Fiscal Multipliers with an Informal Sector *

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Abstract

The shadow economy exaggerates the effect of fiscal policy on both official and true economic activity. We measure these effects using a model with an informal sector in the context of the recent—2010-15—Greek fiscal consolidation experience. We find that formal output declined by 50% more than projected (26% vs 18%) whereas, due to the large increase in the share of the informal sector (by 50%), true output declined by much less (17%). Almost 1/3 of the formal GDP decline was due to income tax rate increases (which failed to raise extra tax revenue). Our model predicts that had the informal sector been contained to its pre-crisis size, at least one-quarter of the decline in GDP could have been averted. And that the capital controls imposed in 2015 may inadvertently have, within a year, contributed to a reduction of the share of the informal sector by almost five percentage points and to an increase in formal GDP by 2.6%.

JEL class: E26, E32, E62, E65, H26, H68

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1 Introduction

Following the outbreak of the sovereign debt crisis, Greece undertook a large, fiscal consolidation program. Official economic activity collapsed and, in spite of the significant, across the board tax increases, tax revenue as a percentage of GDP barely moved whereas the debt to GDP ratio grew. These developments seem to have been by and large unanticipated: the typical scenario put forward by Greece's official creditors (the EZ countries and the IMF) predicted a milder and shorter downturn and the quick achievement of debt sustainability.[1]

Blanchard and Leigh, 2013, find that the errors made in official forecasts of the effects of fiscal consolidation were pervasive across many countries. Pappa et al, 2015, attribute such errors to the fact that standard models do not contain an informal sector.[2] In these models, when taxes go up, agents may choose to work and invest less. But in an economy with an informal sector, economic agents have an additional option, namely, to shift activities to the underground economy. This facilitates a larger outflow of resources from the formal sector, leading to a larger decline in recorded output and tax revenue.

In addition to mis-forecasting, the standard model contains another significant flaw, namely, the mis-measurement of the true (total) level of economic activity. The footloose resources that flee the formal sector are employed in the informal sector, so the decline in true (formal plus informal) output can be quite smaller than that in recorded output: a recession may turn out to be worse than its forecast but it may also not be as bad as it appears.[3]

These points seem uncontroversial. They have not, however, been taken into account in evaluations of actual fiscal consolidation episodes.[4] Their inclusion seems valuable for a number of reasons. First, it produces a more accurate measurement of the effects on true economic activity, "austerity" and welfare. Second, it offers an explanation of why some types of consolidation –spending based– produce better outcomes than other– tax based– (Alesina and Giavazzi, 2015) without requiring implausible labor-leisure elasticities.

[1] Over the 2009-2015 period, Greek GDP declined by 26% and the unemployment rate peaked at 27% in 2013 (from 9.5% in 2009). The official projections at the start of the Economic Adjustment Program postulated a decline of real GDP of 6.6% over the following two years and an increase in the rate of unemployment to 15.3% by 2012 (see European Commission: The Economic Adjustment Programme for Greece, Occasional Papers 61, May, 2010.

[2] For the debt crisis afflicted countries, Schneider, 2015, reports values for the shadow economy that range from 11% for Ireland to 25% for Greece.

[3] Restrepo-Echavarria, 2014, points out another important implication of the mismeasurement of economic activity associated with the informal economy. Namely, that a model can generate a volatility of measured consumption that is higher than that of output, as observed in many developing countries and some developed countries, even though actual consumption is not nearly as volatile.

[4] Pappa et al, 2015, represents an important exception. It produces useful insights but does not address any particular episode/country and only studies mis-forecasting as in their model true and officially measured GDP are identical.

Third, it can be used to run counterfactuals whose results can form the basis for designing more effective policies. For instance, to the extent that the informal sector is elastic in size, policies that increase tax rates and ignore tax enforcement can prove self-defeating. Greece's fiscal odyssey exemplifies this problem.

The objective of this paper is to fill this gap in the literature by studying the experience of the country that has been at center stage on this front during the last decade, namely, Greece.[5] We use the DSGE model of the Bank of Greece (BOGGEM), augmented with an informal sector and the actual fiscal package implemented by Greece over the period 2010-2015 to: a) provide an estimate of the forecast error as well of the size of the mis-measurement of economic activity (compute "true" fiscal multipliers); b) carry out useful counterfactuals regarding how economic conditions in Greece may have evolved differently had the government been able to tame or, simply limit any further expansion in the informal sector following the increases in tax rates during the program. We believe that the results from such counterfactuals will prove useful in the design of better fiscal adjustment programs in countries with a large and/or elastic, informal sector.

The model used is of the New Keynesian DSGE variety. But the results reported are very robust to using simpler –or, more involved– versions of the model, including those with flexible prices.[6]. The main findings are summarized below.

The recorded decline in official GDP in Greece over 2010-15 was 26%. When fed with the actual fiscal package,[7] our model mimics this decline. Neglecting the informal sector leads the model to under-predict the GDP decline (17%) and generates tax multipliers that are about half the size of those in the economy with an informal sector. The mis-measurement of true output is comparably large: for instance, under perfect foresight, the model implies that true output decreases by 17.5%. It is even larger for employment: the reduction in true employment (hours) is 2-3% and that in official employment 14%. The size of the mis-measurement reflects both the large size and the large increase in the share of the black economy, which according to our model, grew from 25% to 37% of official output.

The model also allows us to compute the relative contribution of the individual fiscal instruments to the decline in output. The increase in tax rates accounts for about two-thirds of the decline in GDP, with the income tax being the most significant culprit (about one-third of the total output decline). Remarkably, the increases in the income tax rates seem to have been counterproductive: they landed the biggest blow on economic activity

[5] Chodorow-Reich et al, 2019, study the effects of fiscal consolidation in Greece in a standard model without an informal sector.

[6] This is re-assuring because the normal framework of price rigidity must have been severely disrupted during the crisis.

[7] As the model is dynamic, we need to make assumptions about the nature of expectations about the fiscal instruments. We consider the two polar assumptions of perfect foresight and random walk.

but did not manage to generate higher tax revenue. On this front, consumption taxes seem to have been the most benign, having made a positive contribution to tax revenues with a relatively limited negative impact on economic activity.

Note also that while the flight to informal activities dampened the effects of fiscal adjustment on output and employment, it amplified those on tax revenue. Not taking into account the existence and growth of the informal sector leads to a prediction of a surge in annual tax revenues by about 5% of GDP. When the model takes this into account, it implies a much lower increase, about 1% (a figure close to the actual one).

The picture painted by these findings is that the black economy, in combination with reliance on unsuitable tax instruments (income taxes), was a key factor behind the failure of the fiscal consolidation exercise in Greece. Economic activity collapsed and little extra tax revenue was raised. On the surface, the black economy seems to have mitigated some of the adverse effects on output and employment. But this is misleading because, given the tax revenue requirements associated with the deficit targets imposed by Greece's official creditors, it worsened the vicious circle of continually higher tax rates chasing after a fast shrinking tax base.

How would the outcomes of fiscal consolidation have been different[8] had the official creditors focused more on and the Greek authorities had taken stronger action to contain the informal sector, say to its pre-crisis size? Fixing the size of black economy to its pre-crisis level while implementing the same fiscal package, the model predicts a cumulative decline in GDP by 19.5% and of true output by 16% (instead of 26% and 17.5% when the black economy is allowed to grow). That is, limiting growth in informal activities would have saved 6.5% of official and 1.5% of total output$(17.5\% - 16\%)$.

These figures represent a *lower* bound because they do not take into account the positive effects of a smaller black economy on tax revenue/tax rates. To get a sense of the importance of these effects, we have run counterfactuals where we keep the size of the informal economy fixed at the pre-crisis level, determine the income tax rates that can support certain amounts of extra annual tax revenue –while fixing the values of the other fiscal instruments to their historical values– and use them to calculate the equilibrium allocations. We find that controlling the size of the black economy would have allowed the Greek government to increase tax revenue by about 1% per year with a *decrease* in the income tax rate and in the process only suffer a cumulative output decline of less than 10%!.

The crucial role played by the black sector in the failure of the fiscal consolidation experience in Greece and thus the importance of measures to control tax evasion for tax revenue

[8] There is a large set of other useful counterfactuals that can be readily conducted in our model, such as considering the effects of different fiscal packages.

and economic activity can be further illuminated by considering the natural experiment that took place in Greece in the summer of 2015, namely, the imposition of capital controls. In order to prevent a bank run triggered by doubts about Greece's Eurozone membership, the Greek government imposed limits on the amount of cash that depositors could withdraw from their bank accounts. Hondroyannis and Papaoikonomou, 2017, report that this forced a switch from cash to card payments: the share of card payments in private consumption went from 4% to 12% over the next year. This led to a massive, unexpected increase in VAT revenue. Using their estimated elasticity of VAT revenue to credit card usage, our model implies that the capital controls may have contributed, during the year following their introduction, to an increase in official economic activity by 2.6% and to a reduction in the share of the informal sector by almost five percentage points.[9]

These findings support the following conclusions: First, the large misjudgment of the effects of fiscal consolidation in Greece by its official creditors can be explained by the standard modeling practice of neglecting the informal sector. Second, much (about 1/3) of the reduction in official GDP was offset by the large expansion of the underground sector (by almost 50%). And third, the existence of the black economy played havoc with tax revenues, contributing to both excessively high tax rates and weaker economic activity. Fiscal consolidation in Greece may have proved more successful had more rigorous policies to limit pervasive tax evasion been pursued. Admittedly, the structure of economic activity (a large share of self-employed individuals and small, family operated businesses) and prevailing mentality in Greece made this a daunting task. Nonetheless, the rate of return on tax evasion prevention policies was so high as to have justified the devotion of a much larger amount of resources to it.

The rest of the paper is organized as follows. In section 1 we describe a variant of the DSGE model of the Bank of Greece (henceforth, BOGGEM) that contains an informal sector, and compute "true" fiscal multipliers at the steady state. In section 2, we derive the implications of the actual fiscal instruments employed during the fiscal consolidation in Greece during 2010-2015 for official and total output as well as for tax revenue. In section 3 we carry out a number of relevant counterfactuals.

[9] A caveat is in order. Our model abstracts from possible negative effects of capital controls, such as those arising from their impact on business and import financing. To the extent that capital controls have other negative side effects, the numbers reported here should be interpreted as an upper bound.

2 The BOGGEM with an Informal Sector

2.1 Preliminaries

In our definition, the informal economy consists of activities that lead to economic transactions on the marketplace and which escape government monitoring and taxation and are hence not included in gross national product (home production is thus not part of the shadow economy). Informal and formal goods may be perfect substitutes from the production point of view (produced by the same technology) with the sales of the former being hidden from the state in order to evade taxes and/or regulation; they may be perfect substitutes from the point of view of the buyers (generate the same services) with the former being purchased because of a lower price; and, finally, they may be imperfect substitutes in production and/or in consumption. Imperfect substitutability in production may arise from the interaction of technological and government monitoring considerations. For instance, a firm may choose to operate at an inefficiently low scale in order to reduce the probability of being monitored (make it not worthwhile for tax inspectors to monitor it due to the presence of fixed costs in inspection). This motivation seems to explain the disproportionately large share of very small (family) firms in the total population of firms in Greece. Imperfect substitutability in consumption may arise when informal goods lack certain characteristics of their formal counterparts (such as product repair or return guarantees, quality control certificates, etc.).

There are three alternative ways to guarantee an interior solution in the allocation of resources between the formal and informal sectors. We can assume that the perspective goods are imperfect substitutes in production (so that the production possibility frontier is non-linear). Or, that they are imperfect substitutes in consumption (so that the indifference curves are non-linear). If the goods are perfect substitutes on both the production and consumption (demand) side then in order to get an internal solution we need to introduce curvature somewhere else in the model. A simple way to do so is to assume that the probability of being monitored or the size of the fine paid when caught is increasing in the degree of tax evasion. A suitable choice of the detection/fine schedule can always support an interior equilibrium.

From the point of view of the focus of the present paper, all three ways are interchangeable and lead to similar equilibria. We have opted to make the goods imperfect substitutes in production for two reasons. First, this makes our model more comparable to the extant literature, in particular, to the paper by Pappa et al. (2015). And second, because informality is typically –at least outside the group of rich countries– associated with less efficient production (Murphy et al., 1989, Docquier et al, 2017). Maloney (2004) and de

Paula and Scheinkman (2011) provide evidence that informal firms are managed by less able entrepreneurs, are smaller, and exhibit low capital-labor ratios. Similarly, La Porta and Shleifer (2008) report evidence of a substantial difference between registered and unregistered firms regarding the skills of their managers, quality of inputs and the cost of capital. Greece is characterized by a disproportionately large number of small firms and self-employed agents. It is well understood that many of these economic units select an inefficiently low scale/method of production in order to evade taxation.

2.2 The Model

BOGGEM does not contain an informal sector. It captures well many key features of the official Greek economy (as shown in Papageorgiou, 2014) and thus provides a suitable vehicle for carrying out a quantitative exploration of the effects of fiscal policy. We augment it by including an informal sector and use it to address various questions pertaining to the actual Greek fiscal consolidation experience. In this section, we compute the degree of over-optimism in forecasts about the effects of fiscal policy as well as the degree of mismeasurement of true economic activity in the steady state of the model. In the following section, we study the dynamic path of the Greek economy by feeding into the model the actual paths of the fiscal instruments during the consolidation period 2010-2015. Finally, in section 4 we use the model to carry out counterfactuals about the outcomes of fiscal consolidation if the Greek government had managed to limit informal activities.

We delegate the presentation of the formal model to the Appendix and only offer here a description of its main features as well as its key equations pertaining to the informal sector. In the appendix, we also use a stripped down version of the model with flexible prices and without any bells and whistles to establish that the key properties of the baseline DSGE model are quite general.

Goods

The economy contains firms that operate at different stages of production. In the first stage, we have perfectly competitive firms that use either capital and labour (formal sector) or labour alone (informal sector) to produce a homogeneous, intermediate good. In the second stage, we have imperfectly competitive firms that convert part of the formal, homogeneous intermediate good (the rest being exported) into a formal, *differentiated* intermediate good. At this stage the firms also convert an imported, homogeneous good into a differentiated, foreign, intermediate good. The assumption of imperfect competition is made in order to facilitate the introduction of standard price stickiness. In stage three, firms combine the domestic varieties of intermediate goods with the varieties of the imported good to produce a homogeneous, formal, final good, which can be used for

consumption and investment purposes. The informal, homogeneous, intermediate good is used only for household consumption.

Labour markets

They are assumed to be perfectly competitive.[10]

Price setting

All prices are flexible except for the formal, intermediate goods (both domestic and foreign) that are subject to the standard Calvo pricing friction.

Asset markets

The economy is small. Its residents can hold foreign currency bonds that are subject to adjustment costs that depend on actual relative to steady state holdings. The domestic firms are owned by domestic residents.[11]

Government finances

The government raises revenue through four distortionary taxes, namely, taxes on the revenue of the firms in the sector that produces the formal, homogeneous intermediate good (stage 1); taxes on labour and capital income from the same sector; and taxes on consumption of the formal, final good. The government also raises revenue through a lump sum tax, that is used as a residual to cover any discrepancy that arises between distortionary tax revenue and government spending (recall that we abstract from public debt, see below). Government spending takes four forms: government consumption, public investment, wages paid to public employees and government transfers. In spite of the fact that public debt has played an important role in the Greek crisis, we abstract from it for two reasons: The model lacks proper sovereign debt features; and its level has varied significantly due to factors that are completely outside the model (such as the revision to include previously hidden debt, debt restructuring, bank recapitalization, etc.).

Tax evasion

Tax evasion at the firm level involves the firms not reporting the revenue from the sale of the informal good they produce. If a firm gets caught –which happens with an exogenous probability– then it has to pay a fine, which is a multiple of the revenue tax rate. Similarly,

[10] BOGGEM assumes imperfectly competitive labour markets. We dropped this assumption as it had negligible quantitative effects on our computed multipliers. Nonetheless, in the Appendix we report results for a version of the model with distorted labor markets.

[11] The full version of BOGGEM has two types of households, Ricardian and non-Ricardian. The former own all the firms in the economy and receive their profits as dividends. They can save by investing in physical capital and by buying domestic currency government bonds and foreign currency bonds. The latter do not own any assets and consume their current consumable income. We have also carried out the analysis using the distinction between Ricardian and non-Ricardian households. The results are very similar. We discuss them in the section on robustness.

workers employed in the production of the informal good do not declare their labor income. If they get caught they get fined. While the most natural scenario involves a simultaneous detection of undeclared firm revenue and labour income in the informal sector, we have also considered independent detection of firms and workers tax evasion. The results are not affected by the characteristics of the detection scheme.

Monetary Policy

The exchange rate is fixed. The domestic interest rate equals an exogenously given, risk-free, world interest rate.

Some key equations of the model

Household consumption

$$C_{i,t} = C^p_{i,t} + C^u_{i,t} + \vartheta Y^g_t$$

$C^p_{i,t}$ is consumption goods produced in the formal sector, $C^u_{i,t}$, goods produced in the informal sector and Y^g_t per-capita public goods and services produced by the government. We assume that the household is indifferent between the formally and informally produced private consumption goods, so their relative price is unity.

Household budget constraint

$$\begin{aligned} & (1+\tau^c_t)\, C^p_{i,t} + \frac{P^u_t}{P^c_t} C^u_{i,t} + \frac{P^i_t}{P^c_t} I_{i,t} + \frac{S_t F_{i,t+1}}{P^c_t} = \\ = \; & (1-\tau^l_t)(W^p_t H^p_{i,t} + W^g_t H^g_{i,t}) + (1-\pi\phi^W) W^u_t H^u_{i,t} + (1-\tau^k_t) r^k_t u_{i,t} K^p_{i,t} + \\ & + Div_{i,t} + R^H_{t-1} \frac{S_t F_{i,t}}{P^c_t} - T_t - \Gamma^h_{i,t}, \end{aligned} \tag{1}$$

where P^c_t and P^u_t are respectively the prices of a unit of formal and informal consumption good; P^i_t is the price of a unit of investment good and S_t is the nominal exchange rate expressed in terms of the domestic currency per unit of foreign currency.

