Bank Stability and the European Deposit Insurance Scheme

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ABSTRACT

Empirical evidence shows that a financial distress, faced by a bank or the whole economy, might cause large-scale withdrawals of deposits even when bank deposits are protected by deposit insurance, implicitly or explicitly guaranteed by a government. Building on Kiema – Jokivuolle (2015), we present a new model of such partial bank runs. In our model withdrawals are caused by the fear that both the bank and the government's deposit guarantee might fail in the future. Our focus is on a guarantee rather than on insurance, since the assets of deposit insurance funds might not be sufficient in large-scale systemic crises. Guarantee failure is possible because, being sovereign, the government may choose not to keep its promises. This option causes a fixed welfare cost (e.g., a reputational cost), which in a sufficiently severe crisis may be smaller than the costs from deposit guarantee payments. We also assume that, being welfare-maximizing, the government recapitalizes the bank during the early stage of the bank run. When decisions concerning deposit guarantee payments are made, recapitalization costs are already *sunk costs*, but the partial bank run has reduced the coverage costs that the remaining deposits might cause for the government. In this way, the depositors who withdraw during a partial bank run decrease the danger of a deposit guarantee failure and increase the incentives of the remaining depositors to keep their deposits in the bank. We apply our framework to the European Deposit Insurance Scheme (EDIS), and we view the reliability of the Single Resolution Fund and its backstop as the counterpart of the reliability of the government's promises. It turns out that in an asymmetric shock that affects only a single eurozone country EDIS improves bank stability, but its effects might be ambiguous in a systemic crisis which affects the whole Banking Union.

1. Introduction

Empirical evidence suggests that even if bank deposits are protected by a deposit insurance, implicitly or explicitly guaranteed by a government, a distress that the bank or the government faces might induce depositors to bank run-like large-scale withdrawals of deposits. An example of such behavior was seen in Greece during the period from 2009 to June 2012 as the aggregate amount of Greek bank deposits decreased from €245bn to less than €174bn (Siegel, 2014). It is estimated that only one third of the funds had been withdrawn because of decreasing living standards, and that two thirds either left the country or were stored within Greece outside the Greek banking system (ibid).[1]

The Greek "bank jog", i.e., the withdrawing of deposits only gradually, and only a part of them, would not have made much sense if depositors had during the years 2009-2012 had either no trust at all, or a perfect trust in the deposit guarantee. This is because in the former case it would have been rational to withdraw all deposits immediately, whereas in the latter case there would have been no reason for withdrawing any deposits. These two polar cases are described by the classical bank run model of Diamond and Dybvig (1983), which is a model with three periods (the period T=0 at which the bank makes an investment; the period T=1 at which a bank run might emerge; and the period T=2, at which the return from the investment becomes available). The model has two equilibria: in the bank run equilibrium it is rational for all depositors to withdraw their deposits from the bank at T=1, because all the other depositors do so, while in the other equilibrium (the one without a bank run) there is a sufficient number of depositors (the *patient depositors*) for whom it is optimal to withdraw their deposits only at T=2.

A famous criticism by Goldstein and Pauzner (2005, p. 1294) points to a certain incoherence in the Diamond – Dybvig model: despite of the existence of the bank run equilibrium, in the Diamond – Dybvig model the mutual bank solves the problem of selecting the optimal deposit contract *assuming that a bank run will not occur*. However, the model does not as such answer the question which equilibrium will be realized (or even yield probabilities for the two equilibria).

Goldstein and Pauzner (2005) introduce a global games framework, in which each depositor receives at T=1 an inaccurate signal and uses it for deducing a probability distribution for the correct signal and further, for the revenue from the bank's investment at T=2.[2] The equilibrium of this setting turns out to be unique. A unique equilibrium has been proved to emerge also when the depositors

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[1] Cf also Brown et al. (2016), who have studied bank run-like withdrawals of deposits in Switzerland during the crisis years 2008-2009. They compare the distress which various Swiss banks were facing with the tendency of the depositors of each bank to withdraw their deposits. According to ibid. (pp. 2-3), bank accounts in a highly distressed bank (UBS) were 23 percentage points more prone to experience an outflow of funds than accounts in a non-distressed bank. Cf. discussion below.

