

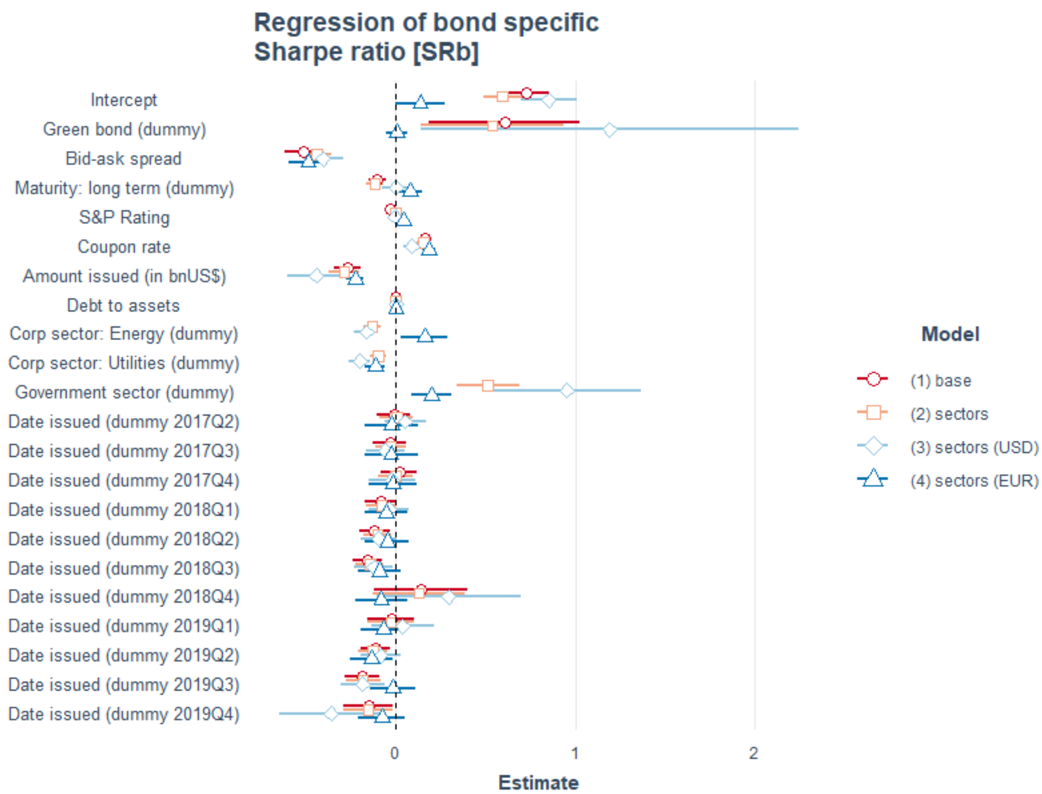

Figure 6: Analysis 1c - Coefplot yield to maturity

5.2 Pairing analysis

To control for issuer-specific effects, we compute a bond pairing algorithm that matches each green bond with a conventional bond. First, we select pairs of similar green and conventional bonds and evaluate the yield differences (green bond returns minus conventional bond returns) on the primary market (yield at issue) and secondary markets (yield to maturity). The aggregate results of our paired sub-set of data confirm the findings of Kapraun & Scheins (2019) which is a negative green premium for yield at issue rates but a positive green premium for yield to maturity rates in comparison to conventional bonds. Additionally, we see higher bond-specific Sharpe ratios for green bonds than for their conventional bond pairings.

Our bond pairing procedure selected green and conventional bonds with the same issuer (and therefore the same sector), currency, maturity, and S&P rating. With this procedure, we create a subset of data that consists of 1,022 paired observations (511 green and 511 conventional bonds). Any green bond of this subset is required to match with a conventional bond based on these five criteria. In many cases, these conditions resulted in more than one conventional matching partners for a green bond. In these cases, we allowed for maximum of 10 conventional bonds for each green bond. The closest matching candidates are identified based on a kNN algorithm that looks for similar coupon rate values. This pairing procedure ensured that similar assets were compared.