In each period t, the household earns labour income by working $H^p_{i,t}$ hours in the formal sector at the real wage rate W^p_t, by working $H^g_{i,t}$ hours in the public sector at the real wage rate W^g_t and $H^u_{i,t}$ hours in the informal sector at the real wage rate W^u_t. It also receives capital income from renting capital services to firms, $u_{i,t}K^p_{i,t}$, where $u_{i,t}$ is the capital utilization rate and $K^p_{i,t}$ is the physical private capital stock; r^k_t is the rental rate of capital. The household can save by investing in physical capital, $I_{i,t}$, and by buying foreign bonds, $F_{i,t+1}$, that pay a nominal gross interest R^H_t. Households own all firms in the economy and receive their profits as dividends, $Div_{i,t}$. A consumption tax, $\tau^c_t \in (0,1)$ is levied on the formal consumption good, while a labour income tax, $\tau^l_t \in (0,1)$, and a capital tax, $\tau^k_t \in (0,1)$, are levied on the income earned in the formal sector. The household may evade the tax on the part of the labour income that is earned by working in the informal

sector. This income can be detected with the constant probability π, in which case a fine is imposed that is a fixed share ϕ^W of the labour income earned informally. Finally, T_t denotes lump-sum taxes/transfers.

Production of homogeneous, intermediate good firms

The homogeneous intermediate goods sector is composed of a continuum of perfectly competitive intermediate good firms indexed by $j \in [0,1]$. Each firm j can produce the intermediate good either formally or informally. The formal good, $Y^p_{j,t}$, is produced by using as inputs capital, $K_{j,t}$, and labour services, $H^p_{j,t}$, and makes use of the average public capital $\overline{K}^g_t$. The informally produced good, $Y^u_{j,t}$, requires only labour, $H^u_{j,t}$. In particular,

$$Y^p_{j,t} = A^p_t \left(H^p_{j,t}\right)^{\alpha} (K_{j,t})^{1-\alpha} \left(\overline{K}^g_t\right)^{\alpha_g} \tag{2}$$

$$Y^u_{j,t} = A^u_t \left(H^u_{j,t}\right)^{\zeta} - F \tag{3}$$

where $\alpha, \zeta, \alpha_g \in (0,1)$ and $A^p_t, A^u_t > 0$ are the exogenous levels of TFP. in formal and informal production, respectively. Public capital provides production externalities.

Firms pay a corporate tax $\tau^f_t \in (0,1)$ per unit of sales of the formal good but evade the tax for the informally produced good. With –a constant– probability π the producer is caught when tax evading and has to pay a fine ϕ^F per unit of informal good sales. The producers choose the scale of production taking as given the prices for the formal, $P^p_{j,t}$, and informal, $P^u_{j,t}$, goods as well as factor prices, r^k_t, W^p_t, W^u_t in order to maximize expected profits:

$$\Pi_{j,t} = \max_{K^p_{j,t}, H^p_{j,t}, H^u_{j,t}} \left\{ \left(1-\tau^f_t\right) \frac{P^p_{j,t}}{P^c_t} Y^p_{j,t} + \left(1-\pi\phi^F\right) \frac{P^u_{j,t}}{P^c_t} Y^u_{j,t} - r^k_t K^p_{j,t} - W^p_t H^p_{j,t} - W^u_t H^u_{j,t} \right\} \tag{4}$$

subject to (24)-(25). The FOCs are:

The government budget constraint

The government levies taxes on consumption, on income from labour and capital earnings, on corporate income and lump-sum taxes. Total tax revenues together with the issue of new government bonds are used to finance government purchases of goods and services, G^c_t, government investment, G^i_t, government transfers, G^{tr}_t, and the wage bill for public sector employees, $W^g_t H^g_t$. Moreover, the government pays interest payments on past domestic public debt, R_t. The within-period government budget constraint is:

$$\tau^c_t C^p_t + \tau^l_t (W^p_t H^p_t + W^g_t H^g_t) + \tau^f_t \tfrac{P^p_t}{P^c_t} Y^p_t + \pi(\phi^F \tfrac{P^u_t}{P^c_t} Y^u_t + \phi^W W^u_t H^u_t) +$$
$$+\tau^k_t r^k_t u_t K^p_t + T_t = \tfrac{P^d_t}{P^c_t} G^c_t + \tfrac{P^d_t}{P^c_t} G^i_t + G^{tr}_t + W^g_t H^g_t$$

Thus, the government has eleven policy instruments: $\tau^c_t, \tau^k_t, \tau^l_t, \tau^f_t, T_t, H^g_t, W^g_t, G^c_t, G^i_t, G^{tr}_t$. Lump-sum taxes adjust to ensure that the budget constraint is satisfied in every period.

For convenience, regarding spending policy instruments, we will work in terms of their shares of steady state GDP, $S_t^c = \frac{P_t^d G_t^c}{P^y Y^{GDP}}$, $S_t^i = \frac{P_t^d G_t^i}{P^y Y^{GDP}}$, $S_t^{tr} = \frac{P_t^c G_t^{tr}}{P^y Y^{GDP}}$, $S_t^w = \frac{W_t^g H_t^g}{P^y Y^{GDP}}$.

2.3 Calibration

The model is calibrated at an annual frequency. Table 1 presents the calibration of the dual economy model with tax evasion. The standard parameter values are as in Papageorgiou (2014). The data source for the fiscal policy variables is Eurostat. The consumption and labour tax rate values have been set to 0.15 and 0.34 respectively. The corporate revenue tax rate has been set so that, given the average profits to sales ratio, it corresponds to a profit tax[12] of around 25%. The share of government consumption in GDP is set equal to its value in the data. The labour share in the formal sector, α, is computed using data from AMECO. The same value is used for the labour share in the informal sector. The fixed cost parameter in the production function of the informal sector is set so that profits are zero in the steady state. The coefficient of risk aversion and the inverse labour elasticity are both set equal to 1.

We have normalized the scale parameter in the production function of the official sector, A^p, to unity and have selected the scale parameters in the informal sector so that the informal sector in the baseline specification is 25% of GDP[13] the share of the shadow economy in Greece as reported in Schneider and Williams (2013).

The detection probability, π, is a difficult number to compute due to the lack of appropriate data. In the model, the probability of detection is uncorrelated with firm characteristics – due to the representative firm assumption– while in the data, a large number of inspections is targeted (directed). In the benchmark case we compute the probability of detection as the ratio of the number of firms inspected for undeclared workers[14] to the total number of firms. This produces a value 0.14. An alternative is to compute the probability of detection as the ratio of the total number of firms inspected by the tax authorities –not just for labor violations– to the total number of firms. The data provided to us by AADE (the Independent Authority for Public Revenue) produce a value of 0.10 (average, 2014-2016). Note that because of the targeted nature of inspections, these numbers underestimate the

[12]Following Collard et al, 2017, we employ a revenue rather than a profit tax because due to our assumptions that firms are competitive and production in the formal sector is Cobb-Douglas, a tax on profits does not affect allocations.

[13]Variation of this share, say in the range between 20% and 30%, does not change the main message as well as the quantitative properties of the results.

[14]We use the ARTEMIS report (September 2013-November 2015) on the (annual average) number of firms inspected for the presence of undeclared employees. The total number of firms with employees under private law contracts is retrieved from the ERGANI annual reports (average of 2014, 2015).See $https://government.gov.gr/wp-content/uploads/2018/06/ARTEMIS-2017.pdf, http://www.ypakp.gr/uploads/docs/7676.pdf, http://www.ypakp.gr/uploads/docs/9205.pdf$.

perceived probability of detection.[15]

The fine on firms is set equal to a fixed fraction of informal output, namely, 10% so that in the baseline case, detection results in a tax liability that is roughly double of that of the law abiding firms. The fine on tax evading households, ϕ^W, is set equal to 0.5 (that is, 50% more than the income tax rate).

The exponent of public capital in production, α_g, is set equal to the average value of the public investment-to-GDP ratio in the data over the period 2000-2009. We set the adjustment cost on private capital, ξ^k, at 2.5 and calibrate the value for the adjustment cost parameter on private foreign assets, ξ^f, to the lowest possible value so as to ensure that the equilibrium solution for foreign assets is stationary. The Calvo parameters, $\theta^d, \theta^x, \theta^m$, are set equal to 0.35, implying that firms adjust their prices every about 6 quarters, which is in the range of estimates for the euro area countries (see e.g. Christoffel et al. 2008).

2.4 Steady-State Fiscal Multipliers

We compute steady state fiscal multipliers for all the fiscal instruments considered. For the sake of space, we only present the effects associated with an increase in the steady state level of the taxes on labour income as well as with a decrease in the level of government consumption. The effects of the remaining instruments are described in Appendix. A full dynamic analysis in the DSGE model is carried out in the following section.

Table 2 shows the effects of increases in the tax rate on labor in the dual economy (formal and informal sectors) that result in higher tax receipts of one percentage point of GDP. Let us focus on the column with the benchmark calibration ($\tau^f = 0.34$). Increasing the tax rate from 34% to 35.64% (second row), increases income tax revenue as a percentage points of GDP, from 19.16% to 20.16% (first row). The share of informal output increases from 25% to 26.4% (row s^u) and informal employment increases from 34.8% to 36.4%. (row sh^u).

Rows $d\tilde{y}^p$ and $d\tilde{y}^T$ report the corresponding effect on official and true output as percentage

[15] The exact value for this parameter alone is not critical in te context of our model in the following sense. The decision to undertake informal activities depends on the relative return of these activities, which in turn, depend on the relative productivities in the two sectors as well as on the probability of detection and the fine imposed when detected. So there exists quite a bit of freedom in choosing values for these individual parameters whose combination can produce a black economy of 25% of GDP. For instance, a lower value of π can be compensated by a lower value of A^u/A^p or a higher value of ϕ in order to keep the share at 25%.

Table 1: Calibration, DSGE

Parameter	Description	Value
A^p	TFP formal sector	1
A^u	TFP informal sector	0.53
α	Labour share formal sector	0.6
ζ	Labour share informal sector	0.6
α_g	Public capital elasticity in production	0.053
F	Fixed cost informal sector	0.060
δ^p	Private capital depreciation rate	0.069
δ^g	Public capital depreciation rate	0.043
ξ^k	Private capital adjustment cost parameter	2.5
ξ^f	Adjustment cost parameter for froreign assets	0.05
ξ^c	Habit persistence	0.6
ψ	Elasticity of marginal depreciation costs	1.5814
β	Discount factor	0.9615
σ	Risk aversion	1
γ	Inverse of labour elasticity	1
ϑ	Substitutability/complementarity between private and public goods	0.05
π	Probability of detection	0.14
ϕ^F	Fine for firms	0.1
ϕ^W	Fine for households	0.5
$\theta^d, \theta^x, \theta^m$	Calvo parameters	0.35
μ^d	Markup - domestic market	1.35
μ^x	Markup - foreign market	1.1
μ^m	Markup - importing firms	1.35
x_d, x_x, x_m	Indexation parameters	0.256
ω_c	Home bias in the production of consumption goods	0.65
ω_i	Home bias in the production of investment goods	0.3
ε_c	Elasticity of substitution between imported and domestic consumption goods	3.351
ε_i	Elasticity of substitution between imported and domestic investment goods	6.352
ε_x	Elasticity of exports	1.463
$\overline{f}$	Target level of net private foreign assets-to-GDP ratio	0
x	Productivity of public spending on goods and services	0.29
G^c/Y	Govt intermediate cons./GDP	0.1024
G^i/Y	Govt investment /GDP	0.057
$W^g H^g/Y$	Govt wage bill /GDP	0.1307
G^{tr}/Y	Govt transfers/GDP	0.2059
τ^f	Tax rate on revenue	0.04
τ^l	Tax rate on labour	0.34
τ^k	Tax rate on capital	0.20
τ^c	Tax rate on consumption	0.15

Note: For the detection probability we use the ARTEMIS report (September 2013-November 2015) data on the (yearly average) number of firms inspected for the presence of undeclared and underdeclared dependent employment. The total number of firms with employees under private law contracts is retrieved from the ERGANI annual reports (average of 2014, 2015).

Table 2: Tax on Labour Income, Dual Economy, DSGE

TR^l/y^p	0.1616	0.1716	0.1816	0.1916	0.2016	0.2116	0.2216
τ^l	0.2893	0.3065	0.3233	0.3400	0.3564	0.3726	0.3884
s^u	0.2117	0.2238	0.2365	0.2500	0.2642	0.2791	0.2948
sh^u	0.3032	0.3179	0.3330	0.3485	0.3644	0.3806	0.3971
$d\tilde{y}^p$		-2.3916	-2.4648	-2.5376	-2.6093	-2.6794	-2.7473
$d\tilde{y}^T$		-1.5408	-1.5691	-1.5956	-1.6201	-1.6421	-1.6614
$d\tilde{y}^u$		3.4076	3.3487	3.2849	3.2163	3.1428	3.0648
$\frac{dTR^l}{d\tau^l}$		0.4128	0.3885	0.3640	0.3396	0.3153	0.2913
$\frac{dTR^t}{d\tau^l}$		0.1206	0.0907	0.0608	0.0313	0.0023	-0.0260

Note: TR^l= narrow (source) tax revenues, τ^l = tax rate, s^u= share of informal to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in narrow tax revenues as a share of GDP, $dTR^l/d\tau^l$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^l$ = change in total tax revenues from change in narrow (source) tax rate.

Table 3: Tax on Labour Income, Single Sector Economy, DSGE

TR^l/y^p	0.1485	0.1585	0.1685	0.1785	0.1885	0.1985	0.2085
τ^l	0.2842	0.3029	0.3215	0.3400	0.3584	0.3768	0.3951
$d\tilde{y}^p$		-1.0756	-1.0980	-1.1213	-1.1456	-1.1709	-1.1974
$\frac{dTR^l}{d\tau^l}$		0.7249	0.7098	0.6945	0.6789	0.6630	0.6467
$\frac{dTR^t}{d\tau^l}$		0.5503	0.5329	0.5151	0.4969	0.4783	0.4593

Table 4: Government Spending, Dual Economy, DSGE

G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
s^u	0.2569	0.2546	0.2523	0.2500	0.2477	0.2454	0.2430
sh^u	0.3539	0.3521	0.3503	0.3485	0.3467	0.3448	0.3430
$d\tilde{y}^p$		0.7619	0.7718	0.7818	0.7919	0.8022	0.8126
$d\tilde{y}^T$		0.6181	0.6273	0.6367	0.6461	0.6557	0.6654
$d\tilde{y}^u$		-0.0676	-0.0673	-0.0670	-0.0668	-0.0667	-0.0665
$\frac{dTR^t}{dG^c}$		0.1306	0.1315	0.1323	0.1331	0.1339	0.1347

Note: G^c = government consumption, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$ percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in government spending as a share of GDP, dTR^t/dG^c = change in total tax revenues from change in government spending.

Table 5: Government Spending, Single Sector Economy, DSGE

G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
$d\tilde{y}^p$		0.6903	0.7009	0.7117	0.7226	0.7336	0.7448
$\frac{dTR^t}{dG^c}$		0.1102	0.1117	0.1131	0.1145	0.1159	0.1172

deviation from the steady state. Recorded output declines by -2.5% and true output by -1.6%, that is, the recorded decline in economic activity exaggerates the true one by more than 50%. Row $d\tilde{y}^u$ shows that the higher tax rate induces and increase of the informal output by 3.3%. Rows $\frac{dTR^f}{d\tau^f}$ and $\frac{dTR^t}{d\tau^f}$ report the effect of the tax increase on narrow (source) and broad (from all sources) tax revenue. Income tax collections increase by 0.36 while total tax revenue increases by 0.060 units. Total revenue increases by less because the switch of activity to the informal sector decreases the tax base for the other taxes too.[16]

It is worth mentioning that the computed multipliers fall in the range reported in the empirical multiplier literature as well as that suggested by quantitative DSGE models (see Schmidt et al. (2015) for a comparative study of multipliers in the euro area), notwithstanding the large variation in existing estimates. For instance, Mertens and Ravn, 2014, report values in the range of 2 and 3.

Table 3 performs the same exercise in the version of the model without an underground economy. In this case, an (analogous) increase in the labor income tax (from a 34% to a 35.8%) exhibits a weaker negative effect on economic activity (-1.12%), a stronger positive effect on tax collection from that source and a positive (but smaller due the negative spillover on the tax base of the other taxes) effect on total tax collection. Comparison of Tables 2 and 3 shows that projections of the effects of a tax rise on output and tax revenue are bound to be significantly more optimistic when the model does not contain a shadow economy, as they do not take into account the migration of economic activity to the underground economy. And that the size of both the forecast error (the difference between Tables 2 and 3 in row $d\tilde{y}^p$) and the exaggeration of the true effect (the difference between rows $d\tilde{y}^p$ and $d\tilde{y}^T$ in Table 2 is increasing in the relative size of the black economy. The same property also characterizes the forecast error in projections of tax revenues (the differences between the –corresponding– last two rows across Tables 2 and 3).

The model's pessimistic predictions about tax revenue collection are clearly in line with the recent Greek experience. According to a recent report[17] "...Since 2010, Athens has introduced revenue boosting measures worth almost 37 billion EUR in total but the result is quite disappointing as the European Commission's official data show that state revenues have declined by 9.2 billion EUR in the same period...In the same period GDP has shrunk about 26%."

Tables 4 and 5 report the effects of changes in government spending, holding the distortionary tax rates constant and using lump sum taxes to balance the budget. Note that the

[16] Note that total tax revenue includes the average value of the fines collected from all the agents –firms and workers– detected.