[2] Cf also e.g. Takeda (2001), who applies a global games model to international capital flows, Moreno and Takalo (2012) who interpret the dispersion in the signals of the global games framework as a measure of bank transparency, and Silva (2008), who analyzes the effects of the design of partial deposit guarantee schemes on bank run probabilities utilizing a global games framework.

coordinate their behavior in an exogenously given manner,[3] and when the demand deposit contracts are suitably modified.[4]

The subsequent literature has also identified a variety of explanations for the *partial* nature of many observed bank runs. For example, Azrieli and Peck (2012) show that a bank run might remain partial when there is more variety in consumer preferences than Diamond and Dybvig (1983) postulated. Ennis and Keister (2010) consider a setup in which depositors withdraw their deposits sequentially and the government can respond to an emerging bank run by changing its policies in order to stop the run.

However, most of the literature has so far focused on bank runs which occur in the absence of a deposit guarantee, or when the deposit guarantee is only partial (cf. Silva, 2008), i.e. guarantees a sum which is smaller than the principal of the deposits. Real-world deposit insurance and government deposit guarantees normally cover the whole deposit, implying that if the depositors had perfect trust in the deposit guarantee, both the behavior of the other depositors and negative economic signals should be irrelevant for the withdrawal decisions of each depositor. If this were the case, bank runs should never occur in the presence of a deposit guarantee.

However, the bank runs in Greece in 2009-2012 suggest that not just a bank run, but also *the trust in a deposit insurance or guarantee* can be partial. Traditional models of bank runs are not well suited for analyzing partiality of trust, although analyzing trust in this context has become increasingly important, also with an eye to the plan to establish a common deposit insurance scheme in the European Union.

The roadmap that the European Commission presented on December 6, 2017 for deepening Europe's Economic and Monetary Union suggests that the European Deposit Insurance Scheme (EDIS) should be implemented already by mid-2019 (European Commission, 2017a, p. 15). Since it is unlikely that the assets of a deposit insurance fund (whether national or union-wide) suffice for reimbursing all insured depositors in a severe, large-scale bank crisis, the availability of other sources of funding is quite essential for the credibility of a deposit insurance. In the case of EDIS, such extra funding would be provided by the Single Resolution Fund and its backstop which, according to the proposal of the European Commission (2017b, p. 6), will be provided by the future European Monetary Fund. As the Commission points out, the backstop "will instil[l] confidence in the banking system by underpinning the credibility of actions taken by the Single Resolution Board" (ibid.).

Clearly, a theoretical analysis of the confidence and the credibility that the Commission wishes to strengthen would be helpful for discussions of these new tools. Wishing to focus on cases in which the assets of insurance funds are insufficient, we shall present a model with a government deposit guarantee rather an insurance. In the model the credibility of the deposit guarantee is a matter of

[3] The equilibrium becomes unique when one postulates that the depositors coordinate their behaviour (in accordance with some exogenously given rule) on the basis of a sunspot signal (see e.g. in Peck - Shell, 2003). Cf. also Engineer et al. (2013, p. 534) and Dermine (2015). Dermine (2015) considers a Diamond-Dybvig style setting and postulates that the bank has also capital and not just deposits, and that a bank run emerges only when the bank´s loan losses are (according to the information which becomes known in the interim period) excessively large, given the bank´s amount of capital.

[4] Cf. Allen - Gale (1998). Allen and Gale point out that a unique equilibrium can be found in a Diamond- Dybvig style model with a shared signal if the bank's investment cannot be liquidated and if the bank is allowed to make the contract conditional on the return, which in their model becomes known already at T=1, that the bank obtains at T=2.

degree. In our framework, the possibility of a deposit guarantee failure emerges naturally as a result of the choices made by a welfare maximizing government, and the model provides a natural explanation for the fact that bank runs have been observed to be partial.

2. Model

Our setting resembles both the framework of Diamond and Dybvig, and the global games framework of Goldstein and Pauzner, in several ways. There are three periods (T=0, T=1, and T=2), consumers who aim at maximizing their expected utility, a single bank which accepts consumer deposits, and a government. There is a riskless liquid asset, which may be used for consumption at any time, and which we picture as cash money for the sake of concreteness. The consumers deposit their liquid assets in the bank at T=0, and they may withdraw their deposits at T=1 or T=2.

Just like in the global games framework, there is a signal η which is observed at T=1, and which provides the actors with information about the state of the economy at T=2. It is quite essential in a global games model that that the possible signals form a *continuum*, since in it the signal of each depositor is an inaccurate estimate of a more accurate (but unknown) average signal. However, we do not need to postulate an infinite number of different signals. To keep things as simple as possible, we shall below assume that that there are just two possible signals $\eta = G$ and $\eta = B$ (G for “Good” and B for “Bad”). Intuitively, the good signal G corresponds to a normal state of affairs, in which depositors believe that bank deposits may be withdrawn at will, whereas after the bad signal B they might lose their trust both in their bank and in government institutions.