We take the green minus conventional (GMC) bond yields and find a positive difference (or "greenium") for issued bonds on the secondary market (higher yield to maturity when comparing green bonds to conventional bonds; see Table 5). For the yield at issue regression, we mostly find no significant effect of the green dummy, except for for the base regression case where a weakly positive yield difference is reported (see Table 4). However, due to the lack of observations, it was not possible to control for all sectors or add a model for EUR bonds, which means that the outcome of the regression should not be over interpreted.

Similar to the non-paired regression we also find a positive impact of green bonds on the bond specific Sharpe ratio, i.e. the yield to maturity rates discounted by asset-specific volatility measures. As Table 7 shows the differences of green minus conventional bond SRb are positive and significant in the case of models (1)-(3) and range from 1.79 to 1.54; only in the case of EUR bonds we do not find a significant positive effect.

Comparing the results from the YTM regression (Table 5) and the SRb regression (Table 7) we see that the positive difference for paired green bonds increases when we add volatility to the indicator. This means that the conventional bond yields that are discounted by their bond-specific volatility measure show a weaker performance than their green pairs. Thus, even if we look at a subset of data with similarly paired bonds, we find evidence that green bonds show lower volatility and are also able to reward the investor with a better Sharpe ratio.

	Dependent variable: Yield at issue		
	(1) Base model	(2) Sector model	(3) USD model
Constant	0.565^{*}	0.157	0.396
	(0.232)	(0.885)	(0.840)
GMC Amount issued	0.066	0.295	0.261
	(0.320)	(1.187)	(1.084)
GMC Time to maturity	-0.154^{***}	-0.145	-0.176
	(0.035)	(0.112)	(0.107)
Rating (numeric)	0.032	0.002	0.004
	(0.016)	(0.015)	(0.015)
Government Sector (dummy)		0.089	0.172
		(0.423)	(0.402)
R^2	0.200	0.282	0.346
Adj. R^2	0.178	0.218	0.284
Num. obs.	115	50	47

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 4: Analysis 2a - Paired bond regression results for yield at issue

	Dependent variable: Yield to maturity			
	(1) Base model	(2) Sector model	(3) USD model	(4) EUR model
Constant	0.080	0.428^{***}	0.792^{***}	0.080
	(0.080)	(0.084)	(0.199)	(0.081)
GMC Volatility 90d	-0.028	-0.010	0.115	-0.092^{*}
	(0.050)	(0.048)	(0.111)	(0.043)
GMC Ask-Bid Spread	-0.465^{*}	-0.400^{*}	-0.353	0.124
	(0.183)	(0.176)	(0.225)	(0.254)
GMC Coupon rate	0.262^{***}	0.246^{***}	0.284^{***}	0.078
	(0.065)	(0.068)	(0.073)	(0.050)
GMC Amount issued	-0.627^{***}	-0.699^{***}	-0.979^{***}	-0.276^{***}
	(0.103)	(0.101)	(0.140)	(0.039)
GMC Time to maturity	-0.029^{*}	-0.077^{***}	-0.117^{***}	-0.027^{*}
	(0.012)	(0.013)	(0.020)	(0.013)
Rating (numeric)	0.089^{***}	0.086^{***}	0.048	0.089^{**}
	(0.022)	(0.019)	(0.040)	(0.031)
Corp Sector: Energy (dummy)		0.187	-0.663^{**}	0.255^{***}
		(0.163)	(0.212)	(0.060)
Corp Sector: Utilities (dummy)		0.344^{***}	0.520^{*}	0.164^{**}
		(0.081)	(0.256)	(0.058)
Government Sector (dummy)		-0.396^{***}	-0.593^{**}	-0.449^{***}
		(0.071)	(0.196)	(0.087)
R^2	0.518	0.575	0.736	0.521
Adj. R^2	0.508	0.562	0.714	0.495
Num. obs.	304	304	116	176

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 5: Analysis 2b - Paired bond regression results for yield to maturity