[17] Hatzinikolaou and Nikas, Kathimerini, November 12, 2016.

share of the formal sector increases with an increase in the share of government spending as the latter type of spending falls exclusively on formal goods. The main finding is that while the effects are qualitatively similar across the two types of fiscal consolidation (like a tax hike, lower public spending increases the share of the informal sector), both the forecast and measurement errors are quite small: model misspecification regarding the shadow economy does not distort quantitatively the implications regarding the effects of changes in government spending (spending multipliers). It suggests that fiscal consolidation is more likely to succeed and its outcome is less uncertain[18] when it relies on spending rather than on tax measures.

These findings are consistent with and can also provide a possible explanation for the those reported in Alesina and Giavazzi, 2015. They argue that "...The accumulated evidence from over 40 years of fiscal adjustments across the OECD speaks loud and clear: ... adjustments achieved through spending cuts are less recessionary than those achieved through tax increases...". We find this to be the case not only in a standard model and for official output but also in the dual economy with an informal sector and for true output. They also argue that "...only spending-based adjustments have eventually led to a permanent consolidation of the budget, as measured by the stabilisation (at least) if not reduction of debt-to-GDP ratios...". Our analysis provides an explanation for this finding that does not hinge on implausibly large labor elasticities. Even when the total supply of labour is inelastic, as is commonly accepted, changes in tax rates and real wages may still matter for employment in the official sector if labor can move across sectors.

Our framework thus implies that from the point of view of debt sustainability, and to a much larger extent than that predicted by the standard, one sector model, spending reductions are more potent means for improving the fiscal position and restraining debt growth than tax increases. Moreover, their effects are less likely to be distorted by the presence of a shadow economy, which helps make policy more reliable.

In the next section, we will use the actual paths of the fiscal instruments in the model during the consolidation period in order to evaluate their quantitative effects on macroeconomic activity and tax revenue in Greece during the crisis.

3 Fiscal Consolidation in Greece

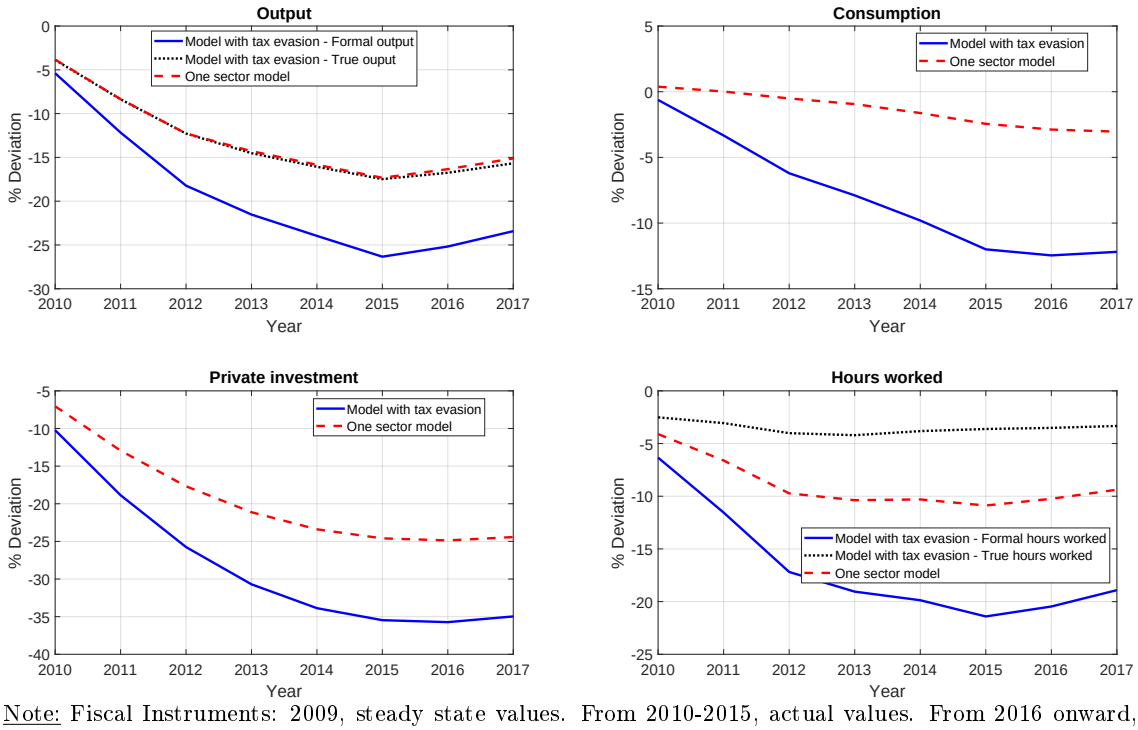
We solve the model under two alternative, informational assumptions about the paths of the fiscal instruments: perfect foresight; and random walk. Under the former, we start

[18] Less uncertain because the computation of spending multipliers is less sensitive to the size of the black economy, which is hard to measure.

the economy in its steady state and then plug in the model the actual values of the fiscal (tax-spending) instruments for the period 2010-2015. After 2015, the fiscal instruments are assumed to gradually return to their pre-crisis 2009 values. In particular, we assume that they follow an autoregressive process using as initial values the 2015 values and an autoregressive coefficient equal to 0.9. We allow lump-sum transfers to fill any government financing gaps. Under the random walk informational assumption, we assume that during the consolidation period, people expect the current fiscal policy stance to remain the same in the next period, so any change is perfectly unanticipated. In reality, some of the changes were known as the plans were drawn for more that one year. But at the same time, there were many ex post, unanticipated changes as often the plans had to be revised mid-course due to failure to achieve the deficit-debt paths and new, harsher fiscal measures had to be introduced. So the true expectations may lie somewhere between these two polar extremes. We present the results from the perfect foresight exercise below and delegate the results from the random walk exercise (which are quite similar) to the Appendix.

The actual paths of the fiscal instruments are depicted in Figure 7 in the Appendix. A discussion of the computation of the tax rates can be found in Appendix 7.1.

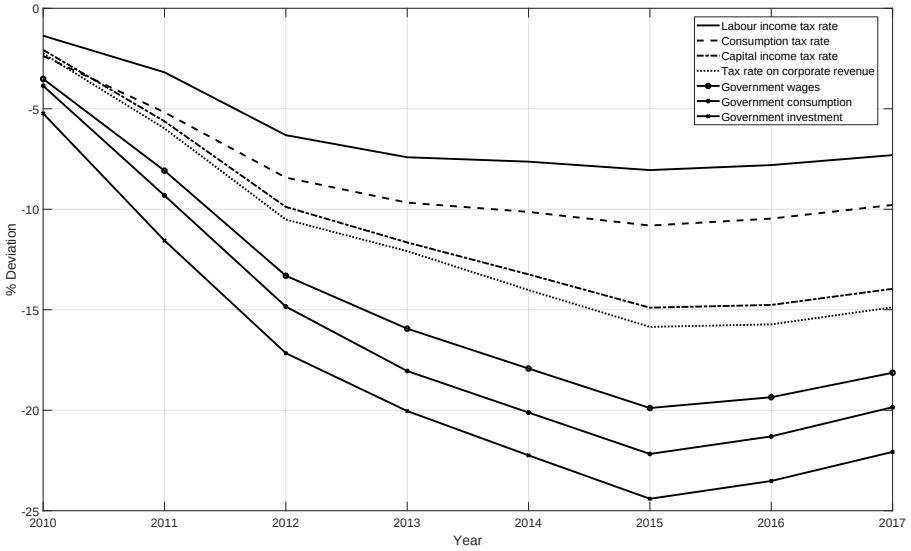
Figure 1: The effects of the Greek fiscal consolidation package on economic activity: Perfect foresight



Note: Fiscal Instruments: 2009, steady state values. From 2010-2015, actual values. From 2016 onward, projected values under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9.

The model is solved non-linearly in Dynare. Figure 1 plots the path of formal and true (total) GDP, consumption, private investment and employment along with the paths of the corresponding variables of the one sector model. The dual economy model implies a cumulative decline in GDP by 2015 that is similar to that observed in the data (26%). The *true* decline, though, is significantly lower (about 17%). Interestingly, the decline predicted by the model that abstracts from the underground economy is fairly close to the true one. The differences across the three measures are greater for employment. The recorded cumulative decrease in employment (hours) is 21%, that implied by the single sector model is 11% and the true one is only 4%. Note that there is nothing in the calibration that targets the decline in economic activity; and that it is not problematic for a model to explain all of the actual decline as this is not a variance decomposition exercise.

Figure 2: Contribution of individual, fiscal instruments: Perfect foresight



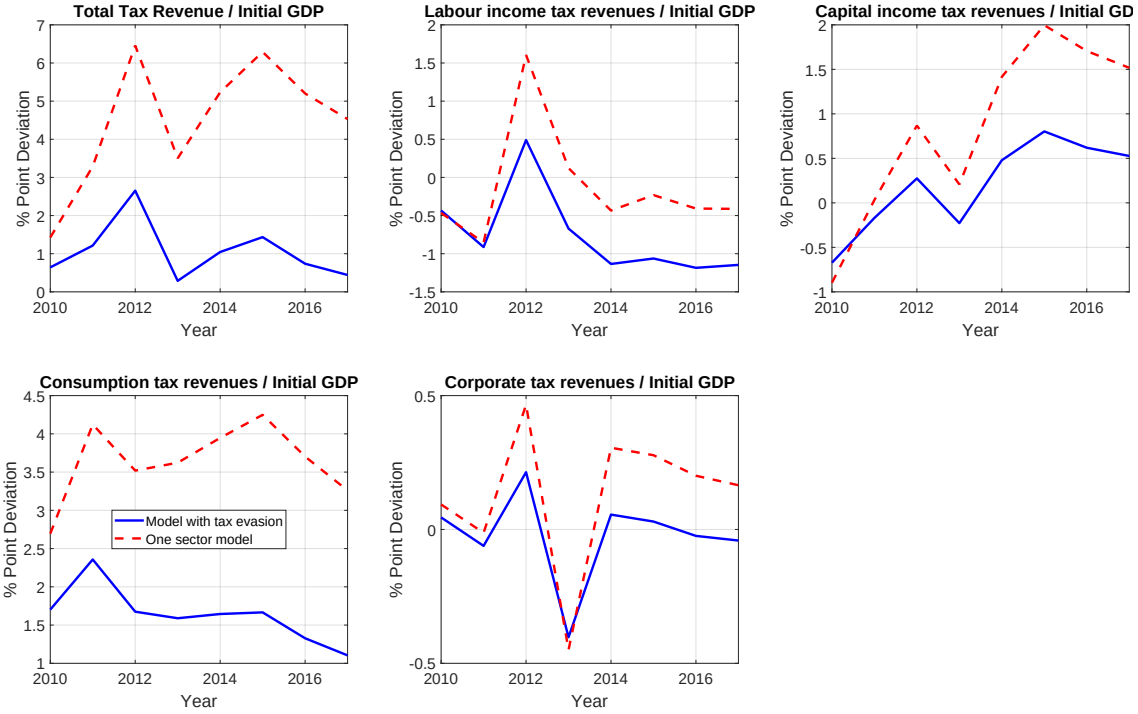
Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

Figure 2 plots the effects of the individual fiscal instruments. It shows that the labour income tax had the biggest effect, accounting for about 1/3 of the decrease in GDP (8%). It is followed by the capital income tax and the decrease in the public employment wage bill (4% each). In general, tax increases accounted for about 2/3 of the total decline in official output and spending cuts for the remaining 1/3.

Figure 3 depicts the paths of the individual tax revenue categories over the period 2010–2015 in the economy with and without an informal sector. Note that these paths are plotted against the path of all the fiscal instruments during the consolidation period. They do not thus capture the corresponding tax Laffer curves as well as Table 3 does. Nonetheless, they are quite revealing regarding the size of over-optimism in tax revenue projections as well

as the limited revenue effectiveness of the package implemented. In particular, Figure 3 shows that while the single sector model predicts that the effect of the tax package adopted would have increased the tax revenue to GDP share in 2012 by more than six percentage points, the increase in the dual economy is less than half of that.

Figure 3: The effects of individual taxes on tax revenue with and without tax evasion: Perfect foresight



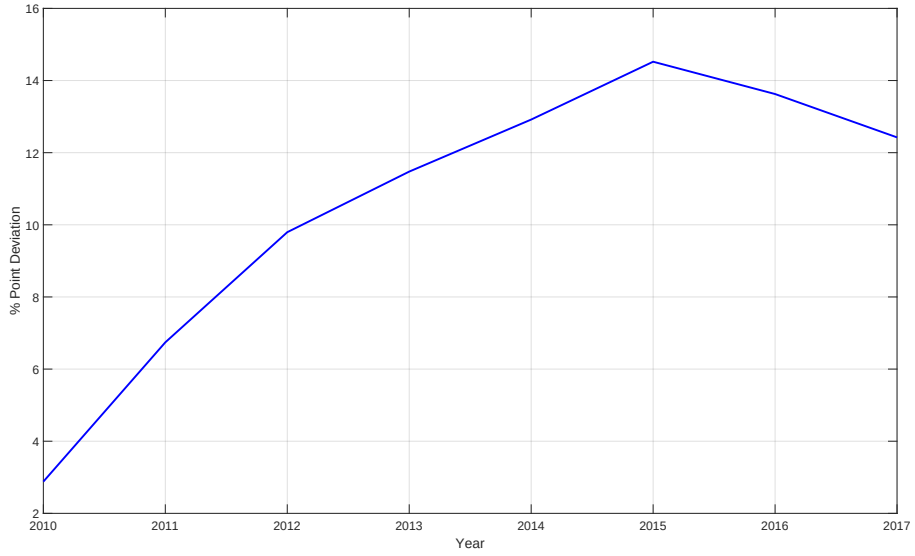
Note: In each graph, the path of the corresponding fiscal variable is as described in the footnote of Figure 1, while the remaining fiscal instruments are held at their steady state values.

Figure 4 plots the implied path of the shadow economy during the consolidation period. The higher taxes led to substantial diversion of activities towards the informal sector. Its share, according to the model, increased by 13 percentage points from 2010 and 2015. The model's prediction about an increase is born out by: a) anecdotal evidence; b) reports on the outcomes of tax inspections by the Independent Agency for Public Finances (see www.aade.gr).

4 Counterfactuals

In order to highlight the role played by the informal sector in shaping the outcomes of the fiscal consolidation experience in Greece, we perform several counterfactuals. We also use the recent imposition of capital controls in the model to get estimates of its effects on the

Figure 4: Informal output as a share of GDP



size of the black economy and GDP growth.

In the first counterfactual, we feed the actual paths of the fiscal instruments into the model with tax evasion –exactly as we did in the previous section– while at the same time varying the probability of detection in order to keep the size of the informal economy constant at its pre crisis level. This exercise is meant to answer the question of how the Greek economy would have fared under the exact same policy package if it had managed to restrict growth in the black economy. The model now predicts a cumulative decline in GDP by 19.5% and of true output by 16% (instead of 26% and 17.5% when the black economy is allowed to grow); see Figure 5. That is, limiting growth in informal activities would have saved 1.5% of *total* output(17.5% − 16%). It would have also led to significantly higher tax revenue collection (Figure 6).

This value represents a *lower* bound because it does not take into account the positive effects of a smaller black economy on tax revenue/tax rates. To get a sense of the importance of these effects, we have run another counterfactual where we keep the size of the informal economy fixed at the pre-crisis level, hold the paths of all fiscal instruments besides the income tax fixed at their historical values and solve for the –constant– income tax rate that can support extra annual tax revenue of 1.15% and 2.5% of steady state GDP respectively. The former figure corresponds to the average change in tax revenue over 2010-2015 implied by the model economy with the black sector when fed by the actual fiscal package and without restricting the size of the informal sector (that is, to the economy studied in section 3). The latter number corresponds to the maximum annual change in tax revenue

Figure 5: Fixing the size of the informal sector: Effects on economic activity

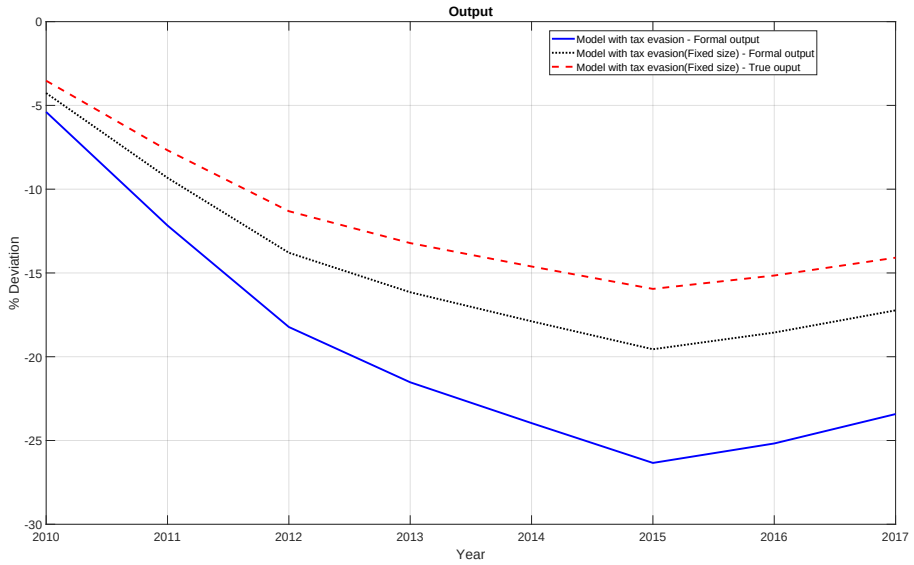


Figure 6: Fixing the size of the informal sector: Effects on tax revenue

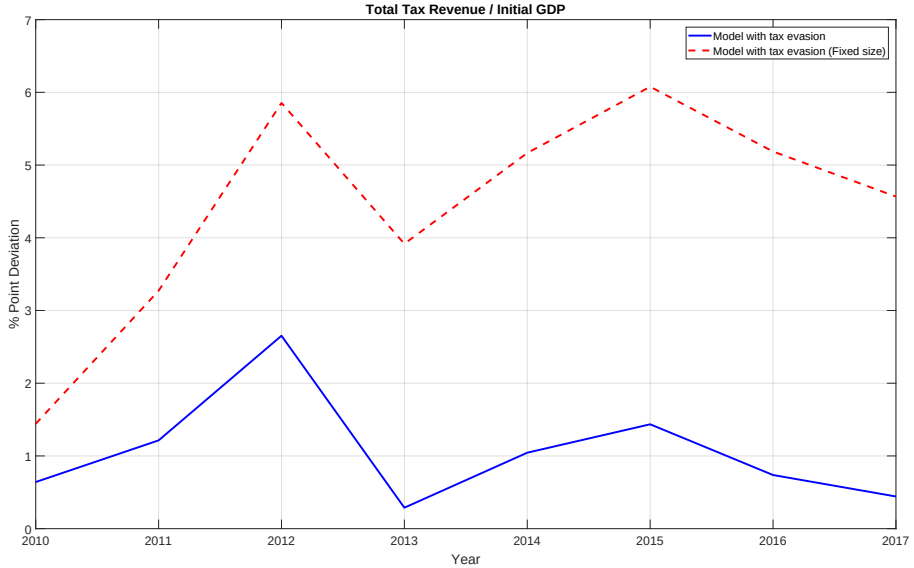


Table 6: Counterfactual: Fiscal consolidation when fixing the size of the informal sector for certain tax revenue requirements

	GDP	true output
Increase in tax revenues by 1.15% $\tau^l = 0.324$	-11.86%	-9.74%
Increase in tax revenues by 2.52% $\tau^l = 0.354$	-14.2%	-11.64

Note: Labor income tax–relative to its steady state value– needed to raise 1.15% and 2.52% extra –relative to the steady state– tax revenue per annum when all other fiscal instruments take their 2010–2015 historical values and the black economy is fixed at its pre-crisis level.

during that period implied by that model.