In our model the bank is owned by banker who aims at maximizing his profit.[5] The government aims at maximizing expected welfare. It makes a promise of a deposit guarantee but, being sovereign, it can choose whether it respects its promise or not. As Figure 1 illustrates, in the presence of three types of actors there are many more choices to be made than in a model in which only the depositors are free to choose between different courses of action. A general analysis of a sequential game which contains all the steps shown in Figure 1 would be quite complicated, but fortunately, it is unnecessary for our current purposes.

The point of our analysis is to study the case in which $\eta = B$, i.e. the case in which the bad signal is observed, and our focus will be on the choices that are made after its occurrence. We think of the bad signal as an adverse, unexpected event, and our approach will be to first solve the model,

[5] Our reasons for introducing a banker into our model, instead of considering the simpler mutual bank of the Diamond-Dybvig model and most of the literature building on it will soon become obvious: we wish to consider bank failures at the last period, T=2, and such failures could not occur in a Diamond-Dybvig style model in which the mutual bank simply divides its wealth between the depositors at T=2.

assuming that the signal is always good, i.e. that $\eta = G$ with probability 1. Keeping the choices made before the signal unchanged, we then consider the choices that are made after it.

This procedure has two interpretations. We may think of it as corresponding to a *restricted rationality* assumption which states that the depositors and the bank behave at T=0 just as if the signal was known to be good for sure. The emergence of the bad signal is under this interpretation an unexpected shock which makes the agents change their strategies.

The other interpretation is based on the fact that – as we shall shortly see – the equilibrium choices at T=0 that we present are *corner solutions*. Even when the possibility of a bad signal is taken into account, they will remain the optimal choice if the bad signal (relative to which they are suboptimal) is sufficiently unlikely. Hence, the solution that that we present must correspond to a Nash equilibrium of the whole game depicted in Figure 1 also without assuming restricted rationality, if the probability of the bad signal $\eta = B$ is sufficiently low.

2.1 The timeline

The consumers form a continuum, whose size we normalize to $1+\mu$, and which consists of μ *impatient* consumers and 1 *patient* consumers. Each consumer is allocated one unit of the riskless, liquid asset in the beginning of period T=0.

Impatient consumers obtain utility only from consumption at T=1, while patient consumers obtain utility from consumption both at T=1 and at T=2. The utility of both patient and impatient consumers is represented by the utility function u which by assumption satisfies the familiar conditions

(1) $$u(0)=0,\ u'(c)>0,\ u''(c)<0$$

and which, by normalization, is also assumed to satisfy the condition[6]

(2) $$u'(0)<1$$

Denoting the consumption in periods T=1 and T=2 by $u(c_1)$ and $u(c_2)$, respectively, the utility of a patient consumer is given by $u(c_1+c_2)$ and the utility of an impatient consumer is given by $u(c_1)$. The characteristics or being patient and impatient are unobservable to others, and not yet known at T=0.

The banker has profitable investment opportunities which are not available to the consumers directly. Motivated by these opportunities, the bank presents the depositors with a demand deposit

[6] The motive for introducing the assumption (2) will be made clear in Section 2.4. There it will be seen that the assumption (2) restricts the weight that the government gives to consumer utility in its welfare function ((25) below).

contract which allows them to withdraw R_1 at T=1 or postpone withdrawal until T=2. The government promotes bank stability with a deposit guarantee which applies to the deposits withdrawn in each period. The deposit guarantee is a promise that the government provides the depositors with the principal of their deposit (i.e., one unit of liquid assets), should the bank fail to do so. We shall discuss the functioning of this guarantee in Sections 2.2. and 2.4 below.

The consumers may choose between depositing and storing their wealth in the form of liquid assets. When the depositors are willing to deposit, the banker may choose any number of depositors between zero and the total number of consumers, $1+\mu$. We denote the number of depositors by *D*. Since the qualities of being patient or impatient are not known, the number of the impatient depositors is

(3) $$D_{IMP} = \frac{\mu}{1+\mu} D$$

and the number of patient depositors is

(4) $$D_{PAT} = \frac{1}{1+\mu} D$$

Having received deposits, the banker uses the sum I_0 (where $0 \le I_0 \le D$) for an investment.

At the beginning of period T=1 the signal η (where $\eta = G, B$) becomes known, and the consumers learn their types (patient or impatient). The banker then specifies the interest factor R_2 that applies to the deposits which are withdrawn only at T=2.[7] Knowing the signal, their own types and the deposit interest factors, the depositors choose whether to withdraw. We refer to the decision not to withdraw as *staying* for short.