	Dependent variable: Bond specific Sharpe ratio (SRb)			
	(1) Base model	(2) Sector model	(3) USD model	(4) EUR model
Constant	2.383^{***}	1.787^{**}	2.454^{*}	-0.153
	(0.620)	(0.644)	(1.154)	(0.115)
GMC Ask-Bid Spread	-2.745^{***}	-2.224^{**}	-1.207	-0.059
	(0.806)	(0.736)	(0.998)	(0.073)
GMC Coupon rate	1.790^{***}	1.907^{***}	2.023^{***}	-0.007
	(0.481)	(0.501)	(0.561)	(0.023)
GMC Amount issued	-5.240^{***}	-5.257^{***}	-7.196^{***}	-0.096^{**}
	(0.985)	(1.002)	(1.161)	(0.036)
GMC Time to maturity	0.012	0.023	0.023	0.013^{*}
	(0.035)	(0.035)	(0.040)	(0.006)
Rating (numeric)	-0.301^{***}	-0.291^{**}	-0.422^{**}	0.011
	(0.084)	(0.091)	(0.141)	(0.014)
Corp Sector: Energy (dummy)		2.046^{**}	-0.248	0.093^{**}
		(0.786)	(0.755)	(0.034)
Corp Sector: Utilities (dummy)		1.434^{**}	1.800	0.052
		(0.479)	(1.000)	(0.029)
Government Sector (dummy)		0.755	-0.112	0.209^{***}
		(0.568)	(0.974)	(0.057)
R^2	0.536	0.549	0.664	0.246
Adj. R^2	0.525	0.531	0.638	0.165
Num. obs.	209	209	113	84

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Figure 7: Analysis 2c - Paired bond regression results for bond specific Sharpe ratio (SRb)

5.3 Volatility analysis

We use the classification and regression tree (CART) method to identify the most essential drivers in the volatility structure of bonds. A CART analysis uses a decision tree that results from a supervised learning predictive model. Based on a set of binary input variables (our categorical regressors) we are predicting the value of our target variable, which is a bond volatility measure. Some benefits of this type of analysis is that CART is non-parametric and therefore does not rely on data belonging to a particular type of distribution, CART is not significantly impacted by outliers in the input variables, and CART can use the same variables more than once in different parts of the tree and therefore uncover complex inter-dependencies between sets of variables (Nisbet et al., 2018).

By running a CART analysis we find further validation that green bonds have lower volatilities than conventional bonds and also find that the sectorial attribute plays a very important role in predicting volatilities, e.g. bonds in the energy sector have higher volatilities than bonds in other sectors. The widely-used machine learning CART technique is a helpful tool to identify complex inter-dependencies between different sets of variables and, due to being a non-parametric method, it also does not rely on data belonging to a particular type of distribution.

We apply the CART method in R with the rpart command to predict bond volatilities based on four relevant categorical variables: (i) bond type (i.e., green or conventional), (ii) sectors, (iii) maturities, and (iv) ratings. Based on these variables we find that the top discriminating factor is sectorial classification: the sectorial affiliation matters most when it comes to predicting high bond volatilities. Using this command in conjunction with our data set on conventional and green bond (from 2017 to 2020) yields a huge tree (see Figure 8). For better readability, we separately depict the left branch (which includes the energy and utilities sector) below in Figure 9 and the right branch (which includes the finance and government sector) in 10.

These figures show that bonds in the energy and utilities sector have higher volatilities than those in the finance and government sector. Additionally, the CART analysis validates our prior findings that green bonds are associated with lower volatilities than conventional bonds. The CART results in Figures 8, 9, and 10 show a decision tree that ranks the sectorial specification as the most important property regarding the volatility prediction. Bonds from the energy and utilities sector open the top left branch, which means that bonds in these sectors will predict higher volatility than bonds in other sectors (i.e., the finance and government sector). A bond in the energy (and utility) has a higher predicting power for volatility (graphically: energy is listed as the top classifier). Another good validation of our results is that bonds that are categorized as green are mostly shown to the right of conventional bond branches, which means that being a green bond predicts in most cases lower volatilities than being a conventional bond.