Table 4 shows that controlling the size of the black economy would have had strong beneficial effects on the tax rate (for any tax revenue target) and thus on economic activity. In particular, in the former case, the implied income tax rate is 32.4% that is, it represents a tax reduction relative to the pre-crisis rate of 34% and is far below the rate contained in the actual fiscal package (41% by 2015). In the latter case, there is a slight increase from 34% to 35.4%. The implications for economic activity are astounding. Official output drops by only 11.6% (14.2%) and true output by 9.7% (11.6%), that is, the implied recession in official activity is quite milder than that in the benchmark economy (26%).

We have also carried out similar counterfactuals involving the complete elimination of the shadow economy in the beginning of the program. The results are quite similar –with stronger positive effects– and are reported in the appendix.

The crucial role played by the black sector in the failure of the fiscal consolidation experience in Greece and thus the importance of measures to control tax evasion for tax revenue and economic activity can be further illuminated by considering the natural experiment that took place in Greece in the summer of 2015, namely, the imposition of capital controls. In order to prevent a bank run triggered by doubts about Greece's Eurozone membership, the Greek government imposed limits on the amount of cash that depositors could withdraw from their bank accounts. Hondroyannis and Papaoikonomou, 2017, report that this forced a switch from cash to card payments: the share of card payments in private consumption went from 4% to 12% over the next year. This led to a massive, unexpected increase in VAT revenue. Using their estimated elasticity of VAT revenue to credit card usage, our model implies that the capital controls may have contributed during the year following their introduction to an increase in official economic activity by 2.6% and to a reduction in the share of the informal sector by almost five percentage points. The cumulative effect

Table 7: Counterfactual experiment: Impact of the imposition of capital controls

	GDP		True output		Informal/formal		Prob. detection	
	2015	2016	2015	2016	2015	2016	2015	2016
Baseline	-2.38	-1.16	-1.41	0.72	1.60	-0.90	0.14	0.14
Counterfactual	2.64	4.23	-0.51	1.63	-4.78	-4.09	0.33	0.40

Note: GDP, true output: percentage change from previous year;informal/formal share: percentage points, change relative to previous year.

after two years is 7% and 9 percentage points, respectively.[19]

These results are striking. They help establish quantitatively that the existence of a large shadow economy made fiscal consolidation in Greece a much more challenging endeavor than it would have otherwise been: it led to "excessive" increases in tax rates that did not translate into large increases in tax revenue, but led instead to a severe and protracted downturn. Had the informal economy been better controlled, the fiscal consolidation could have been milder both in terms of the tax burden for households and firms and in terms of the implied output loss.

4.1 Sensitivity

We have carried out an extensive set of sensitivity exercises. In particular, we have also considered specifications of the model with the following features: a) Ricardian and non-Ricardian households; b) imperfect competition in the labour markets with labour unions setting wages; c) nominal and real wage rigidity; d) inclusion of the public wage bill in government consumption; e) variation of the following parameters: (i) value of 0.5 –instead of 0.9– for the persistence parameter in the fiscal instruments autoregressive rules after 2015; (ii) a value of zero for the fine on workers who are caught working in the informal sector (ϕ^W); (iii) a value of two for the coefficient of relative risk aversion σ; (iv) a value of two for the Frisch elasticity γ; (v) we also varied the Calvo parameters, the disutility from working in the informal sector as well as the fine imposed on tax evading firms.

Some of these exercises are reported in the Appendix while the remaining are available from the authors. Figures 9 and 12 and Tables 19 and 22 show the results associated with the model that contains features (a)-(d). This version of the model implies a decline in GDP of about 26% too. One can thus conclude that the quantitative results reported above are very robust to changes in the specification of the model. We also find that the

[19] A caveat is in order. Our model abstracts from negative effects of capital controls, such as their impact on business and import financing. To the extent that capital controls have other negative side effects, the numbers reported here should be interpreted as the upper bound of the possible effects.

greatest sensitivity is with regard to the coefficient of relative risk aversion.

5 Conclusion

Following the outbreak of the sovereign debt crisis in Greece, the country undertook an ambitious fiscal consolidation program. At the outset of the program, Greece's official creditors were predicting that the adjustment, while substantial, would be manageable and that the resulting recession would be limited and short lived. The actual experience defied these predictions by a wide margin. Tax rates kept on increasing, yet tax revenue grew little with the public debt to GDP ratio exploding. And the economy plunged into a deep and protracted recession.

Our paper has provided an explanation for these facts that centers on the existence of a substantial informal sector in Greece. Failure to account for this sector led to overoptimistic projections about the size of the required fiscal adjustment as well as about the severity of GDP and employment contraction. Had the model underlying the projections contained a dual economy, the predictions would have been quite more pessimistic (and more realistic) both on the tax revenue and the economic activity front.

We have also argued that the true (total) decline in economic activity may have been considerably smaller than that reported about official activity. That is, recorded "austerity" may have appeared more severe than it actually was because of the omission of the informal economy.

Finally, we have highlighted the important, negative role played by the black economy for the outcomes of in the fiscal consolidation experience in Greece by arguing that, had the Greek government been able to prevent further growth in the black economy, the recession would have been significantly milder. The key policy lesson learned from this is that a better designed fiscal adjustment program should focus more on tax compliance issues. The unintended positive effects on tax revenue and official activity of the capital controls imposed in 2015 in Greece and which led to a large switch from cash to credit card payments, thus reducing black market activities, provides strong support to this assertion.

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7 Appendix

7.1 Tax rate computation, 2010-2015

The methodology followed for constructing the effective tax rates is based on the work of Mendoza et al. (1994). Broadly speaking, the effective tax rates are estimated as the ratios between the tax revenues from particular taxes and the corresponding tax bases using information from the National Accounts. The data set comprises of annual data that cover the period 2000-2015. The data source is Eurostat. The macroeconomic variables used for the computation of the effective tax rates are:

- HY: Taxes on individual or household income including holding gains
- $WSSE$: Compensation of employees
- $SSCER$: Employers' actual social security contributions
- $SSCH$: Households' actual social security contributions
- $GOSH$: Gross operating surplus and mixed income of households
- $CFCH$: Consumption of fixed capital of households
- $IYRH$: Interest income received by households
- $IYPH$: Interest income paid by households
- $CORY$: Taxes on the income or profits of corporations including holding gains
- $STAMP$: Stamp taxes
- $TFCT$: Taxes on financial and capital transactions
- TLG: Taxes on winnings from lottery or gambling
- CTC: Current taxes on capital
- CAT: Capital taxes
- OTP: Other taxes on production
- $NFYT$: Taxes on income paid by non-financial corporations
- FYT: Taxes on income paid by financial corporations
- GOS: Gross operating surplus, Total economy
- CFC: Consumption of fixed capital, Total economy
- TPI: Taxes on production and imports
- GIC: Intermediate consumption, government
- HC: Household and NPISH final consumption expenditure
- $GOSNFC$: Gross operating surplus and mixed income of non-financial corporations
- $GOSFC$: Gross operating surplus and mixed income of financial corporations
- $CFCNFC$: Consumption of fixed capital of non-financial corporations
- $CFCFC$: Consumption of fixed capital of financial corporations

7.1.1 Tax rate on personal income

Tax revenue data do not provide a breakdown of tax revenue from individual labour and capital income. In order to decompose tax revenue from labour and capital income of households, we follow Mendoza et al. (1994) and compute a personal income tax rate that applies both to labour and capital income of households:

$$\tau^h = \frac{HY}{(WSSE - SSCH - SSCER) + (GOSH - CFCH) + (IYRH - IYPH)} \tag{5}$$

Effective tax rate on employed labour income

The effective tax rate on labour income is computed as the ratio of labour income taxes to the labour income of employees:

$$\tau^l = \frac{\tau^h (WSSE - SSCH - SSCER) + SSCH + SSCER}{WSSE} \tag{6}$$

Effective tax rate on capital income

The effective tax rate on capital income is computed as the ratio of capital income taxes to the capital income:

$$\tau^k = \frac{\tau^h (GOSH - CFCH + IYRH - IPRH) + CAPT}{GOS - CFC} \tag{7}$$

where $CAPT = TFCT + CAT + TLG + OTP + STAMP + CTC + CORY$ are capital income tax revenue.

Effective tax rate on consumption

The effective tax rate on consumption corresponds to the difference between the post-tax consumer price and the pre-tax price at which firms supply the consumption good.

$$\tau^c = \frac{CT}{HC + GIC - CT} \tag{8}$$

where $CT = TPI - TFCT - TLG - OTP$ are total tax revenue from indirect taxation, which by definition are equal to the difference between the nominal value of aggregate consumption at post-tax and pre-tax prices. Note that we deduct the categories $TFCT, TLG$ and OTP, from TPI since these categories include mainly capital and labour income taxes. The denominator is the base of the consumption tax, which is the pre-tax value of consumption.

Tax rate on corporate revenue

In computing the tax rate on corporate revenue (sales) we assume that the tax rate is proportional to the tax rate on corporate income. Specifically we calculate the tax rate on corporate revenue as:

$$\tau^s = \tau^{corp} \frac{profits}{sales} \tag{9}$$

where τ^{corp} is the effective tax rate on corporate income estimated as:

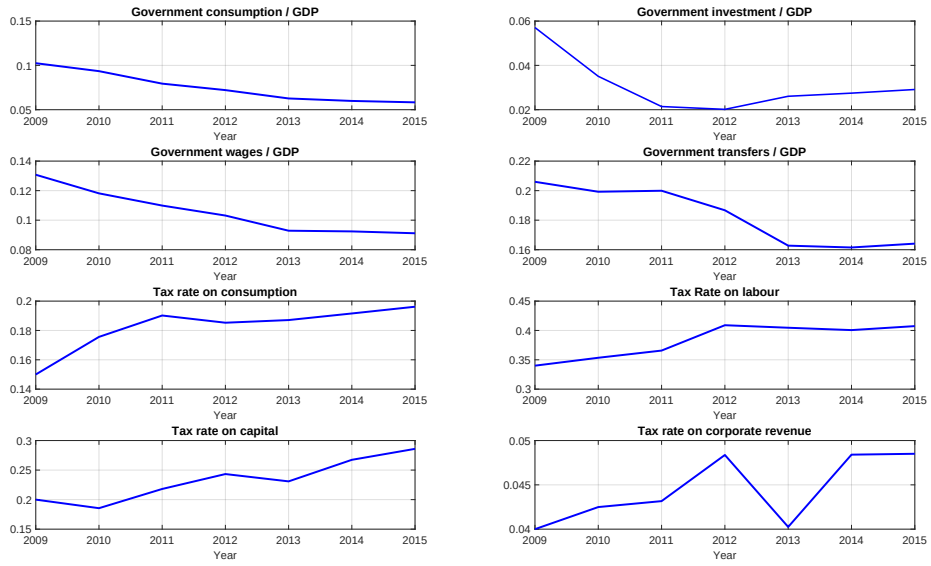
$$\tau^{corp} = \frac{FYT + NFYT}{GOSNFC + GOSFC - CFCNFC - CFCF} \tag{10}$$

The data source for total *profits* (profits before taxes and depreciation) and total *sales* correspond to the aggregate of these measures for the Greek listed firms for the years 2007-2008. The datasource is DataStream.

7.2 Actual Fiscal Instrument Paths, 2010-15

Here we report the actual paths taken by all the fiscal instruments used in the model over the period 2009-2015. The construction of the tax rates follows the description above.

Figure 7: Actual fiscal instrument paths



Note:Government consumption, investment and the public sector wage bill are expressed as shares of the 2009 GDP.

7.3 BOGGEM: Additional Fiscal Instruments, Baseline Version

7.3.1 Steady State Results

Table 8: Tax on Firm Revenue, Dual Economy, BOGGEM

TR^f/y^p	0.0109	0.0209	0.0309	0.0409	0.0509	0.0609
τ^f	0.0112	0.0256	0.0400	0.0545	0.0691	0.0837
s^u	0.2242	0.2367	0.2500	0.2641	0.2790	0.2949
sh^u	0.3217	0.3349	0.3485	0.3625	0.3768	0.3915
$d\tilde{y}^p$		-2.7325	-2.8065	-2.8829	-2.9614	-3.0421
$d\tilde{y}^T$		-1.8445	-1.8683	-1.8915	-1.9141	-1.9360
$d\tilde{y}^u$		3.0281	2.9928	2.9554	2.9157	2.8737
$\frac{dTR^f}{d\tau^f}$		0.7305	0.6900	0.6498	0.6100	0.5707
$\frac{dTR^t}{d\tau^f}$		-0.0648	-0.0907	-0.1156	-0.1395	-0.1622

Note: TR^f= narrow (source) tax revenues, τ^f = tax rate on firm revenue, s^u= share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenues as share of GDP, $dTR^f/d\tau^f$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^f$ = change in total tax revenues from change in the tax rate.

Table 9: Tax on Firm Revenue, Single Sector Economy, BOGGEM

TR^f/y^p	0.0174	0.0274	0.0374	0.0474	0.0574
τ^f	0.0254	0.0400	0.0547	0.0694	0.0842
$d\tilde{y}^p$		-1.4480	-1.4663	-1.4854	-1.5053
$\frac{dTR^f}{d\tau^f}$		1.0352	1.0051	0.9753	0.9456
$\frac{dTR^t}{d\tau^f}$		0.3116	0.2918	0.2721	0.2526

Table 10: Consumption Tax, Dual Economy, BOGGEM

TR^c/y^p	0.0692	0.0792	0.0892	0.0992	0.1092	0.1192	0.1292
τ^c	0.1037	0.1191	0.1345	0.1500	0.1656	0.1814	0.1972
s^u	0.2189	0.2290	0.2393	0.2500	0.2610	0.2724	0.2842
sh^u	0.3231	0.3315	0.3400	0.3485	0.3570	0.3655	0.3740
$d\tilde{y}^p$		-1.3899	-1.3939	-1.3979	-1.4017	-1.4055	-1.4091
$d\tilde{y}^T$		-0.8817	-0.8781	-0.8745	-0.8709	-0.8672	-0.8634
$d\tilde{y}^u$		1.8531	1.8090	1.7661	1.7243	1.6836	1.6440
$\frac{dTR^c}{d\tau^c}$		0.6410	0.6204	0.6002	0.5804	0.5611	0.5421
$\frac{dTR^t}{d\tau^c}$		0.4604	0.4437	0.4275	0.4117	0.3963	0.3813

Note: TR^c = narrow (source) tax revenues,τ^c = tax rate on consumption, s^u= share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenue as share of GDP, $dTR^c/d\tau^c$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^c$ = change in total tax revenues from change in the tax rate.

Table 11: Consumption Tax, Single Sector Economy, BOGGEM

TR^c/y^p	0.0692	0.0792	0.0892	0.0992	0.1092	0.1192	0.1292
τ^c	0.1043	0.1195	0.1347	0.1500	0.1653	0.1807	0.1960
$d\tilde{y}^p$		-0.5569	-0.5500	-0.5433	-0.5368	-0.5304	-0.5242
$\frac{dTR^c}{d\tau^c}$		0.9938	0.9821	0.9708	0.9597	0.9489	0.9384
$\frac{dTR^t}{d\tau^c}$		0.8711	0.8619	0.8529	0.8441	0.8356	0.8273

Table 12: Capital Income Tax, Dual Economy, BOGGEM

TR^k/y^p	0.0226	0.0326	0.0426	0.0526	0.0626	0.0726	0.0826
τ^k	0.0849	0.1229	0.1613	0.2000	0.2391	0.2788	0.3189
s^u	1.2940	1.2728	1.2506	1.2273	1.2027	1.1767	1.1492
sh^u	0.3093	0.3217	0.3348	0.3485	0.3630	0.3782	0.3943
$d\tilde{y}^p$		-3.7266	-3.8919	-4.0751	-4.2797	-4.5102	-4.7731
$d\tilde{y}^T$		-2.7450	-2.8192	-2.8992	-2.9863	-3.0817	-3.1878
$d\tilde{y}^u$		3.2066	3.2476	3.2909	3.3371	3.3872	3.4431
$\frac{dTR^k}{d\tau^k}$		0.2735	0.2511	0.2286	0.2061	0.1836	0.1611
$\frac{dTR^t}{d\tau^k}$		0.0086	-0.0144	-0.0373	-0.0600	-0.0826	-0.1051

Note: TR^k = narrow (source) tax revenues, τ^k = tax rate on capital income, s^u = share of informal output to GDP, sh^u = ratio of informal to total hours worked, $d\tilde{y} = d\log y$ is the percentage change in output (formal, p, informal, u, or true, T) for a one percentage point change in the narrow tax revenue as share of GDP, $dTR^k/d\tau^k$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^k$ = change in total tax revenues from change in the tax rate.