It is obvious that all the impatient depositors always choose to withdraw. We denote the share of the staying, and of the withdrawing depositors among all patient depositors by χ and λ, respectively. Clearly,

(5) $$\lambda + \chi = 1$$

We could choose either λ or χ to be the variable which represents the choice made by the depositors. It has turned out that using χ leads to less clumsy notation. While λ would be a measure of the size of the bank run, χ can be thought of as a measure of the stability of the banking system, and we refer to it as *bank stability* for short. Clearly, the value $\chi = 0$ corresponds to the full-scale

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[7] Observe that under these assumptions the banker cannot make at T=1 a binding commitment $(R_1, R_2(\eta))$ which would specify also the payoff at T=2, R_2, and make it depend on the signal. The exclusion of this possibility is motivated not just by realism (i.e., the fact that actual demand deposit contracts do not make interest rates contingent on receiving negative economic signals) but also by our interpretation of the signal $\eta = B$. Its real-world counterparts are not e.g. well-defined economic indicator values that one could make contracts contingent upon, but various kinds of negative developments which cannot be characterized precisely in advance.

bank run of most bank run models, while the maximum value $\chi = 1$ corresponds to a no-bank-run equilibrium, in which all patient depositors stay.

If the withdrawal at T=1 exceeds the liquid assets of the bank, the bank can get funding through government *recapitalization*. By recapitalization we mean a procedure in which the government provides the bank with the extra liquid assets that it needs for the withdrawn deposits and in exchange receives the ownership of some share s_G of the bank. This ownership gives the government the right to receive a part of the payoff of the bank at T=2.

If government recapitalization was the only source of funding for the banker in case of liquidity shortage, our model would not yield a well-defined equilibrium value for s_G. However, we postulate that the banker has also the possibility to *disinvest*. More specifically, if the banker makes at T=0 the investment I_0 and liquidates the part ΔI ($0 \le \Delta I \le I_0$) of it at T=1, the liquidation immediately produces $\gamma(\Delta I)$, where $\gamma < 1$. Disinvestment reduces welfare, and the government prefers recapitalizing the bank to letting the banker disinvest. The outside option of disinvestment affects the equilibrium of the model via the value of s_G, which is determined by the condition that the banker would choose to disinvest if recapitalization reduced his profits more than disinvesting. This is discussed in more detail in Section 2.2.

If the investment which remains at T=2 is I, it produces ρI where ρ is a random variable. The probability distribution of ρ is influenced by the signal η. We assume that after each signal η ($\eta = G$ or $\eta = B$) the distribution of ρ is characterized by the density function $h_\eta(\rho)$. It turns out in order to make our model yield interesting comparative static results, it is practical to assume that $h_B(\rho)$ is positive in the whole interval $[0,1]$, i.e. that after the "bad" signal arbitrarily small returns for the investment occur with a positive probability. On the other hand, as we already explained, the "good" signal corresponds to a case in which the depositors do not fear to lose their deposits. Below it will turn out that this will be the case when

(6) $\quad h_G(\rho) = 0$ when $\rho < 1 + \varepsilon$ for some positive ε

i.e. when the investment *I* produces after the "good" signal at least the slightly more than the value of the invested liquid assets.[8]

At T=2 the assets of the bank consist of the return γI from the remaining investment and the liquid assets, if any,[9] that remain after the investment of T=0 and the withdrawals of T=1, and its liabilities consists of χ deposits of value R_2. If the assets suffice for the withdrawals, the depositors receive their deposits and the bank's owners (the banker, the government, or both) get the difference of its assets and liabilities. When the assets of insufficient, the bank fails. In this case the bank is taken over

[8] Our analysis would, as a matter fact, be valid under the simpler assumption which states that $h_G(\rho) = 0$ when $\rho < 1$, but if we did not introduce the slightly stronger version (6), the discussion in Section 2.3 would become quite clumsy.

[9] We shall shortly see that at T=2 there are, as a matter of fact, no such remaining liquid assets in the equilibria of the model.

by the government. As we have seen, the government has given a *deposit guarantee,* which obliges the government to provide each of the staying depositors with the principal (i.e., 1) of their deposits. As the last move of the game (which occurs only in case of bank failure), the government chooses whether to honor its promise. We postpone the more detailed discussion of bank failure, and the welfare function that the government maximizes while making its choice, to Section 2.4 below.