The only case where we observe the opposite is for government bonds (see bottom right) - but also Kapraun and Scheins (2019) found an opposite trend for green bonds when they looked at the government sector. For most of the decision nodes, we also see that non-investment grade bonds predict higher volatilizes than investment grade bonds, which is analogous to the comparison of long-term bonds and short-term bonds. This CART analysis was done across all currencies, but currency-specific CART analyses for the USD and EUR confirm the general findings: we see that (i) the energy sector also appears as a high volatility classifier for both currencies (not necessarily as the top one but still appearing as a high-volatility predictor), and that (ii) the bond type category shows lower volatilities for green bonds in the USD case but does not appear as a strong predictor for the EUR case (a currency anomaly that we already see in respect to the multivariate regressions)

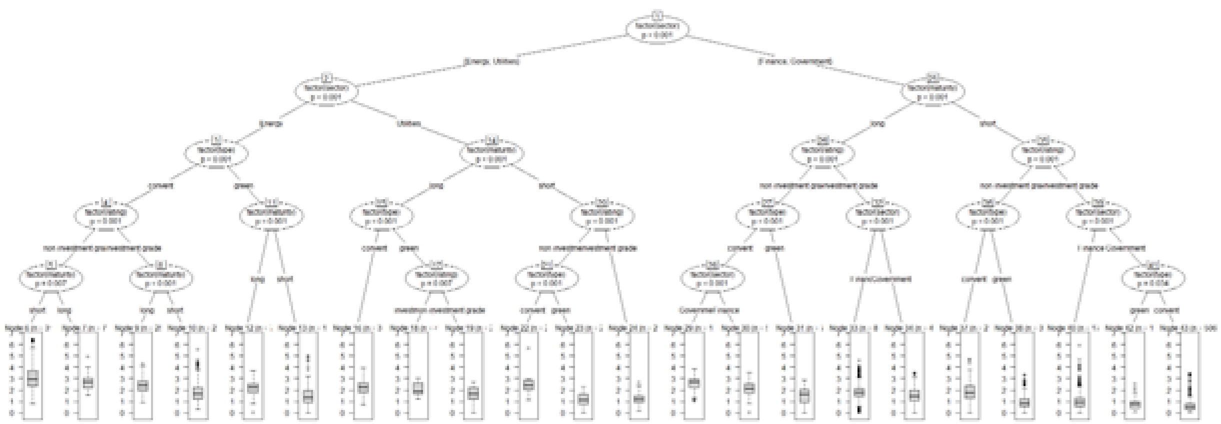

Figure 8: Analysis 3 - CART results (big chart)

Figure 9: Analysis 3a - CART results, left branch (energy and utility sector)

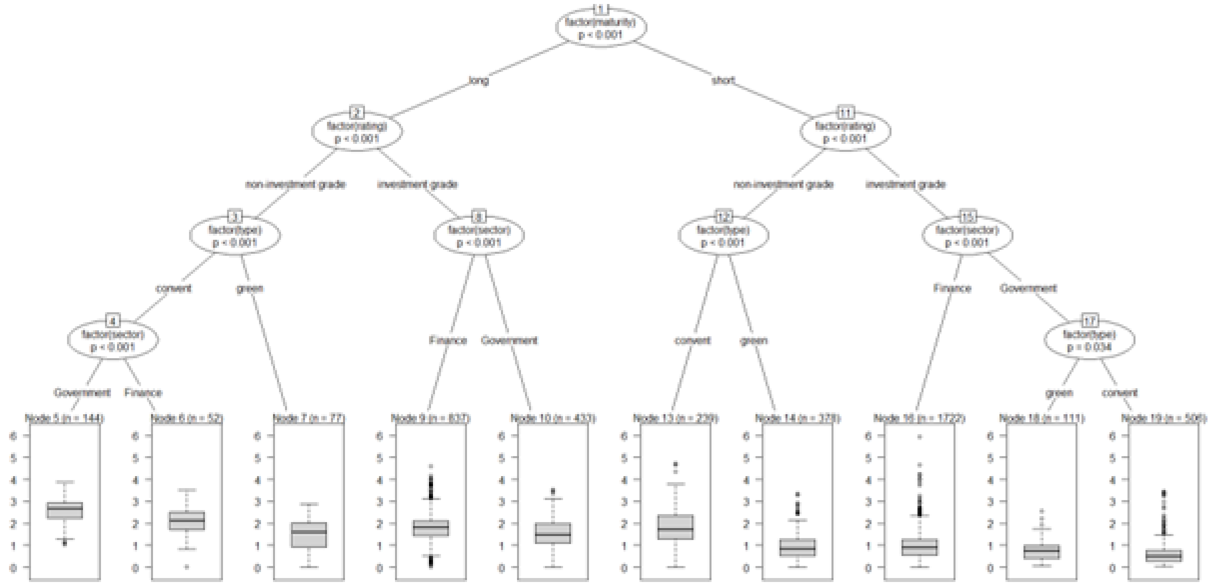

Figure 10: Analysis 3b - CART results, right branch (finance and government sector)