Table 13: Capital Income Tax, Single Sector Economy, BOGGEM

TR^k/y^p	0.0226	0.0326	0.0426	0.0526	0.0626	0.0726	0.0826
τ^k	0.0852	0.1233	0.1615	0.2000	0.2387	0.2777	0.3170
$d\tilde{y}^p$		-2.4169	-2.4972	-2.5865	-2.6864	-2.7989	-2.9265
$\frac{dTR^k}{d\tau^k}$		0.4054	0.3848	0.3641	0.3430	0.3217	0.2999
$\frac{dTR^t}{d\tau^k}$		0.1702	0.1472	0.1237	0.0996	0.0747	0.0488

Table 14: Government Investment, Dual Economy, BOGGEM

G^i/y^p	0.0270	0.0370	0.0470	0.0570	0.0670	0.0770
s^u	0.2986	0.2778	0.2623	0.2500	0.2398	0.2311
sh^u	0.3852	0.3700	0.3582	0.3485	0.3402	0.3329
$d\tilde{y}^p$		4.6122	3.6295	3.0293	2.6233	2.3295
$d\tilde{y}^T$		3.1137	2.5176	2.1463	1.8915	1.7048
$d\tilde{y}^u$		-2.9782	-2.3565	-1.9613	-1.6865	-1.4832
$\frac{dTR^t}{dG^i}$		1.1249	0.8677	0.7097	0.6028	0.5254

Note: G^i = government consumption, y^u/y^p = ratio of informal to formal output, h^u/h^p = = ratio of informal to formal employment, $d\widehat{y} = d\log y$ percentage change in output (formal, p, informal, u, or total, T) for a one percentage point change in government spending as a share of GDP, dTR^t/dG^i = change in total tax revenues from change in government spending.

Table 15: Government Investment, Single Sector Economy, BOGGEM

G^i/y^p	0.0270	0.0370	0.0470	0.0570	0.0670	0.0770
$d\tilde{y}^p$		2.8508	2.3239	1.9993	1.7782	1.6174
$\frac{dTR^t}{dG^i}$		0.7274	0.5730	0.4779	0.4133	0.3663

Table 16: Government Wage Bill, Dual Economy, BOGGEM

$W^g H^g / y^p$	0.1007	0.1107	0.1207	0.1307	0.1407	0.1507	0.1607
s^u	0.2596	0.2564	0.2532	0.2500	0.2468	0.2436	0.2404
sh^u	0.3491	0.3489	0.3487	0.3485	0.3483	0.3481	0.3478
$d\tilde{y}^p$		1.2283	1.2436	1.2593	1.2755	1.2920	1.3090
$d\tilde{y}^T$		1.0118	1.0267	1.0419	1.0575	1.0735	1.0900
$d\tilde{y}^u$		-0.0076	-0.0077	-0.0079	-0.0081	-0.0083	-0.0085
$\frac{dTR^t}{dW^g H^g}$		0.3547	0.3548	0.3548	0.3548	0.3548	0.3548

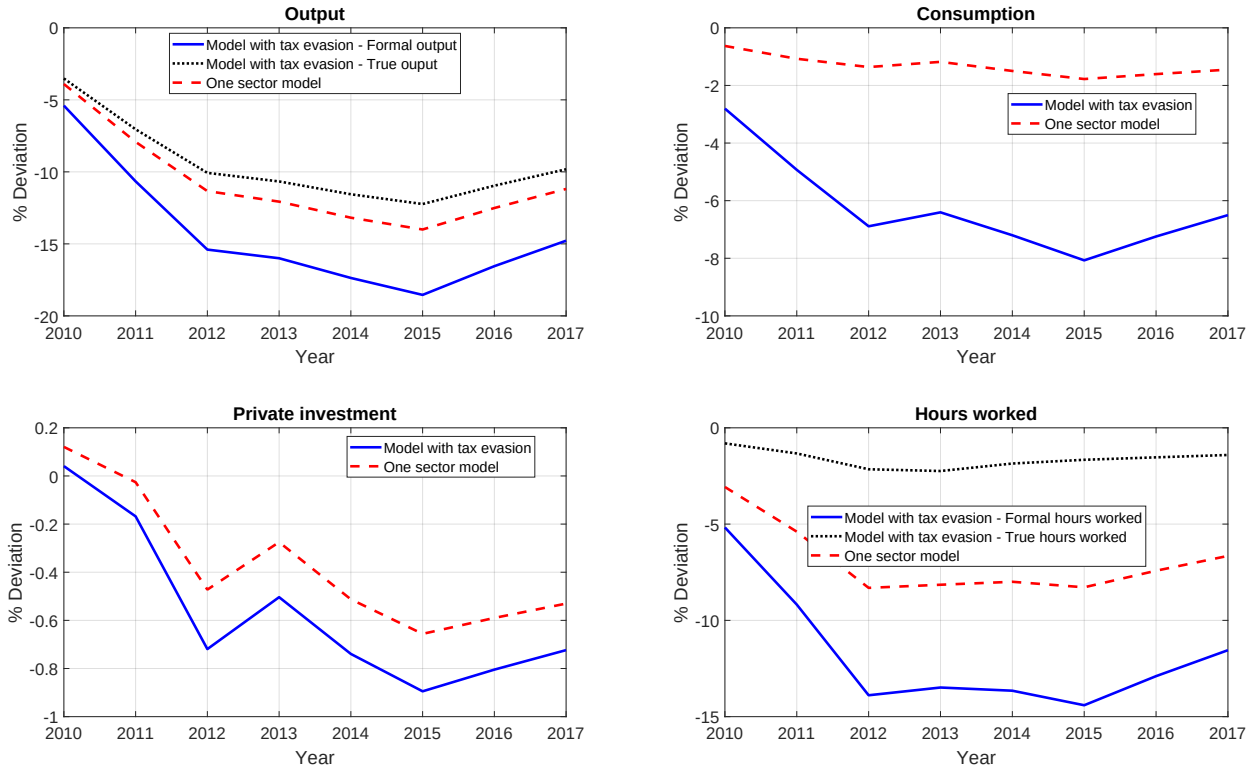
Note: $W^g H^g$ = public employment wage bill, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d \log y$ percentage change in output (formal, p, informal, u, or total, T) for a one percentage point change in government spending as a share of GDP, $dTR^t / dW^g H^g$ = change in total tax revenues from change in government wage bill.

Table 17: Government Wage Bill, Single Sector Economy, BOGGEM

$w^g h^g / GDP$	0.1007	0.1107	0.1207	0.1307	0.1407	0.1507	0.1607
$d\tilde{y}^p$		1.2192	1.2343	1.2499	1.2658	1.2821	1.2988
$\frac{dTR^t}{dW^g H^g}$		0.3525	0.3526	0.3526	0.3526	0.3526	0.3526

7.3.2 Dynamics Under a Random Walk

Figure 8: The effects of the Greek fiscal consolidation package on economic activity: Random walk



Note: 2010-2015: Actual values of fiscal instruments; 2015 onward: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

7.4 Counterfactuals: Complete Elimination of the Informal Sector in the Beginning of the Consolidation

We calculate the paths of the *extra* revenue raised in the dual economy in Greece from 2010 to 2015 (relative to the steady state). We then ask how much labour and capital income taxes would have to increase in each period – relative to their steady state values – in the economy that is free of an informal sector in order to generate this extra revenue. We carry this exercise using the average annual (about 1 billion EUR) and the maximum annual value (about 2 billion EUR) of the change in the tax revenues in the dual economy observed over the 2010-2015 period.

Table 18 reports the corresponding average tax rates in the single sector model as well as the cumulative total change in GDP. As can be seen in the Table, the changes in the tax rates are quite small. For instance, raising the tax rates from $\tau^l = 0.34$ to 0.342, and from $\tau^k = 0.20$ to 0.201 could generate about 1 more billion EUR.

The general equilibrium effects on economic activity are also significant. The cumulative decline in economic activity would have been 10.2% rather than the –true– decline of 17% in the dual economy. That is, in the absence of an informal sector, Greece would have been able to raise the revenue it actually did by relying in much smaller increases in tax rates and would have experienced a recession of the order of -2% per year, much less than the -5% recorded. This is a big difference.

Table 18: A Counterfactual: Fiscal consolidation without a black economy

		Impact on GDP
1 billion pa	$\tau^l : 0.34 \to 0.342$, $\tau^k : 0.20 \to 0.201$	-10.2%
2 billions pa	$\tau^l : 0.34 \to 0.365$, $\tau^k : 0.20 \to 0.215$	-12.4%

Note: Labor and capital income taxes–relative to their steady state values– needed to raise the amount of tax revenue raised per annum (pa) by the actual Greek fiscal instruments over 2010–2015 in the dual economy.

7.5 Sensitivity Analysis

7.5.1 All inclusive BOGDSGE: Ricardian and non Ricardian Agents, Labour Unions, Wage Rigidity

Table 19: Tax on Labour Income, Dual Economy

TR^l/y^p	0.1616	0.1716	0.1816	0.1916	0.2016	0.2116	0.2216
τ^l	0.2890	0.3062	0.3232	0.3400	0.3566	0.3729	0.3890
s^u	0.2127	0.2245	0.2369	0.2500	0.2638	0.2783	0.2934
sh^u	0.3039	0.3184	0.3332	0.3485	0.3641	0.3801	0.3963
$d\tilde{y}^p$		-2.3059	-2.3659	-2.4256	-2.4845	-2.5453	-2.5984
$d\tilde{y}^T$		-1.4695	-1.4869	-1.5029	-1.5171	-1.5924	-1.5396
$\frac{dTR^l}{d\tau^l}$		0.3497	0.3315	0.3133	0.2952	0.2773	0.2597
$\frac{dTR^t}{d\tau^l}$		0.1031	0.0815	0.0601	0.3090	0.0184	-0.0016

Note: TR^l= tax revenue from labor income,τ^l = tax rate on labor income, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, or true, T) for a one percentage point change in the narrow (source) tax revenue as share in GDP, $dTR^l/d\tau^l$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^l$ = change in total tax revenue (from all sources) from change in the tax rate.

Table 20: Tax on Labour Income, Single Sector Economy

TR^l/y^p	0.1485	0.1585	0.1685	0.1785	0.1885	0.1985	0.2085
τ^l	0.2743	0.2956	0.3175	0.3400	0.3631	0.3870	0.4116
$d\tilde{y}^p$		-1.3125	-1.3752	-1.4453	-1.5243	-1.6142	-1.7175
$\frac{dTR^l}{d\tau^f}$		0.5975	0.5830	0.5679	0.5522	0.5356	0.5182
$\frac{dTR^t}{d\tau^l}$		0.4336	0.4177	0.4010	0.3836	0.3651	0.3456

Table 21: Government Spending, Dual Economy

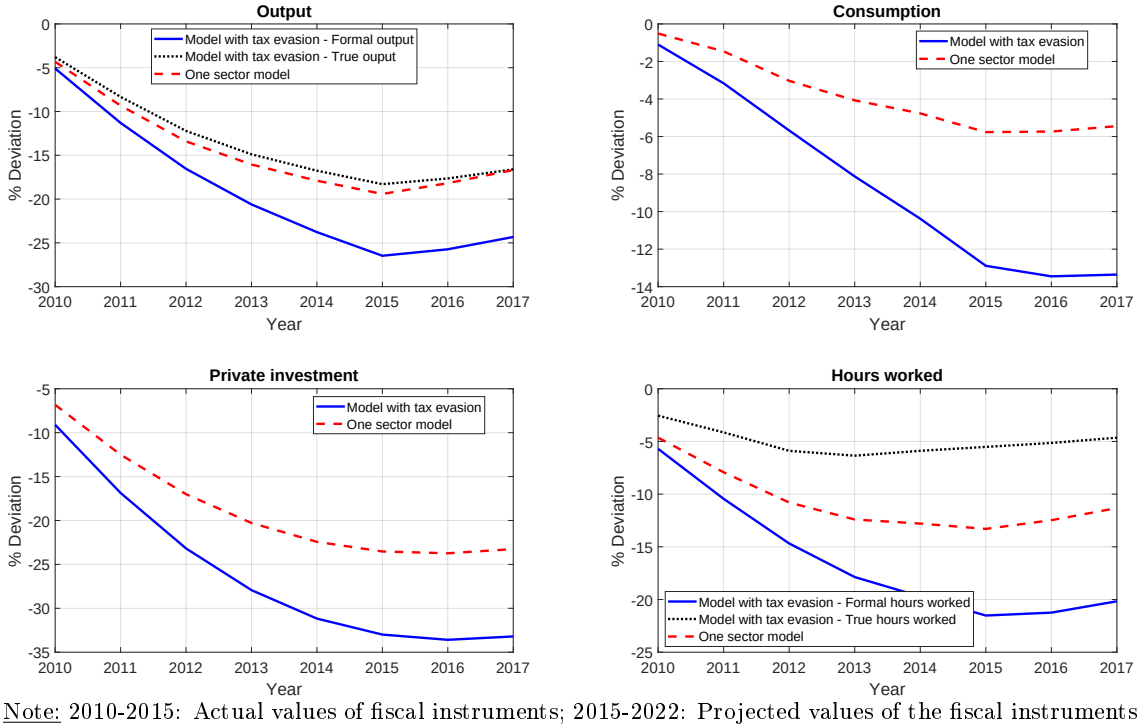
G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
s^u	0.2571	0.2548	0.2524	0.2500	0.2476	0.2452	0.2428
sh^u	0.3540	0.3522	0.3504	0.3485	0.3466	0.3447	0.3428
$d\tilde{y}^p$		0.8491	0.8608	0.8726	0.8846	0.8969	0.9094
$d\tilde{y}^T$		0.6990	0.7100	0.7211	0.7325	0.7441	0.7559
$\frac{dTR^t}{dG^c}$		0.1568	0.1577	0.1585	0.1594	0.1602	0.1610

Note: See note in Table 19. G^c = government spending (consumption).

Table 22: Government Spending, Single Sector Economy

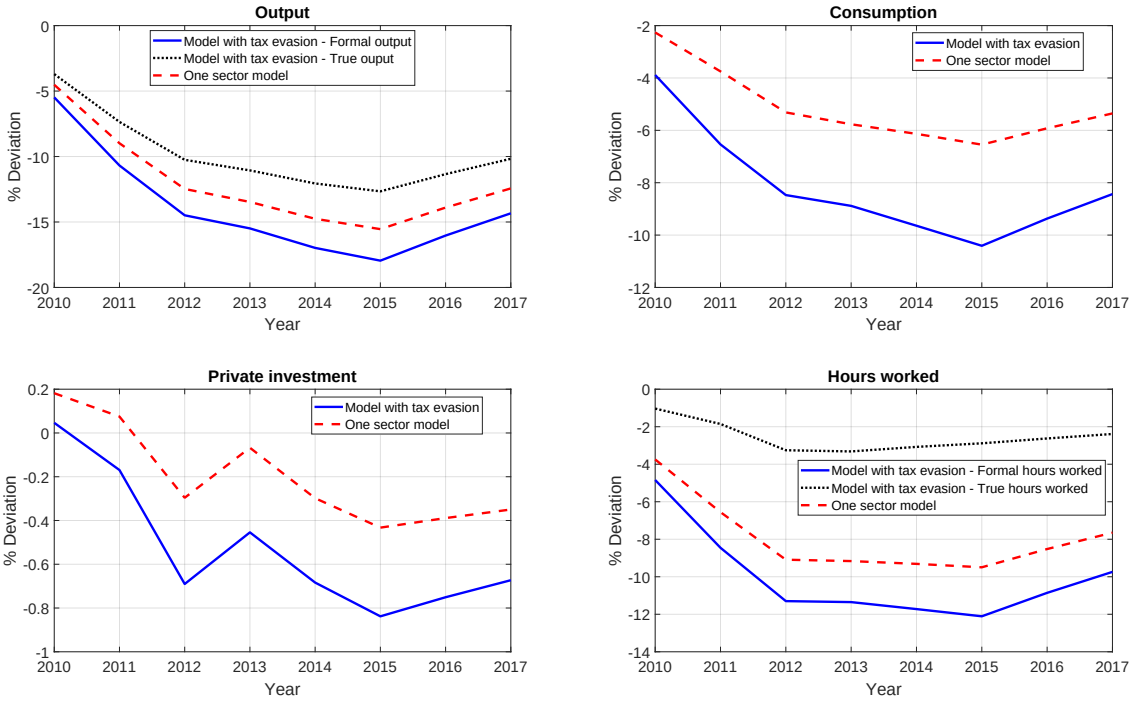
G^c/y^p	0.0724	0.0824	0.0924	0.1024	0.1124	0.1224	0.1324
$d\tilde{y}^p$		0.7937	0.8066	0.8197	0.8331	0.8468	0.8608
$\frac{dTR^t}{dG^c}$		0.1408	0.1424	0.1439	0.1454	0.1469	0.1484

Figure 9: The effects of the Greek fiscal consolidation package on economic activity: full BOGDSGE model, Perfect foresight