2.2. Recapitalization and the bank's final payoff

We now return to the discussion of period T=1. As we have seen, all the D_{IMP} impatient depositors will withdraw at T=1, and in our notation the number of withdrawing and staying patient depositors are denoted by λD_{PAT} and by χD_{PAT}, respectively. Remembering (3), (4), and (5), we see that the withdrawals amount up to

$$(7) \qquad R_1\left(D_{IMP}+\lambda D_{PAT}\right)=R_1\left[\frac{\mu}{1+\mu}+\frac{1}{1+\mu}\lambda\right]D=R_1\left[1-\frac{\chi}{1+\mu}\right]D$$

We denote the difference of the liquid assets of the bank (in the absence of a disinvestment) and the withdrawals by *L*, so that

$$(8) \qquad L=D-I_0-\left(1-\frac{\chi}{1+\mu}\right)DR_1$$

Simple algebra shows that the liquid assets of the bank suffice for the withdrawals (i.e. that $L\geq 0$) even without any disinvestment if the bank stability χ satisfies $\chi\geq\bar{\chi}$, where

$$(9) \qquad \bar{\chi}=\frac{1+\mu}{R_1}\left(\frac{I_0}{D}+R_1-1\right)$$

By definition, the bank's *net worth* at T=2 is the difference between its assets and liabilities, and as we have seen, the bank fails when this difference is negative. The bank's *final payoff* is equal with the net worth when the bank does not fail, and zero when it does. We denote the bank's final payoff by π_{BANK} and the banker's profit by π_{BANKER}. These are identical when the bank's liquid assets suffice for the withdrawals at T=1, and we may now conclude that they are in this case given by

$$(10) \qquad \begin{aligned}\pi_{BANK}=\pi_{BANKER}&=\max\left\{L+\rho I_0-R_2\frac{\chi D}{1+\mu},0\right\}\\&=\max\left\{(\rho-1)I_0-(R_1-1)D-(R_2-R_1)\frac{\chi D}{1+\mu},0\right\}\end{aligned} \qquad (\chi\geq\bar{\chi})$$

When $\chi < \bar{\chi}$, the liquid assets of the bank are insufficient for the withdrawals. In this case there are two strategies to be considered, disinvestment and recapitalization. In a disinvestment a part ΔI of the bank's investment changed into $\gamma(\Delta I)$ (where $\gamma < 1$) in liquid assets. We assume that the government prefers recapitalization to disinvestment independently of which one of the signals $\eta = B, G$ is realized, and independently of the size of the bank run. We also assume that, in case of recapitalization, the government prefers larger values of its share s_G as an owner of the bank to smaller ones.

The latter assumption means, simply, that the government prefers obtaining the bank's payoff to giving it to the banker. Also the intuition behind the former assumption is easy to see. Disinvestment reduces the profits when ρ is sufficiently large to prevent the bank from failing, and when ρ is smaller and the bank fails, a smaller revenue from the remaining investment might correspond to larger deposit guarantee payments by the government at T=2. Hence, assuming that γ is sufficiently small, it makes sense for the government to recapitalize the bank instead of letting the banker destroy a part or whole of the investment.

When extra liquidity is needed, the value of L (defined by (8)) is negative, and the necessary extra liquidity amounts up to $|L|$. As our next step, we shall explain how the outside option of disinvesting determines the share s_G of the bank that the government can demand for itself in exchange for providing $|L|$. In general, a disinvestment of size ΔI reduces the remaining investment to $I = I_0 - \Delta I$ and produces $\gamma(\Delta I)$ in liquid assets at T=0. Using the disinvestment strategy, the liquid assets that are available at T=1 consist of the liquid assets $D - I_0$ that remain after T=0 plus the liquid assets $\gamma(\Delta I)$ from the disinvestment. These assets equal the withdrawals only after the whole investment has been disinvested (i.e. when $\Delta I = I_0$ and $I = 0$) if χ equals

$$\underline{\chi} = \frac{1+\mu}{R_1}\left[(1-\gamma)\frac{I_0}{D} + R_1 - 1\right] \tag{11}$$

If $\chi \leq \underline{\chi}$, the disinvestment strategy would lead to the elimination of the whole investment, and if $\chi < \underline{\chi}$, it would cause bank failure already at T=1. Between the two extremes $\chi = \bar{\chi}$ (for which no disinvestment is needed and the remaining investment is $I = I_0$) and $\chi = \underline{\chi}$, the investment that remains under the disinvestment strategy is a linear function of χ. Hence, we may express the investment that still remains at T=2 under the disinvestment strategy as

$$I_{DIS}(\chi) = \begin{cases} 0, & \chi < \underline{\chi} \\ \left[(\chi - \underline{\chi})/(\bar{\chi} - \underline{\chi})\right] I_0, & \underline{\chi} \leq \chi \leq \bar{\chi} \end{cases} \tag{12}$$