5.4 Energy sector analysis

In order to investigate differences between green and "brown" forms of investment we compare the bond performance of green and conventional bonds in the energy sector. Energy specific fixed income securities in Bloomberg are grouped into several subcategories but green bonds are mostly found under "Renewable Energy". All other energy subcategories are mostly fossil fuel related ("Pipeline", "Oil & Gas Services & Equipment", "Integrated Oils", "Exploration & Production", "Refining & Marketing", "Coal Operations"). For the energy specific analysis, we include all observations that are categorized under "Energy" in Bloomberg and add observations that are categorized as "Power generation". Since our sample does not include issuers who sold both types of bonds, in the green and "brown" energy universe, it is not possible to carry out a pairing analysis and control for issuer specific effects in the energy sector. Therefore, we run a non-matching analysis to compare the specificities of the energy sectors.

The regression results show that there is no clear evidence of a green premium for bonds in the energy sector. Table 6 shows no significant effects of the green dummy on yield to maturity rates. Yet, there is weak evidence of higher Sharpe ratios for green bonds. Table 7 shows that in the general case (first three columns with no currency restriction, models 1a-3a) the bond specific Sharpe ratio (SRb) is higher by 27.2 bps for a green bond when the 30-day volatility is used for the SR calculation; there is also a weak positive effect on the SR in the case of green bonds issued in USD (see the second three rows, models 2a-2c) in the 30d and 90d case.

The evidence of higher SRb's for green bonds is weak. However, it has to be noted that all significant green dummy effects are positive and that no negative effects of greenness on the SRb have been recorded.

	Dependent variable: Yield to maturity								
	(1a) M.30d	(1b) M.90d	(1c) M.260d	(2a) M-USD.30d	(2b) M-USD.90d	(2c) M-USD.260d	(3a) M-EUR.90d	(3b) M-EUR.90d	(3c) M-EUR.260d
Intercept	-2.988^{***}	-2.558^{***}	-4.103^{***}	-3.239^{***}	-2.768^{***}	-4.654^{***}	-3.446^{***}	-3.359^{***}	-3.918^{***}
	(0.219)	(0.256)	(0.899)	(0.275)	(0.282)	(1.141)	(0.275)	(0.349)	(0.734)
Green bond (dummy)	−0.258	−0.275	−0.078	−0.286	−0.364	−0.317	−0.174	−0.577	0.414
	(0.179)	(0.251)	(0.436)	(0.202)	(0.264)	(0.611)	(0.255)	(0.629)	(0.290)
Volatility 30d	0.264^{***}	0.298^{***}	0.482						
	(0.018)	(0.020)	(0.279)						
Volatility 90d				0.172^{***}	0.215^{***}	0.380			
				(0.019)	(0.022)	(0.284)			
Volatility 260d							0.060^{***}	0.067^{***}	0.054
							(0.009)	(0.013)	(0.039)
Bid-ask spread	0.043	-0.441^{**}	0.844	0.060	-0.488^{**}	1.756	0.308^{*}	0.123	1.100
	(0.157)	(0.140)	(0.784)	(0.207)	(0.185)	(1.359)	(0.142)	(0.204)	(0.584)
Maturity: long term (dummy)	-0.707^{***}	-0.750^{***}	−1.283	-0.591^{***}	-0.697^{***}	−1.356	−0.125	−0.178	−0.624
	(0.147)	(0.147)	(0.764)	(0.173)	(0.181)	(1.016)	(0.131)	(0.165)	(0.353)
Rating	0.303^{***}	0.249^{***}	0.292^{**}	0.315^{***}	0.237^{***}	0.303^{*}	0.291^{***}	0.242^{***}	0.178^{*}
	(0.020)	(0.026)	(0.106)	(0.027)	(0.035)	(0.116)	(0.026)	(0.043)	(0.067)
Coupon rate	0.595^{***}	0.713^{***}	0.498	0.586^{***}	0.710^{***}	0.240	0.646^{***}	0.801^{***}	0.735^{**}
	(0.039)	(0.070)	(0.287)	(0.046)	(0.070)	(0.476)	(0.052)	(0.103)	(0.220)
Amount issued (in bnUSD)	-0.706^{***}	-0.941^{***}	0.477	-0.359^{*}	-0.540^{***}	0.957^{*}	-0.515^{***}	-0.733^{***}	0.828^{**}
	(0.127)	(0.139)	(0.419)	(0.145)	(0.151)	(0.443)	(0.154)	(0.183)	(0.307)
Debt to assets	0.009^{***}	0.002	0.009	0.010^{***}	0.004	0.010	0.014^{***}	0.010^{**}	0.016^{***}
	(0.002)	(0.003)	(0.008)	(0.002)	(0.003)	(0.008)	(0.002)	(0.003)	(0.004)
Year issued (dummy 2018)	−0.138	−0.168	−0.164	−0.277	-0.317^{*}	−0.379	−0.083	−0.117	0.105
	(0.118)	(0.131)	(0.333)	(0.144)	(0.155)	(0.514)	(0.131)	(0.171)	(0.309)
Year issued (dummy 2019)	−0.005	0.025	−0.559	−0.209	−0.177	−0.917	0.135	0.183	−0.072
	(0.111)	(0.129)	(0.552)	(0.137)	(0.153)	(0.896)	(0.138)	(0.183)	(0.285)
R^2	0.837	0.843	0.895	0.811	0.824	0.840	0.796	0.780	0.894
Adj. R^2	0.834	0.840	0.879	0.808	0.820	0.815	0.792	0.774	0.876
Num. obs.	653	485	77	640	503	76	534	386	70