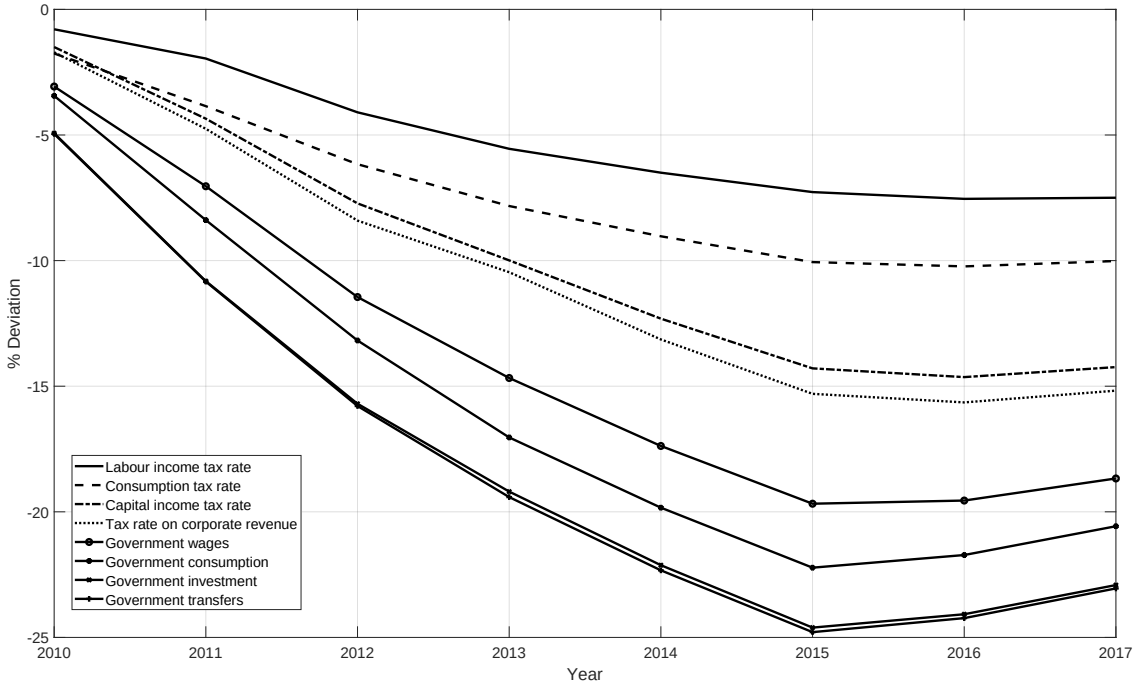
Note: 2010-2015: Actual values of fiscal instruments; 2015-2022: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 10: The effects of the Greek fiscal consolidation package on economic activity: full BOGDSGE model, Random walk



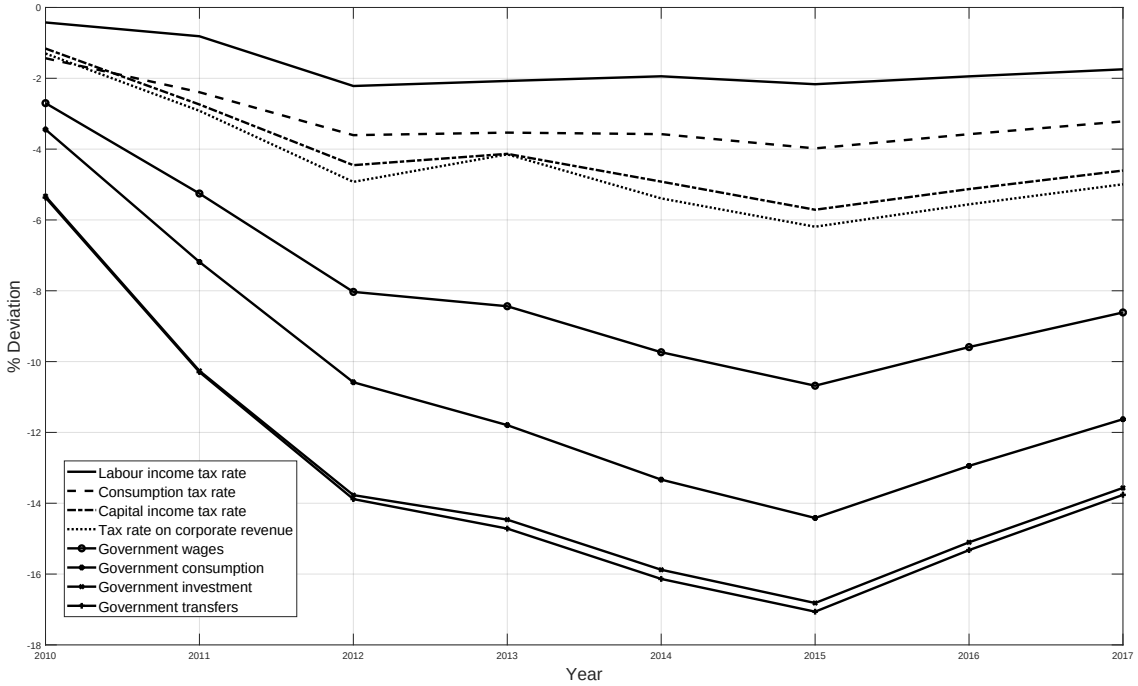
Note: 2010-2015: Actual values of fiscal instruments; 2015-2022: Projected values of the fiscal instruments under the assumption that they return to their 2009 values according to an AR(1) process with persistence 0.9; 2009: steady state values.

Figure 11: Contribution of individual, actual fiscal instruments: full BOGDSGE model, Perfect foresight



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

Figure 12: Contribution of individual, actual fiscal instruments: full BOGDSGE model, Random walk



Note: Each fiscal instrument is introduced sequentially. Note that the contributions are not orthogonal.

7.5.2 Robustness with Regard to Variation in the Model Parameters

We consider variation in the values of various parameters (see the box in the first gigure of each row). Figures 13-14 report the results.

Figure 13: Sensitivity analysis with respect to various parameters (perfect foresight)

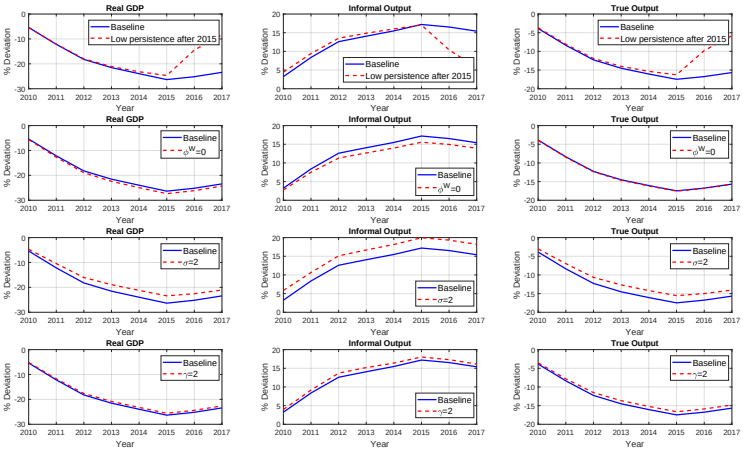
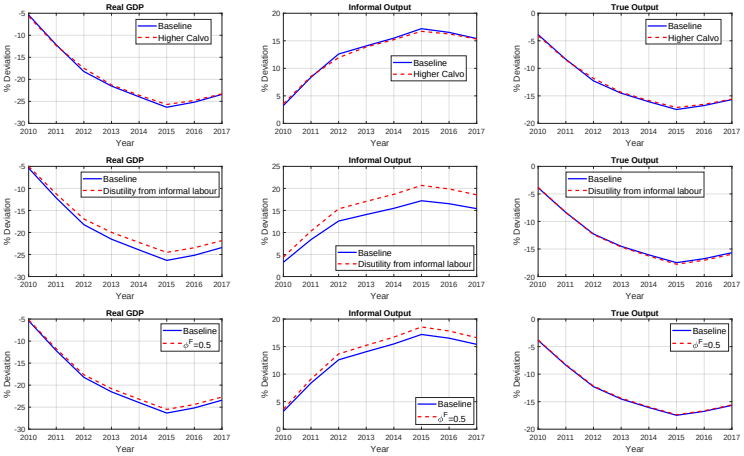


Figure 14: Sensitivity analysis with respect to various parameters (perfect foresight)



7.6 Formal Description of thhe DSGE Model

7.6.1 Households

The preferences for each household i are given by the utility function:

$$u\left(C_{i,t} - \xi^c \overline{C}_{t-1}, H_{i,t}\right) \tag{11}$$

where $C_{i,t}$ is effective consumption (defined below), $H_{i,t}$ is total hours worked, $\xi^c \in [0,1)$ is a parameter that measures the degree of external habit formation in consumption and $\overline{C}_{t-1}$ is the average (across households) lagged once private consumption. The instantaneous utility function is of the form:

$$u\left(C_{i,t} - \xi^c \overline{C}_{t-1}, H_{i,t}\right) = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \kappa \frac{H_{i,t}^{1+\gamma}}{1+\gamma}, \tag{12}$$

where $\kappa > 0$ is a preference parameter. Effective consumption is defined as

$$C_{i,t} = C_{i,t}^p + C_{i,t}^u + \vartheta Y_t^g$$

where $C_{i,t}^p$ are consumption goods produced in the formal sector, $C_{i,t}^u$, are those produced in the informal sector and Y_t^g are the per-capita public goods and services produced by the government. We assume that the household is indifferent between the formally and informally produced private consumption goods. The parameter $\vartheta \in [-1,1]$ governs the impact of public goods and services on household utility.

The household's budget constraint in period t is

$$\begin{aligned} & (1+\tau_t^c)\, C_{i,t}^p + \frac{P_t^u}{P_t^c} C_{i,t}^u + \frac{P_t^i}{P_t^c} I_{i,t} + \frac{B_{i,t+1}}{P_t^c} + \frac{S_t F_{i,t+1}}{P_t^c} = \\ = \; & (1-\tau_t^l)(W_t^p H_{i,t}^p + W_t^g H_{i,t}^g) + (1-\pi\phi^W) W_t^u H_{i,t}^u + (1-\tau_t^k) r_t^k u_{i,t} K_{i,t}^p + \\ & + Div_{i,t} + R_{t-1} \frac{B_{i,t}}{P_t^c} + R_{t-1}^H \frac{S_t F_{i,t}}{P_t^c} - T_t - \Gamma_{i,t}^h, \end{aligned} \tag{13}$$

where P_t^c and P_t^u are respectively the prices of a unit of formal and informal consumption good; P_t^i is the price of a unit of investment good and S_t is the nominal exchange rate expressed in terms of the domestic currency per unit of foreign currency.

In each period t, the household earns labour income by working $H_{i,t}^p$ hours in the formal sector at the real wage rate W_t^p, by working $H_{i,t}^g$ hours in the public sector at the real wage rate W_t^g and $H_{i,t}^u$ hours in the informal sector at the real wage rate W_t^u. It also receives capital income from renting capital services to firms, $u_{i,t} K_{i,t}^p$, where $u_{i,t}$ is the capital utilization rate and $K_{i,t}^p$ is the physical private capital stock; r_t^k is the rental rate of capital. The household can save by investing in physical capital, $I_{i,t}$, and by buying domestic government bonds, $B_{i,t+1}$, that pay a gross nominal interest R_t at time $t+1$ and

foreign bonds, $F_{i,t+1}$, that pay a nominal gross interest R_t^H. Households own all firms in the economy and receive their profits as dividends, $Div_{i,t}$. A consumption tax, $\tau_t^c \in (0,1)$ is levied on the formal consumption good, while a labour income tax, $\tau_t^l \in (0,1)$, and a capital tax, $\tau_t^k \in (0,1)$, are levied on the income earned in the formal sector. The household may evade the tax on the part of the labour income that is earned by working in the informal sector. It is detected with the constant probability π, in which case it pays out a fine that is a fixed share ϕ^W of the labour income earned informally. Finally, T_t denotes lump-sum taxes/transfers.

The households face costs when they adjust their private foreign asset holdings, $\Gamma_{i,t}^h$, if the private foreign assets-to-GDP ratio, $\frac{S_t F_{i,t+1}}{P_t^y Y_t^{GDP}}$, deviates from its long-run level, $\overline{f} \geq 0$. In particular:

$$\Gamma_{i,t}^h = \frac{\xi^f}{2} \frac{P_t^y Y_t^{GDP}}{P_t^c} \left(\frac{S_t F_{i,t+1}}{P_t^y Y_t^{GDP}} - \overline{f} \right)^2, \tag{14}$$

where Y_t^{GDP} is the economy's real GDP, P_t^y is the GDP deflator, and $\xi^f \geq 0$ is an adjustment cost parameter.

The law of motion of the private capital stock is:

$$K_{i,t+1}^p = \left(1 - \delta^p\left(u_{i,t}\right)\right) K_{i,t}^p + \left[1 - \Psi\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right] I_{i,t}, \tag{15}$$

where Ψ is a convex adjustment cost function, $\Psi(0) = 0, \Psi' > 0, \Psi'' > 0$ of the form proposed by Christiano et al. (2005). In particular, we assume

$$\Psi_{i,t} = \frac{\xi^k}{2} \left(\frac{I_{i,t}}{I_{i,t-1}} - 1 \right)^2, \tag{16}$$

where $\xi^k \geq 0$ is an adjustment cost parameter. We assume that the depreciation rate of private capital depends on the rate of capacity utilization according to a convex function that satisfies $\delta^{p\prime} > 0$, $\delta^{p\prime\prime} > 0$. The depreciation function is of the form: $\delta^p\left(u_{i,t}\right) = \delta^p u_{i,t}^{\psi}$, where $\delta^p \in (0,1)$ and $\psi > 0$ are, respectively, the average rate of depreciation of private capital and the elasticity of marginal depreciation costs.

The household chooses consumption of formal and informal good, labour supply in the formal and informal sectors, savings in physical capital, domestic and foreign bonds and capital utilization rate to maximize expected lifetime utility subject to (13)-(16). The FOCs are:

$$\Lambda_{i,t} = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{-\sigma}}{1 + \tau_t^c} = \frac{\left(C_{i,t} - \xi^c \overline{C}_{t-1}\right)^{-\sigma}}{P_t^u / P_t^c} \tag{17}$$

$$\Lambda_{i,t} = \beta E_t \left[\Lambda_{i,t+1} \frac{R_t}{\Pi_{t+1}^c} \right] \tag{18}$$

$$\Lambda_{i,t}\left[1+\xi^{f}\left(\frac{S_{t}F_{i,t+1}}{P_{t}^{y}Y_{t}^{GDP}}-\overline{f}\right)\right]=\beta E_{t}\left[\Lambda_{i,t+1}\frac{R_{t}^{H}}{\Pi_{t+1}^{c}}s_{t+1}\right] \tag{19}$$

$$\left(1-\tau_{t}^{k}\right)r_{t}^{k}=q_{i,t}\delta^{p\prime}(u_{i,t}) \tag{20}$$

$$\begin{aligned}\frac{P_{t}^{i}}{P_{t}^{c}} &= q_{i,t}\left[1-\Psi_{i,t}-\Psi_{i,t}^{\prime}\frac{I_{i,t}}{I_{i,t-1}}\right]+\\ &\quad+\beta E_{t}\left[q_{i,t+1}\frac{\Lambda_{i,t+1}}{\Lambda_{i,t}}\Psi_{i,t}^{\prime}\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^{2}\right]\end{aligned} \tag{21}$$

$$q_{i,t}=\beta E_{t}\frac{\Lambda_{i,t+1}}{\Lambda_{i,t}}\left[\left(1-\tau_{t+1}^{k}\right)r_{t+1}^{k}u_{i,t+1}+q_{i,t+1}\left(1-\delta^{p}\left(u_{i,t+1}\right)\right)\right] \tag{22}$$

$$\left(1-\tau_{t}^{l}\right)W_{t}^{p}=W_{t}^{u}=\frac{\kappa H^{\gamma}}{\Lambda_{i,t}} \tag{23}$$

where β is the discount factor, $\Lambda_{i,t}$ is the marginal utility of a unit of consumption good, $\Pi_{t}^{c}=P_{t}^{c}/P_{t-1}^{c}$ is the gross rate of the consumption price index (CPI) inflation of the formal consumption good, $s_{t}=S_{t}/S_{t-1}$ is the gross growth rate of the nominal exchange rate and $q_{i,t}=Q_{i,t}/\Lambda_{i,t}$ is the shadow price of a unit of capital.

7.6.2 Production

Homogeneous intermediate good firms

The homogeneous intermediate goods sector is composed of a continuum of perfectly competitive intermediate good firms indexed by $j\in[0,1]$. Each firm j can produce the intermediate good either formally or informally. The formal good, $Y_{j,t}^{p}$, is produced by using as inputs capital, $K_{j,t}$, and labour services, $H_{j,t}^{p}$, and makes use of the average public capital $\overline{K}_{t}^{g}$. The informally produced good, $Y_{j,t}^{u}$, requires only labour, $H_{j,t}^{u}$. In particular,

$$Y_{j,t}^{p} = A_{t}^{p}\left(H_{j,t}^{p}\right)^{\alpha}\left(K_{j,t}\right)^{1-\alpha}\left(\overline{K}_{t}^{g}\right)^{\alpha_{g}} \tag{24}$$

$$Y_{j,t}^{u} = A_{t}^{u}\left(H_{j,t}^{u}\right)^{\zeta}-F \tag{25}$$

where $\alpha,\zeta,\alpha_{g}\in(0,1)$ and $A_{t}^{p},A_{t}^{u}>0$ are the exogenous levels of productivity in formal and informal production, respectively. Public capital provides production externalities, so that the production function of the formal sector exhibits increasing returns to scale with respect to all inputs.