After disinvestment the assets of the bank would at T=2 amount up to $\rho I_{DIS}(\chi)$ and the liabilities would amount up to R_2 for each of the χD_{PAT} remaining deposits. Remembering (4), it is seen that the final payoff from the bank would be

(13) $$\pi_{DIS} = \max\{\rho I_{DIS}(\chi) - R_2\chi D/(1+\mu), 0\} \qquad (\chi \le \overline{\chi})$$

and this final payoff would at the same time express the profit of the banker.

The disinvestment strategy affects the equilibrium of the model, in which extra liquidity is provided by recapitalization, via the result (13). Under the recapitalization strategy, in which the government provides the missing liquidity and demands in exchange the ownership of the share s_G of bank, the final payoff from the bank is

(14) $$\pi_{BANK} = \max\{\rho I_0 - R_2\chi D/(1+\mu), 0\} \qquad (\chi \le \overline{\chi})$$

The share s_G of this payoff goes to the government and the share $1-s_G$ to the banker. Hence, in this case the banker's profit is

(15) $$\pi_{BANKER} = (1-s_G)\pi_{BANK} \qquad (\chi \le \overline{\chi})$$

while the final payoff that the government receives from the bank is

(16) $$\pi_{GOV} = s_G\pi_{BANK} \qquad (\chi \le \overline{\chi})$$

The banker will not accept recapitalization if the expected profit from it is smaller than the expected profit from disinvestment. Introducing the notation

(17) $$E_{\rho|\eta}G(\rho) = \int_0^\infty G(\rho)h_\eta(\rho)$$

for the expectation value of any function of $G(\rho)$ of ρ, assuming that the signal is η (where either $\eta = B$ or $\eta = G$), we may formulate the condition which determines the government ownership s_G as

(18) $$(1-s_G)E_{\rho|\eta}(\pi_{BANK}) = E_{\rho|\eta}(\pi_{DIS}) \qquad (\chi \le \overline{\chi})$$

We conclude from (12) and (13) that the result (18) is formally valid also when $\chi < \underline{\chi}$ (i.e., in which the disinvestment strategy leads to the elimination of the whole investment and bank failure already at T=1) since in this case disinvestment corresponds to zero profit, implying that the government can demand the whole bank for itself and that $s_G = 1$. Our analysis of the banker's strategy is based on the result, which implied by (15) and (18), that

(19) $$E_{\rho|\eta}\pi_{BANKER} = (1-s_G)E_{\rho|\eta}\pi_{BANK} = E_{\rho|\eta}\pi_{DIS}$$

so that the banker's expected-profit-maximizing choices are identical with the ones that correspond to the disinvestment strategy (despite of the fact that the recapitalization strategy is always chosen).

2.3 The signal G and some simplifications

We shall now consider the case in which the signal η turns out to be G. According to (6), this implies that at T=2 the investment produces at least slightly more than the value of the invested assets. Our analysis of this case justifies a number of simplifications to our model.

Although we have already explained why we may leave the banker's choice between disinvestment and recapitalization out of the game that we consider (and assume that recapitalization is always chosen), bewilderingly many choices still seem to exist in the model. At T=0 the banker chooses R_1; the depositors choose whether to deposit; if they do, the banker chooses the amount of deposits D and the size of the investment I_0; after the signal η the banker chooses the interest factor R_2; the depositors choose whether to stay or withdraw; and at T=2, in case of bank failure, the government chooses whether to provide the promised deposit guarantee.

However, our approach is to solve the equilibrium values R_1, D, I_0, R_2 and χ assuming that the good signal $\eta = G$ is observed, to assume that the choices R_1, D, and I_0 (which are made before observing the signal) correspond to the good signal, and to investigate the game that takes place after the signal when the signal is $\eta = B$. When the case with the "good" signal is investigated, it is not necessary to consider the choice of the government at T=2, because this choice (i.e., whether to provide deposit guarantee payments) is made only in case of bank failure, and it turns out that after $\eta = G$ the bank never fails in equilibrium. As we stated above, under its obvious interpretation our model describes a case in which the signal $\eta = B$ is a shock which the actors have not considered while choosing their strategies at T=0, but the same equilibrium emerges also when the probability of the signal $\eta = B$ is sufficiently small, given the information of period T=0.