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 6: Analysis 4a - Energy sector regression: yield to maturity

	Dependent variable: Bond specific Sharpe ratio (SRb)								
	(1a) M.30d	(1b) M.90d	(1C) M.260d	(2a) M-USD.30d	(2b) M-USD.90d	(2C) M-USD.260d	(3a) M-EUR.30d	(3b) M-EUR.90d	(3C) M-EUR.260d
Intercept	0.409^{***}	0.581^{***}	-1.165	0.196^{***}	0.304^{***}	-1.367^{**}	0.083^{*}	0.125^{***}	-0.555^{**}
	(0.117)	(0.099)	(0.739)	(0.057)	(0.041)	(0.417)	(0.036)	(0.037)	(0.185)
Green bond (dummy)	0.272^{*}	0.391	-0.119	0.098^{*}	0.085^{*}	-0.226	0.015	0.002	0.018
	(0.132)	(0.263)	(0.315)	(0.048)	(0.042)	(0.205)	(0.030)	(0.040)	(0.060)
Bid-ask spread	-0.005	-0.316^{***}	0.895	0.010	-0.119^{***}	0.777^{*}	-0.015	-0.052^{**}	0.512^{*}
	(0.085)	(0.050)	(0.708)	(0.043)	(0.021)	(0.377)	(0.026)	(0.016)	(0.221)
Maturity: long term (dummy)	0.017	0.003	0.093	0.020^{**}	0.005	0.116^{**}	0.013^{***}	0.004	0.037
	(0.012)	(0.008)	(0.070)	(0.006)	(0.004)	(0.039)	(0.004)	(0.003)	(0.020)
Rating	-0.355^{***}	-0.117^{**}	-0.411^{*}	-0.182^{***}	-0.091^{***}	-0.282	-0.052^{**}	-0.020	-0.159^{*}
	(0.064)	(0.038)	(0.192)	(0.033)	(0.022)	(0.144)	(0.020)	(0.017)	(0.064)
Coupon rate	0.137^{***}	0.116^{***}	0.104	0.043^{***}	0.063^{***}	-0.123	0.028^{***}	0.035^{***}	-0.039
	(0.025)	(0.019)	(0.197)	(0.011)	(0.010)	(0.093)	(0.008)	(0.009)	(0.042)
Amount issued (in bnUSD)	-0.414^{***}	-0.301^{***}	0.274	-0.088^{**}	-0.091^{***}	0.645^{*}	-0.109^{***}	-0.086^{***}	0.170^{**}
	(0.068)	(0.052)	(0.368)	(0.033)	(0.020)	(0.287)	(0.019)	(0.015)	(0.056)
Debt to assets	0.000	-0.000	-0.001	0.001	0.000	0.001	0.000	0.000	0.001
	(0.001)	(0.001)	(0.003)	(0.001)	(0.000)	(0.002)	(0.000)	(0.000)	(0.001)
Year issued (dummy 2018)	0.071	0.012	0.335	-0.040	-0.070^{**}	0.206	-0.003	-0.029	0.086
	(0.070)	(0.045)	(0.278)	(0.031)	(0.022)	(0.165)	(0.018)	(0.018)	(0.057)
Year issued (dummy 2019)	-0.095	-0.004	-0.099	-0.085^{**}	-0.058^{*}	-0.234	-0.022	-0.016	-0.097
	(0.056)	(0.044)	(0.214)	(0.026)	(0.023)	(0.121)	(0.019)	(0.018)	(0.052)
R^2	0.250	0.312	0.612	0.253	0.385	0.569	0.252	0.285	0.648
Adj. R^2	0.239	0.299	0.558	0.243	0.374	0.507	0.239	0.268	0.593
Num. obs.	650	485	75	637	503	73	528	386	67