Firms pay a corporate tax $\tau_{t}^{f}\in(0,1)$ per unit of sales of the formal good but evade the tax for the informally produced good. With –a constant– probability π the producer is caught

when tax evading and has to pay a fine ϕ^F per unit of informal good sales. Without loss of generality, we assume the same probability of tax evasion for firms and workers. The producers choose the scale of production taking as given the prices for the formal, $P^p_{j,t}$, and informal, $P^u_{j,t}$, goods as well as factor prices, r^k_t, W^p_t, W^u_t in order to maximize expected profits:

$$\Pi_{j,t} = \max_{K^p_{j,t}, H^p_{j,t}, H^u_{j,t}} \left\{ \left(1-\tau^f_t\right) \frac{P^p_{j,t}}{P^c_t} Y^p_{j,t} + \left(1-\pi\phi^F\right) \frac{P^u_{j,t}}{P^c_t} Y^u_{j,t} - r^k_t K^p_{j,t} - W^p_t H^p_{j,t} - W^u_t H^u_{j,t} \right\} \tag{26}$$

subject to (24)-(25). The FOCs are:

$$\begin{aligned} W^p_t &= \left(1-\tau^f_t\right) \alpha \frac{P^p_{j,t}}{P^c_t} \frac{Y^p_{j,t}}{H^p_{j,t}} \\ W^u_t &= \left(1-\pi\phi^F\right) \zeta \frac{P^u_{j,t}}{P^c_t} \frac{Y^u_{j,t}}{H^u_{j,t}} \\ r^k_t &= \left(1-\tau^f_t\right) (1-\alpha) \frac{P^p_{j,t}}{P^c_t} \frac{Y^p_{j,t}}{K^p_{j,t}} \end{aligned}$$

The absence of market power ensures that $P^p_{j,t} = P^p_t$ and $P^u_{j,t} = P^u_t$ and that all producers choose the same level of production in both goods, i.e. $Y^p_{j,t} = Y^p_t$, $H^p_{j,t} = H^p_t$ and $Y^u_{j,t} = Y^u_t$, $H^u_{j,t} = H^u_t$.

Differentiated Intermediate Goods

We have imperfectly competitive domestic firms, indexed $f \in [0,1]$, that convert the formal, homogeneous good into formal, differentiated intermediate goods that are either used for the domestic final good production, or are exported. There are also importing firms, indexed $f^m \in [0,1]$, that convert the imported, homogeneous good into a differentiated, foreign intermediate good. The assumption of imperfect competition is made in order to facilitate the introduction of standard price stickiness.

We assume price stickiness à la Calvo (1983) for the formal, differentiated intermediate goods, whether produced domestically or abroad. In particular, in each period t, a firm f can optimally reset its price with a constant probability $1-\theta^d$ when it sells domestically formal, differentiated goods and $1-\theta^x$ when it exports these goods. Similarly, for importing goods there is a constant probability $1-\theta^m$ that the importing firm f^m optimally resets its price in each period. An intermediate good firm that cannot reoptimize its price, will partially index their prices to the aggregate past inflation according to the following price indexation schemes if it produces domestic, exported, or imported goods, respectively:

$$\begin{aligned} P^d_{f,t} &= P^d_{f,t-1} \left(\Pi^d_{t-1}\right)^{x_d}, \\ P^x_{f,t} &= P^x_{f,t-1} \left(\Pi^x_{t-1}\right)^{x_x}, \\ P^m_{f^m,t} &= P^m_{f^m,t-1} \left(\Pi^m_{t-1}\right)^{x_m}, \end{aligned}$$

where $\Pi_t^d = P_t^d/P_{t-1}^d$, $\Pi_t^x = P_t^x/P_{t-1}^x$ and $\Pi_t^m = P_t^m/P_{t-1}^m$ and the indexation parameters $x_d, x_x, x_m \in [0,1]$ determine the weights given to past inflation (unit value denotes full indexation).

The price of a firm producing formal, differentiated goods that are sold domestically and has not been able to reoptimize for τ periods in the domestic market is $P_{f,t+\tau}^d = P_{f,t}^d \Pi_{s=1}^{\tau} \left(\Pi_{t+s-1}^d\right)^{x_d}$. Each firm that reoptimizes its price in the domestic market in period t, chooses the optimal price $P_{f,t}^{*d}$ to maximize the discounted sum of expected real profits (in terms of the formal, final consumption good P_t^c), by taking aggregate domestic demand Y_t^d and the aggregate price index in the domestic market, P_t^d, as given. Thus, each firm maximizes:

$$\max_{P_{f,t}^d} E_t \sum_{t=0}^{\infty} \left(\beta\theta^d\right)^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \frac{P_{t+\tau}^d}{P_{t+\tau}^c} \left[\Pi_{s=1}^{\tau} \left(\Pi_{t+s-1}^d\right)^{x_d} \frac{P_{f,t}^d}{P_{t+\tau}^d} - mc_{t+\tau}^d \right] Y_{f,t+\tau}^d \right\} \tag{27}$$

subject to

$$Y_{f,t}^d = \left(\Pi_{s=1}^{\tau} \left(\Pi_{t+s-1}^d\right)^{x_d} \frac{P_{f,t}^d}{P_{t+\tau}^d} \right)^{-\frac{\mu_{t+\tau}^d}{\mu_{t+\tau}^d - 1}} Y_{t+\tau}^d \tag{28}$$

where $mc_t^d = P_t^c mc_t^p / P_t^d$ is the average real marginal cost in terms of the domestic price index of the formal, differentiated goods, $mc_t^p = P_t^p/P_t^c$ is the marginal cost expressed in terms of the formal consumption good, $\Lambda_{t+\tau}/\Lambda_t$ is the intertemporal marginal rate of substitution of the households according to which firms value future profits and $\frac{\mu_t^d}{\mu_t^d - 1} > 1$ is the elasticity of substitution between the varieties of formal intermediate goods. Given that all firms face the same marginal cost and take aggregate variables as given it follows that they set the same optimal price $P_{f,t}^{*d} = P_t^{*d}$. Thus, the first order condition of the above problem is:

$$E_t \left\{ \left(\beta\theta^d\right)^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left[\Pi_{s=1}^{\tau} \frac{\left(\Pi_{t+s-1}^d\right)^{x_d}}{\Pi_{t+s}^d} \frac{P_t^{*d}}{P_t^d} \right]^{-\frac{\mu_{t+\tau}^d}{\mu_{t+\tau}^d - 1}} Y_{t+\tau}^d \frac{P_{t+\tau}^d}{P_{t+\tau}^c} \Pi_{s=1}^{\tau} \frac{\left(\Pi_{t+s-1}^d\right)^{x_d}}{\Pi_{t+s}^d} \frac{P_t^{*d}}{P_t^d} - \right.$$

$$\left. - \mu_{t+\tau}^d mc_{t+\tau}^d \right\} = 0 \tag{29}$$

According to (29), firms set nominal prices so as to equate the average future expected marginal revenues to average future expected costs. To facilitate solution, we express (29) in recursive form, so that we define:

$$
\begin{aligned}
g_t^{d_1} &= \mu_t^d mc_t^d Y_t^d \Lambda_t \frac{P_t^d}{P_t^c} + \beta\theta^d E_t \left[\frac{\left(\Pi_t^d\right)^{x_d}}{\Pi_{t+1}^d} \right]^{-\frac{\mu_t^d}{\mu_t^d - 1}} g_{t+1}^{d_1} \\
g_t^{d_2} &= \Pi_t^{*d} Y_t^d \Lambda_t \frac{P_t^d}{P_t^c} + \beta\theta^{pd} E_t \left[\frac{\left(\Pi_t^d\right)^{x_d}}{\Pi_{t+1}^d} \right]^{1-\frac{\mu_t^d}{\mu_t^d - 1}} \frac{\Pi_t^{*d}}{\Pi_{t+1}^{*d}} g_{t+1}^{d_2} \\
g_t^{d_3} &= g_t^{d_2}
\end{aligned}
$$

where $\Pi_t^{*d} = P_t^{*d}/P_t^d$.

The aggregate domestic index evolves according to:

$$
P_t^d = \left\{ \left(1-\theta^d\right) \left(P_t^{*d}\right)^{\frac{1}{1-\mu_t^d}} + \theta^d \left[P_{t-1}^d \left(\Pi_{t-1}^d\right)^{x_d} \right]^{\frac{1}{1-\mu_t^d}} \right\}^{1-\mu_t^d}
$$

The maximization problem for the part of the formal, differentiated intermediate goods that are exported is defined in a similar manner. Each firm maximizes:

$$
\max_{P_{f,t}^x} E_t \sum_{t=0}^{\infty} \left(\beta\theta^x\right)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \frac{P_{t+\tau}^x}{P_{t+\tau}^c} \left[\Pi_{s=1}^\tau \left(\Pi_{t+s-1}^x\right)^{x_x} \frac{P_{f,t}^x}{P_{t+\tau}^x} - mc_{t+\tau}^x \right] Y_{f,t+\tau}^x \right\} \tag{30}
$$

subject to

$$
Y_{f,t}^x = \left(\Pi_{s=1}^\tau \left(\Pi_{t+s-1}^x\right)^{x_x} \frac{P_{f,t}^x}{P_{t+\tau}^x} \right)^{-\frac{\mu_{t+\tau}^x}{\mu_{t+\tau}^x - 1}} Y_{t+\tau}^x \tag{31}
$$

where $mc_t^x = P_t^c mc_t^p / P_t^x$ is the average real marginal cost in terms of the aggregate export price index and $\frac{\mu_t^x}{\mu_t^x - 1} > 1$ is the elasticity of substitution between the varieties of formal intermediate goods that are exported. The first order condition of the above problem is:

$$
\begin{aligned}
E_t \Bigg\{ \left(\beta\theta^x\right)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left[\Pi_{s=1}^\tau \frac{\left(\Pi_{t+s-1}^x\right)^{x_x}}{\Pi_{t+s}^x} \frac{P_t^{*x}}{P_t^x} \right]^{-\frac{\mu_{t+\tau}^x}{\mu_{t+\tau}^x - 1}} Y_{t+\tau}^x \frac{P_{t+\tau}^x}{P_{t+\tau}^x} \Pi_{s=1}^\tau \frac{\left(\Pi_{t+s-1}^x\right)^{x_x}}{\Pi_{t+s}^x} \frac{P_t^{*x}}{P_t^x} - \\
- \mu_{t+\tau}^x mc_{t+\tau}^x \Bigg\} = 0
\end{aligned}
$$

and the aggregate domestic index is:

$$
P_t^x = \left\{ \left(1-\theta^x\right) \left(P_t^{*x}\right)^{\frac{1}{1-\mu_t^x}} + \theta^x \left[P_{t-1}^x \left(\Pi_{t-1}^x\right)^{x_x} \right]^{\frac{1}{1-\mu_t^x}} \right\}^{1-\mu_t^x}
$$

Finally, each importing firm f^m buys differentiated intermediate goods at the international price $S_t P_t^{*y}$ and sells them domestically at a price $P^m_{f^m,t}$. Each firm maximizes:

$$\max_{P^m_{f^m,t}} E_t \sum_{t=0}^{\infty} (\beta\theta^m)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left\{ \frac{P^m_{t+\tau}}{P^c_{t+\tau}} \left[\Pi^\tau_{s=1} \left(\Pi^m_{t+s-1}\right)^{x_m} \frac{P^m_{f^m,t}}{P^m_{t+\tau}} - mc^m_{t+\tau} \right] Y^m_{f^m,t+\tau} \right\} \tag{32}$$

subject to

$$Y^m_{f^m,t} = \left(\Pi^\tau_{s=1} \left(\Pi^m_{t+s-1}\right)^{x_m} \frac{P^m_{f^m,t}}{P^m_{t+\tau}} \right)^{-\frac{\mu^m_{t+\tau}}{\mu^m_{t+\tau}-1}} Y^m_t \tag{33}$$

where $mc^m_t = S_t P^{*y}_t / P^m_t$ is the average real marginal cost and $\frac{\mu^m_t}{\mu^m_t - 1} > 1$ is the elasticity of substitution between the varieties of imported intermediate goods. The first order condition is:

$$E_t \Bigg\{ (\beta\theta^m)^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} \left[\Pi^\tau_{s=1} \frac{\left(\Pi^m_{t+s-1}\right)^{x_m}}{\Pi^m_{t+s}} \frac{P^{*m}_t}{P^m_t} \right]^{-\frac{\mu^m_{t+\tau}}{\mu^m_{t+\tau}-1}} Y^m_{t+\tau} \frac{P^m_{t+\tau}}{P^m_{t+\tau}} \Pi^\tau_{s=1} \frac{\left(\Pi^m_{t+s-1}\right)^{x_m}}{\Pi^m_{t+s}} \frac{P^{*m}_t}{P^m_t} - \\ - \mu^m_{t+\tau} mc^m_{t+\tau} \Bigg\} = 0$$

The aggregate domestic index evolves according to:

$$P^m_t = \left\{ (1-\theta^m) \left(P^{*m}_t\right)^{\frac{1}{1-\mu^m_t}} + \theta^m \left[P^m_{t-1} \left(\Pi^m_{t-1}\right)^{x_m} \right]^{\frac{1}{1-\mu^m_t}} \right\}^{1-\mu^m_t} \tag{34}$$

Final good firms

Perfectly competitive final good firms combine domestic and imported formal, differentiated intermediate goods to produce final consumption and investment goods. The informally produced homogeneous intermediate good is used for consumption purposes only, so that $Y^u_t = C^u_t$.

The representative producer of the private consumption good combines a bundle of formal, differentiated intermediate consumption goods, $C^d_t = \left(\int_0^1 \left(C^d_{f,t} \right)^{\frac{1}{\mu^d_t}} df \right)^{\mu^d_t}$, with a bundle of differentiated intermediate consumption goods that are imported, $C^m_t = \left(\int_0^1 \left(C^m_{f^m,t} \right)^{\frac{1}{\mu^m_t}} df^m \right)^{\mu^m_t}$, to generate a composite consumption final good, C^p_t, by using a constant elasticity of substitution (CES) production function:

$$C^p_t = \left[\omega_c^{\frac{1}{\varepsilon_c}} \left(C^d_t \right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + (1-\omega_c)^{\frac{1}{\varepsilon_c}} \left(C^m_t \right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]$$

where $\omega_c \in [0,1]$ measures the home bias in the production of the final consumption good that determines the degree of openness in the long run, and $\varepsilon_c > 0$ is the elasticity of substitution between domestic and imported consumption goods.

The final consumption good producer solves a two-stage problem. In the first stage, he takes as given the retail prices of the varieties of the domestic formal intermediate goods, $P^d_{f,t}$, and imported intermediate goods, $P^m_{f^m,t}$, and optimally chooses his demand of differentiated varieties $C^d_{f,t}$, $C^m_{f^m,t}$ to minimize expenditures: $\int_0^1 P^d_{f,t}C^d_{f,t}df + \int_0^1 P^m_{f^m,t}C^m_{f^m,t}df^m$, subject to the respective CES bundle technologies. Optimal demand satisfies $C^d_{f,t} = \left(\frac{P^d_{f,t}}{P^d_t}\right)^{-\frac{\mu^d_t-1}{\mu^d_t}} C^d_t$ and $C^m_{f^m,t} = \left(\frac{P^m_{f^m,t}}{P^m_t}\right)^{-\frac{\mu^m_t-1}{\mu^m_t}} C^m_t$, where $P^d_t = \left(\int_0^1 \left(P^d_{f,t}\right)^{\frac{1}{1-\mu^d_t}} df\right)^{1-\mu^d_t}$ and $P^m_t = \left(\int_0^1 \left(P^m_{f^m,t}\right)^{\frac{1}{1-\mu^m_t}} df^m\right)^{1-\mu^m_t}$ are the aggregate price indexes of the domestic formal and imported, differentiated intermediate goods, respectively. In the second stage, the final consumption good producer chooses his output, C^c_t, and inputs, C^d_t, C^m_t to maximize profits $\Pi^c_t = P^c_t C^p_t - P^d_t C^d_t - P^m_t C^m_t$, taking aggregate prices P^d_t, P^m_t, and P^c_t, as given, which implies:

$$\begin{aligned} C^d_t &= \omega_c \left(\frac{P^d_t}{P^c_t}\right)^{-\varepsilon_c} C^p_t, \\ C^m_t &= (1-\omega_c)\left(\frac{P^m_t}{P^c_t}\right)^{-\varepsilon_c} C^p_t \end{aligned}$$

and

$$P^c_t = \left[\omega_c \left(P^d_t\right)^{1-\varepsilon_c} + (1-\omega_c)\left(P^m_t\right)^{1-\varepsilon_c}\right]^{\frac{1}{1-\varepsilon_c}}.$$

The production of the final private investment good is modeled in an analogous manner. In particular, the representative producer combines a bundle of domestically produced intermediate investment varieties, $I^d_t = \left(\int_0^1 \left(I^d_{f,t}\right)^{\frac{1}{\mu^d_t}} df\right)^{\mu^d_t}$, with a bundle of imported intermediate varieties, $I^m_t = \left(\int_0^1 \left(I^m_{f^m,t}\right)^{\frac{1}{\mu^m_t}} df^m\right)^{\mu^m_t}$, to generate a composite final private investment good, I^p_t, using the CES technology:

$$I^p_t = \left[\omega_i^{\frac{1}{\varepsilon_i}} \left(I^d_t\right)^{\frac{\varepsilon_i-1}{\varepsilon_i}} + (1-\omega_i)^{\frac{1}{\varepsilon_i}} \left(I^m_t\right)^{\frac{\varepsilon_i-1}{\varepsilon_i}}\right]$$

where $\omega_i \in [0,1]$ measures home bias in the production of the final investment good, $\varepsilon_i > 0$ is the elasticity of substitution between domestic and imported investment goods. The demand functions for the bundles of the domestically produced intermediate goods and imported intermediate goods are respectively $I^d_t = \omega_i \left(\frac{P^d_t}{P^c_t}\right)^{-\varepsilon_i} I^p_t$, and $I^m_t = (1-\omega_i)\left(\frac{P^m_t}{P^c_t}\right)^{-\varepsilon_i} I^p_t$. The aggregate price index of the final investment good is $P^i_t = \left[\omega_i \left(P^d_t\right)^{1-\varepsilon_i} + (1-\omega_i)\left(P^m_t\right)^{1-\varepsilon_i}\right]^{\frac{1}{1-\varepsilon_i}}$.