As our first step, we observe that a choice $R_2 < R_1$ would lead to a full-scale bank run, since for a patient consumer the utility of withdrawing is always $u(R_1)$, but the utility from staying is maximally $u(R_2)$. Accordingly, from now on we shall assume that $R_2 \geq R_1$. Our analysis is complicated by the fact we have not yet stated what happens in case of bank failure (although we know from (6) that when $\eta = G$, there are no bank failures when the values of R_1 and R_2 are not excessively large). However, we can already now present a somewhat technical result, which provides a maximum for the banker's expected profit and which allows us to solve for R_1, D, I_0, and R_2 in the "good signal equilibrium".

Remark 1.[10] Assume that observed signal is $\eta = G$, and consider the choices of R_1, I_0, R_2 by the banker, viewing D as fixed.

(a) If the interest factors chosen by the banker are R_1, R_2 (where $0 < R_1 \le R_2$), the banker's expected profit is not larger than $E_{\rho|G}\left(\pi^*_{BANKER}\right)$, where

$$\pi^*_{BANKER} = \max\left\{D\left[\rho\left(1-\frac{\mu R_1}{1+\mu}\right)-\frac{\overline{\overline{R}}_2}{1+\mu}\right],0\right\}$$

and $\overline{\overline{R}}_2 = \min\left\{R_2,(1+\varepsilon)R_1\right\}$. The maximum value can only be achieved if the investment is

$$I_0^* = D\left(1-\frac{\mu R_1}{1+\mu}\right)$$

and all patient depositors choose to stay.

(b) There are interest factor values $\overline{R}_1, \overline{R}_2$ with $\overline{R}_2 > \overline{R}_1 > 1$ which are such that, as long as $R_1 \le \overline{R}_1$ and $R_2 \le \overline{R}_2$, the bank never fails. If the banker chooses interest factors R_1, R_2 for which $R_1 < R_2$, $R_1 \le \overline{R}_1$ and $R_1 < R_2 \le \overline{R}_2$, and the investment is I_0^*, all patient depositors stay and the banker's expected profit has the value $E_{\rho|G}\left(\pi^*_{BANKER}\right)$ defined in part (a).

Remark 1 leads to a simple characterization of the expected-profit-maximizing choices R_1, I_0 and R_2. The upper limit of expected profit, π^*_{BANKER} is decreasing in R_1, and also in R_2 when $R_2 < (1+\varepsilon)R_1$, and hence, it is an immediate consequence of Remark 1 that the optimal values of R_1 and R_2 must satisfy $R_1 < \overline{R}_1$ and $R_2 < \left\{\overline{R}_2,(1+\varepsilon)R_1\right\}$.

Assuming these conditions to be valid, we now consider the choice that the patient consumers make at T=1 after the signal $\eta = G$. Clearly, the banker's expected profit – which now equals $E_{\rho|G}\left(\pi^*_{BANKER}\right)$ – is increased by a decrease in R_2 if $R_2 > R_1$ but – as we have seen – banker's profit is always zero if $R_2 < R_1$. It follows that the only Nash equilibrium is the limiting case in which $R_2 = R_1$, it is immaterial to the patient depositors whether to stay or withdraw (since they know that the bank never fails and their utility is in both cases $u(R_1)$), and they all choose to stay so that $\chi = 1$.

We now consider banker's choice of R_1 at T=0. If $R_1 < 1$, the consumers know that their expected utility from depositing must be $u(R_1)$, i.e. smaller than the consumers' utility from storing wealth in the form of liquid assets. Hence, the banker cannot attract any depositors if he chooses $R_1 < 1$, and

[10] An appendix containing the proofs of the Remarks and Theorems is available upon request from Ilkka Kiema (ilkka.kiema@labour.fi)

we can now conclude that $R_1 \geq 1$ in equilibrium. Further, since in equilibrium $R_2 = R_1$, we observe that

$$\pi^*_{BANKER} = \max\left\{ D\left[\rho\left(1 - \frac{\mu R_1}{1+\mu}\right) - \frac{R_1}{1+\mu}\right], 0 \right\}$$

Now the choice $R_1 > 1$ cannot maximize expected profit, since $E_{\rho|G}\left(\pi^*_{BANKER}\right)$ is decreasing in R_1, while the choice $R_1 < 1$ yields zero profit. Hence, the only Nash equilibrium is the limiting case in which $R_1 = 1$, it is immaterial for the consumers whether to deposit since it yields the same utility as holding liquid assets would yield, and the number *D* of consumers, as desired by the banker, choose to deposit. The maximum expected profit $E_{\rho|G}\left(\pi^*_{BANKER}\right)$ that we just deduced increases linearly in *D*, implying that the expected-profit-maximizing value of *D* is its maximal value, i.e.