Standard errors are heteroskedasticity robust. $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table 7: Analysis 4b - Energy sector regression: Bond specific Sharpe ratio (SRb)

6 Conclusion

The model-driven results showed us that the fossil fuel based assets - in our case bonds - should, if the negative externalities are properly priced in, exhibit empirically low returns and possibly higher volatility. On the other hand, one would expect, for the green bonds, delivering positive externalities in the long-run, higher returns and lower volatility. The second type of assets should lead to superior asset and wealth accumulation as compared to the first type of assets. This should also lead -- as our dynamic portfolio model predicts -- to a change of portfolio holdings of the two types of assets.

The empirical analysis shows that green bonds show lower volatilities and achieve higher Sharpe ratios (in the paired as well as in the unpaired analysis) which reflects an improved wealth accumulating characteristic of green bonds. Looking at the results of the regression analysis of paired bonds we also see significant and positive secondary market yield effects of green bonds. Yet, this effect is somehow ambiguous since unmatched yield to maturity rates show significant negative yield differentials of green bonds (the conclusion for primary market yields is due to a weaker data availability less clear).

There are certain reasons for the ambiguities and why there is still a gap between the model-driven results and the empirics in some instances: there are different preferences of market participants, only little information on the positive and negative externalities is integrated into the actual trading, the green bond market is still small and evolving market and a lot of learning is still going on.

Our empirical results, supported by previous findings of the literature, seem to show evidence of negative yield differential of green minus conventional bonds in the primary market, but positive yield differentials for a paired bond analysis in the case of the secondary market. Additionally our study shows that green bonds show a higher bond specific Sharpe ratio in several regression designs, which makes green bonds an interesting investment form. Especially due to lower volatilities, as identified with the CART analysis, green bonds can help to improve the portfolios of investors and help to achieve sustainable wealth accumulation. Also the energy sector specific analysis showed weak evidence of an improved Sharpe ratio of green bonds, which points towards better volatility discounted returns of green bonds compared to brown, i.e. fossil fuel based, bonds.

Generically, the higher potential of asset accumulation is shown by our model-driven results where certain preferences of individual bond holders (socially-oriented investors, ESG investors, and social impact investors) can allow the additional social returns to arise in the long run, however not necessarily showing up in the shorter run in the trading of assets, driven by short-termism and other forces. We can therefore have larger asset accumulation in the case of renewable energy assets, as compared to the case of fossil fuel energy, see the upper graph in Figure 2. On the other hand, there should be a (shadow) tax on fossil fuel energy which makes the return lower, see lower two graphs. This resembles earlier studies put forward in microeconomics as positive and negative externality effects which has been used in climate-oriented models by Acemoglu et al. (2012), but here now applied to bond prices and yields.[29]

We also want to note that here as in Semmler et al. (2020), by using the NMPC program, the returns for risky assets are made time dependent, but they can also be defined as impacted by stochastic shocks along their paths. Empirically estimated low frequency movements in returns are estimated and built into a dynamic portfolio model, see Chiarella et al. (2015, ch. 4).