Foreign firms and export market

The exporting firm takes the prices of the exported differentiated goods $P^x_{f,t}/S_t$ as given, and chooses the optimal amounts of differentiated inputs to minimize the total input costs,

$\int_0^1 \left(P_{f,t}^x/S_t\right) Y_{f,t}^x df$, subject to the technology that bundles the differentiated varieties of intermediate goods exported. The solution of the cost minimization problem gives the demand for each input $Y_{f,t}^x = \left(\frac{P_{f,t}^x}{P_t^x}\right)^{-\frac{\mu_t^x}{\mu_t^x-1}} Y_t^x$, and the aggregate price index of the exported domestic intermediate goods $P_t^x = \left(\int_0^1 \left(P_{f,t}^x\right)^{\frac{1}{1-\mu_t^x}} df\right)^{1-\mu_t^x}$, where Y_t^x is total foreign demand for domestic intermediate goods. The latter is assumed to be given by an equation analogous in structure to the demand equations for the domestic and imported intermediate goods

$$Y_t^x = \left(\frac{P_t^x/S_t}{P_t^{x*}}\right)^{-\varepsilon_x} Y_t^*,$$

where P_t^{x*} the price of foreign competitors in the export markets and Y_t^* is a measure of aggregate foreign demand.

7.6.3 Government

The government levies taxes on consumption, on income from labour and capital earnings, on corporate income and lump-sum taxes and issues one-period government bonds in the domestic bond market, B_{t+1}. Total tax revenues together with the issue of new government bonds are used to finance government purchases of goods and services, G_t^c, government investment, G_t^i, government transfers, G_t^{tr}, and the wage bill for public sector employees, $W_t^g H_t^g$. Moreover, the government pays interest payments on past domestic public debt, R_t. The within-period government budget constraint is:

$$\frac{B_{t+1}}{P_t^c} + \tau_t^c C_t^p + \tau_t^l (W_t^p H_t^p + W_t^g H_t^g) + \tau_t^f \frac{P_t^p}{P_t^c} Y_t^p + \pi(\phi^F \frac{P_t^u}{P_t^c} Y_t^u + \phi^W W_t^u H_t^u) +$$
$$+\tau_t^k r_t^k u_t K_t^p + T_t = \frac{P_t^d}{P_t^c} G_t^c + \frac{P_t^d}{P_t^c} G_t^i + G_t^{tr} + W_t^g H_t^g + R_{t-1} \frac{B_t}{P_t^c}$$

Thus, the government has eleven policy instruments: $\tau_t^c, \tau_t^k, \tau_t^l, \tau_t^f, T_t, H_t^g, W_t^g, G_t^c, G_t^i, G_t^{tr}, B_{t+1}$. We assume that the government debt level is zero, $B_{t+1} = B_t = 0$, and lump-sum taxes adjust to ensure that the budget constraint is satisfied in every period. For convenience, regarding spending policy instruments, we will work in terms of their shares of steady state GDP, $S_t^c = \frac{P_t^d G_t^c}{P^y Y^{GDP}}$, $S_t^i = \frac{P_t^d G_t^i}{P^y Y^{GDP}}$, $S_t^{tr} = \frac{P_t^c G_t^{tr}}{P^y Y^{GDP}}$, $S_t^w = \frac{W_t^g H_t^g}{P^y Y^{GDP}}$.

In addition, the government produces the public good, Y_t^g, by combining public spending on goods and services, G_t^c, and public employment, H_t^g:

$$Y_t^g = A_t^g \left(G_t^c\right)^\chi \left(H_t^g\right)^{1-\chi}$$

where $\chi \in (0,1)$ is a technology parameter and $A_t^g > 0$.

The law of motion of public capital is:

$$K_{t+1}^g = (1-\delta^g) K_t^g + G_t^i$$

where $\delta^g \in (0,1)$ is the depreciation rate for public capital stock and $K_0^g > 0$ is given.

7.6.4 Monetary policy regime

The domestic economy is modeled as a member of a currency union in the sense that the nominal exchange rate, S_t, is exogenously set and there is no monetary policy independence. We assume that that the domestic nominal interest rate, R_t^H, equals an exogenously given risk-free world interest rate, R_t^*.

7.6.5 Market clearing conditions

$$\begin{aligned}
H_t &= H_t^p + H_t^u + H_t^g \\
K_t &= u_t K_t^p \\
Y_t^p &= u_t^d Y_t^d + u_t^x Y_t^x \\
Y_t^d &= C_t^d + I_t^d + G_t^c + G_t^i \\
Y_t^u &= C_t^u \\
M_t &= u_t^m Y_t^m \\
Y_t^m &= C_t^m + I_t^m \\
p_t^y Y_t^{GDP} &= p_t^y Y_t^p + W_t^g H_t^g \\
p_t^y Y_t^{GDP} &= p_t^d Y_t^d + p^x Y_t^x
\end{aligned}$$

where prices in small letters denote prices in terms of the formal consumption good, e.g. $p_t^y \equiv P_t^y / P_t^c$, and u_t^d, u_t^x and u_t^m are price dispersion measures, defined as:

$$\begin{aligned}
u_t^d &= \int_0^1 \left(\frac{P_{f,t}^d}{P_t^d} \right)^{-\frac{\mu_t^d}{\mu_t^d - 1}} df \\
u_t^x &= \int_0^1 \left(\frac{P_{f,t}^x}{P_t^x} \right)^{-\frac{\mu_t^x}{\mu_t^x - 1}} df \\
u_t^m &= \int_0^1 \left(\frac{P_{f^m,t}^m}{P_t^m} \right)^{-\frac{\mu_t^m}{\mu_t^m - 1}} df^m
\end{aligned}$$

that evolve respectively according to:

$$u_t^d = \left(1-\theta^d\right)\left(\Pi_t^{*d}\right)^{-\frac{\mu_t^d}{\mu_t^d-1}} + \theta^d\left[\frac{\left(\Pi_{t-1}^d\right)^{x_d}}{\Pi^d}\right]^{-\frac{\mu_t^d}{\mu_t^d-1}} u_{t-1}^d$$

$$u_t^x = \left(1-\theta^x\right)\left(\Pi_t^{*x}\right)^{-\frac{\mu_t^x}{\mu_t^x-1}} + \theta^x\left[\frac{\left(\Pi_{t-1}^x\right)^{x_x}}{\Pi^x}\right]^{-\frac{\mu_t^x}{\mu_t^x-1}} u_{t-1}^x$$

$$u_t^m = \left(1-\theta^m\right)\left(\Pi_t^{*m}\right)^{-\frac{\mu_t^m}{\mu_t^m-1}} + \theta^m\left[\frac{\left(\Pi_{t-1}^m\right)^{x_m}}{\Pi^m}\right]^{-\frac{\mu_t^m}{\mu_t^m-1}} u_{t-1}^m$$

Exports market equilibrium is:

$$Y_t^x = \left(\frac{p_t^x/p_t^y}{q_t^{ex}p_t^{x*}}\right)^{-\varepsilon_x} Y_t^*$$

where $q_t^{ex} = \frac{S_t P_t^{*y}}{P_t^y} = q_{t-1}^{ex}\frac{s_t\Pi_t^{*y}}{\Pi_t^y}$ is the real effective exchange rate.

The evolution of net foreign assets includes the adjustment costs suffered by the domestic agents on their foreign asset holdings. The balance of payments is given by the following equation:

$$\frac{S_t F_{t+1}}{P_t^c} = R_{t-1}^H\frac{S_t F_t}{P_t^c} + \frac{P_t^x}{P_t^c}Y_t^x - q_t^{ex}\frac{P_t^y}{P_t^c}M_t - \frac{\xi^f}{2}\frac{P_t^y Y_t^{GDP}}{P_t^c}\left(\frac{S_t F_{t+1}}{P_t^y Y_t^{GDP}} - \overline{f}\right)^2$$

Finally, we define true output as

$$Y_t^T \equiv Y_t^{GDP} + Y_t^u$$

Profits of the intermediate good producers, retailers and importing firms are allocated as dividends:

$$\begin{aligned}
Div_t &= \Xi_t + \Xi_t^d + \Xi_t^m \\
\Xi_t &= \Pi_t = \left(1-\tau_t^f\right)p_t^p Y_t^p + \left(1-\pi\phi\right)p_t^u Y_t^u - r_t^k K_t - W_t^p H_t^p - W_t^u H_t^u \\
\Xi_t^d &= Y_t^d\left(p_t^d - mc_t^p\right) + Y_t^x\left(p_t^x - mc_t^p\right) \\
\Xi_t^m &= p_t^m Y_t^m - q_t^{ex}p_t^y M_t
\end{aligned}$$

7.7 A stripped down model version with flexible prices

In this section we consider a very simple DSGE model with an informal sector and with flexible prices. We assume that the underground activity centers on the firms' attempt to evade the firm revenue tax.

Households

The representative household has preferences over consumption and hours worked captured by the intertemporal utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right), \tag{35}$$

where E_0 is the expectation operator, $\beta \in (0,1)$ is the discount factor, C_t is private consumption and H_t is total hours worked in period t. We specialize the instantaneous utility function as follows:

$$u\left(C_t, H_t\right) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{H_t^{1+\gamma}}{1+\gamma}, \tag{36}$$

where $\sigma \geq 0$ is the inverse of the intertemporal elasticity of consumption and $\gamma > 0$ is the inverse of the Frisch elasticity of labour supply. Without loss of generality, we will assume that the household is indifferent between the consumption good produced in the formal sector, C_t^p, and that produced informally, C_t^u, so that $C_t = C_t^p + C_t^u$.

The budget constraint of the household in period t is

$$\left(1+\tau_t^c\right) P_t^p C_t^p + P_t^u C_t^u + P_t^p I_t = (1-\tau_t^l) W_t^p H_t^p + W_t^u H_t^u + P_t^p r_t^k K_t + Div_t - T_t \tag{37}$$

where P_t^p and P_t^u are respectively the prices of a unit of formal final good and a unit of informal consumption good. The household earns a wage W_t^p in the formal sector and a wage W_t^u in the informal sector (with H_t^p and H_t^u hours worked, respectively, in these sectors). It also receives capital income from renting out capital services to firms, K_t, at the rate r_t^k. Here, anticipating the assumption that only the formal sector uses capital and in order to lighten the exposition, we are dropping the "p" superscript from investment and capital. Each period, the household saves by investing in physical capital, I_t. Finally, households own the firms in the economy and receive their profits as dividends, Div_t. The government transfers T_t to the household in a lump-sum manner. A consumption tax, $\tau_t^c \in (0,1)$, is levied on the formal consumption good and a labour income tax, $\tau_t^l \in (0,1)$, is levied on the income earned in the formal sector.

The law of motion of the capital stock is:

$$K_{t+1} = (1-\delta) K_t + I_t, \tag{38}$$

where $\delta \in (0,1)$ is the rate of depreciation of capital.

The household chooses consumption of the formal and informal goods, labour supply in the formal and informal sector, and savings in order to maximize utility (35)-(36) subject to (37) and (38). The FOCs are as follows:

$$(1+\tau_t^c)P_t^p\Lambda_t = C_t^{-\sigma} = P_t^u\Lambda_t \tag{39}$$

$$\Lambda_t = \beta E_t\left[\Lambda_{t+1}\left(r_{t+1}^k + 1 - \delta\right)\right] \tag{40}$$

$$\left(1-\tau_t^l\right)W_t^p\Lambda_t = W_t^u\Lambda_t = H^\gamma \tag{41}$$

where Λ_t is the Lagrange multiplier.

Note that (39) implies $P_t^u = (1+\tau_t^c)P_t^p$ and (41) implies $W_t^u = (1-\tau_t^l)W_t^p$.

Production

Each representative firm has two alternative means of producing a homogeneous good. It can produce the good either formally or informally. For the formal good, Y_t^p, it combines capital, K_t, and labour services, H_t^p. For the informal good, Y_t^u, it uses only labour, H_t^u. More specifically:

$$Y_t^p = A_t^p(H_t^p)^\alpha(K_t)^{1-\alpha} \tag{42}$$

$$Y_t^u = A_t^u(H_t^u)^\zeta - F \tag{43}$$

where $\alpha, \zeta \in (0,1)$, and A_t^p, $A_t^u > 0$ are the exogenous levels of productivity in the formal and informal sector, respectively. F is the fixed cost of operating in the informal sector.[20]

What distinguishes the formal from the informal good is that the production, sale and consumption of the latter may not always be detected and taxed. In particular, we assume that firms pay a revenue tax $\tau_t^f \in (0,1)$ per unit of formal good produced. For the informal good, the firm can evade the revenue tax if the activity does not get detected. But if the activity gets detected by the tax authorities, an event that occurs with an exogenous probability π, then the producer pays a fine. Without loss of generality, we will assume that this fine takes the form of confiscating a fixed share, ϕ, of the value of the informal output. We also assume that the firm bears the entire burden in the case of detection, the

[20] A fixed cost is the simplest way to guarantee that the informal sector only comes into existence due to taxes. Due to the decreasing returns in the informal sector, without the fixed cost there would always be a bit of shadow economy even when taxes were set to zero. Having constant returns to scale also in the informal sector would result in a corner solution for the composition of output between formal and informal output.

workers who produced (and did not declare their labour income) and the consumers who consumed the informal good do not suffer any consequences. In the next section and for the larger model, we allow fines to be imposed also on other participating agents besides firms.

The producers choose the scale and composition of production in order to maximize expected profits, taking as given final output prices for formal, P_t^p, and informal, P_t^u, goods as well as factor prices, r_t^k, W_t^p, W_t^u:

$$\Pi_t = \max_{K_t^p, H_t^p, H_t^u} \left\{ \left(1 - \tau_t^f\right) P_t^p Y_t^p + (1 - \pi\phi) P_t^u Y_t^u - r_t^k K_t - W_t^p H_t^p - W_t^u H_t^u - J F_t \right\} \tag{44}$$

subject to (42)-(43). J is an indicator function that takes the value of 1 if Y_t^u is positive and zero if Y_t^u is zero. The FOCs are as follows:

$$\begin{aligned} W_t^p &= \left(1 - \tau_t^f\right) \alpha P_t^p \frac{Y_t^p}{H_t^p} \\ W_t^u &= (1 - \pi\phi) \zeta P_t^u \frac{Y_t^u}{H_t^u} \\ r_t^k &= \left(1 - \tau_t^f\right) (1 - \alpha) P_t^p \frac{Y_t^p}{K_t} \end{aligned}$$

The informal goods are only used for consumption, hence

$$Y_t^u = C_t^u$$

The formal goods are used for consumption, C_t^p, investment, I_t, and public consumption, G_t^c, purposes.

$$Y_t^p = C_t^p + I_t^p + G_t^c$$

Government

The government budget constraint is

$$\tau_t^c P_t^p C_t^p + \tau_t^l W_t^p H_t^p + \tau_t^f P_t^p Y_t^p + \pi\phi_t P_t^u Y_t^u + T_t = P_t^p G_t^c$$

We assume that when the shadow activity is detected, any amount of the informal good that is confiscated is returned to the households via a lump sum transfer. That is, the government only consumes the formal good. As a result the private consumption of the informal good is equal to the production of that good (net of the fixed cost, F).

7.7.1 Solution-Calibration-Results

We illustrate the main mechanisms at work by focusing on the steady state solution and undertaking comparative statics. We use the same parameter values as in the BOGGEM.

Table 23: Tax on Firm Revenue, Dual Economy

TR^f/y^p	0.0135	0.0235	0.0335	0.0435	0.0535	0.0635	0.0735
τ^f	0.0106	0.0204	0.0302	0.0400	0.0498	0.0595	0.0693
s^u	0.2096	0.2220	0.2355	0.2500	0.2657	0.2827	0.3011
sh^u	0.2910	0.3037	0.3169	0.3307	0.3451	0.3601	0.3757
$d\tilde{y}^p$		-2.5310	-2.6161	-2.7075	-2.8060	-2.9122	-3.0269
$d\tilde{y}^T$		-1.6357	-1.6615	-1.6885	-1.7169	-1.7465	-1.7777
$\frac{dTR^f}{d\tau^f}$		1.0587	1.0024	0.9453	0.8875	0.8288	0.7692
$\frac{dTR^t}{d\tau^f}$		0.0494	-0.0046	-0.0596	-0.1159	-0.1735	-0.2326

Note: TR^f= tax revenue from tax on firm revenues, τ^f = tax rate, s^u= share of informal output to GDP, sh^u = share of informal to total hours worked, $d\tilde{y} = d\log y$: percentage change in output (formal, p, or true, T) for a one percentage point change in the narrow (source) tax revenue as share in GDP, $dTR^f/d\tau^f$ = change in narrow tax revenue from change in the tax rate, $dTR^t/d\tau^f$ = change in total tax revenue (from all sources) from change in the tax rate.

Table 24: Tax on Firm Revenue, Single Sector Economy

TR^f/y^p	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700
τ^f	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700
$d\tilde{y}^p$		-1.3615	-1.3728	-1.3844	-1.3962	-1.4083	-1.4206
$\frac{dTR^f}{d\tau^f}$		1.2344	1.2002	1.1662	1.1325	1.0990	1.0659
$\frac{dTR^t}{d\tau^f}$		0.5198	0.4941	0.4686	0.4434	0.4184	0.3936

Table 25: Government Spending, Dual Economy

G^c/GDP	0.1700	0.1800	0.1900	0.2000	0.2100	0.2200	0.2300
s^u	0.2558	0.2539	0.2519	0.2500	0.2480	0.2461	0.2441
sh^u	0.3358	0.3341	0.3324	0.3307	0.3290	0.3272	0.3254
$d\tilde{y}^p$		0.7535	0.7643	0.7753	0.7867	0.7984	0.8106
$d\tilde{y}^T$		0.6164	0.6261	0.6360	0.6462	0.6568	0.6677
$\frac{dTR^t}{dG^c}$		0.0852	0.0865	0.0879	0.0893	0.0907	0.0921

Note: See note in Table 23. G^c = government spending (consumption).

As can be seen, the quantitative implications of the simple model are quite similar to those of BOGGEM.

Table 26: Government Spending, Single Sector Economy

G^c/GDP	0.1700	0.1800	0.1900	0.2000	0.2100	0.2200	0.2300
$d\tilde{y}^p$		0.8156	0.8291	0.8431	0.8576	0.8725	0.8880
$\frac{dTR^t}{dG^c}$		0.1021	0.1039	0.1057	0.1076	0.1094	0.1114