Our approach can also be used to control sovereign debt as much as sovereign bonds are convertible into some equity. This is an important issue, since many observers express the criticism that issuing green bonds for climate protection will make the debt to GDP ratio rising leading to unsustainable debt. This does not need to occur if (shadow) tax rates are properly levied on returns from fossil fuel assets and aid to give incentives to create positive externalities of renewable energy. The use of convertible bonds, that turn into equity holdings when renewable energy firms are successful, aids to reduce debt of the sovereign issuer of bonds.

[29]Of course, as also Kapraun and Scheins (2019) argue, once the bonds are traded there might be a multiplicity of drivers of actual bond prices and yields, for example relevant are the actual activities in portfolio management, monetary policy and interest rates, varying risk premia, also for other assets, then the varying Distance to Default, and more.

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Appendix A - Fast fourier transform (FFT)

In Section 3, we use a Fast fourier transform (FFT) to estimate the green and fossil fuel returns applied to the dynamic portfolio model, as in Chiarella et al. (2016) and Semmler & Hsiao (2011). The FFT filters short-term shocks, estimating the coefficients of a sine-cosin function that represents the asset performance - see eq. A.1. In Section 3, it generates low-frequency movements using the monthly annual total returns for green and fossil fuel monthly returns from December/2015 to December/2020 for the US.

$$y(t) = \sum_{i=1}^{k}(a_i sin(\frac{2\pi}{\tau_i}(t - t_o)) + b_i cos(\frac{2\pi}{\tau_i}(t - t_o))) \tag{A.1}$$

As described by Semmler & Hsiao (2011), the first step to apply the FFT method is to de-trend the real returns for each index in the time series. We thus subtract a linear trend from the real returns:

$$Detrendedreturns = realreturns - (b_1(t - t_0) + b_2) \tag{A.2}$$

The linear trend coefficients ($b_1$and b_2) are obtained by a polynomial curve $p(x)$ of degree 1 that returns the best fit (in a least-square sense) for the real returns data. The values for t and $t_0$depend on the period covered by the time series. The FFT method picks up the periods with the highest power (τ_i). We obtain then the coefficients a_i and b_i for the harmonic fit for the different values of k and the τ_i for the different periods (in months), as in eq. A.1.

The harmonic regression model is estimated for different values of k, from 1 to 6, which represents different frequencies - from low to high frequency data. For our analysis, we select the estimation with the lower sum of squared errors (see estimations in Figure A.1). We select $k = 4$ for green bonds and $k = 1$ for fossil fuel bonds.

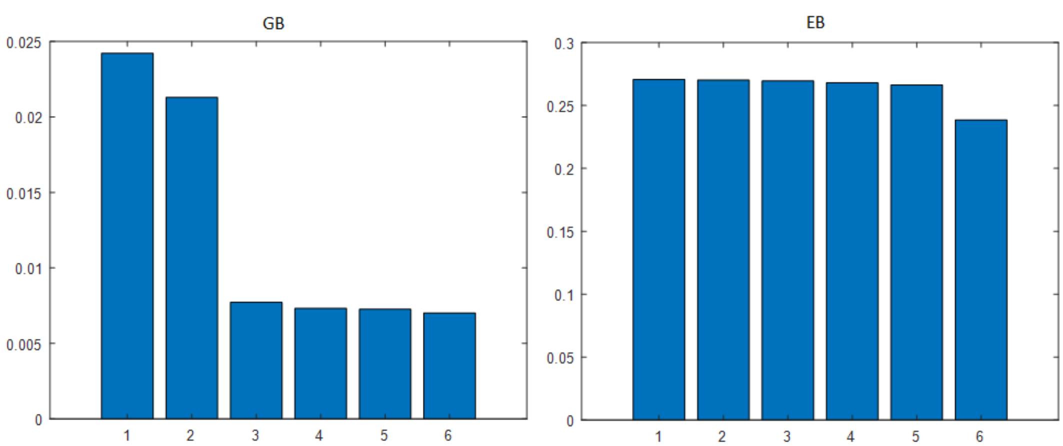

Figure A.1: Harmonic estimations: Sum of squared errors for green bonds (GB) and fossil fuel bonds (EB) in the US