$$D = 1 + \mu$$

Finally, we may now conclude from Remark 1(b) that the optimal investment is

$$I_0 = D\left(1 - \frac{\mu R_1}{1+\mu}\right) = 1$$

Except for the result concerning the interest factor R_2, which is chosen only after the signal has been observed, these results remain valid also in the equilibrium in which the signal unexpectedly turns out to be $\eta = B$. Remembering (3) and (4), the simplifications that apply also to this case can now be summarized as follows:

$$(20) \qquad \begin{cases} D = 1 + \mu \\ D_{PAT} = 1 \\ D_{IMP} = \mu \\ R_1 = 1 \\ I_0 = 1 \end{cases}$$

In particular, these conditions imply that the investment I_0 is identical with the number D_{PAT}, further implying that the bank has never extra liquidity after the period T=1. In other words, the value of *L* defined by (8) (which expresses the difference between the liquidity that the bank needs at T=1 and its actual liquidity) is never positive. Indeed, *L* is now given by

$$(21) \qquad L = \chi - 1$$

We saw above that the case with extra liquidity corresponds to χ values with $\chi > \overline{\chi}$, and we can now conclude also from (9) and (11) that

(22) $$\begin{cases} \underline{\chi} = 1 - \gamma \\ \overline{\chi} = 1 \end{cases}$$

which also shows that the case with extra liquidity is impossible.

Finally, remembering (19) and (13), we observe that that the expected profit of the banker can (in general, and not just after the "good" signal) be now expressed as

(23) $$E_{\rho|\eta}\pi_{BANKER} = E_{\rho|\eta}\pi_{DIS} = E_{\rho|\eta}\max\left\{\rho I_{DIS}(\chi) - R_2\chi, 0\right\}$$

where according to (12), (20), and (22)

(24) $$I_{DIS}(\chi) = \max\left\{0, (\chi + \gamma - 1)/\gamma\right\}$$

Armed with these simplifications, we now move to the discussion of the case in which the signal turns out to "bad", i.e. $\eta = B$. There are three choices that remain to be considered in this case: the choice of R_2 at T=1 by the banker; the choice whether to withdraw or to stay, made at T=1 by the depositors; and the choice whether to provide the promised deposit guarantee, made at T=2 by the government. To proceed, we must now discuss bank failure and the government's choice in more detail.

2.4 The deposit guarantee and the welfare function

By assumption, the welfare function which the government wishes to maximize is

(25) $$W = \tilde{U} + \xi\pi_{BANKER} + \pi_{GOV} - (1-\chi) - \chi\tau - \hat{F}$$

where the first term

(26) $$\tilde{U} = (D-\chi)u(1) + \chi u_S$$

is the aggregate utility of the depositors, u_S being the utility of each staying depositor. (The withdrawing $D-\chi$ depositors include, of course, both the impatient depositors and the withdrawing patient depositors.) The next two terms correspond to the payoff that bank ownership yields to the banker and to the government. The constant multiplier ξ satisfies $\xi < 1$, which means, intuitively, that the government sees less welfare value in the assets obtained by the banker than in the assets it gets for itself.

The fourth term represents the costs of recapitalization. We saw in Section 2.2 that the needed recapitalization is always $|L|$ which according to (21) equals

$$|L| = -L = 1 - \chi$$

To explain the remaining two terms, it is necessary to discuss deposit guarantee in more detail. In case of bank failure the assets of the bank – which amount up to $\rho I_0 = \rho$, since the bank cannot have any excessive liquid funds at T=1 in equilibrium - are divided equally between the χ staying depositors. By assumption, the government makes an additional transfer $\tau \geq 0$ to each staying depositor in case of bank failure. The choice of the government in the game that we consider consists in choosing the value of τ. This implies that the utility of each staying depositor is

(27) $$u_S = u(\rho / \chi + \tau)$$

We model the deposit guarantee as the promise that the payments to each staying depositor,

$$\rho / \chi + \tau$$

will altogether amount up to at least 1. In other words, the government promises that transfer τ amounts up to at least

(28) $$\tau_{DEP} = \max\{0, 1 - \rho / \chi\}$$

The quantity $\hat{F}$ is the counterpart of reliability of the government's promise. Being sovereign, the government can also choose not to honor its promise, but this choice causes a fixed welfare cost $F > 0$. The welfare cost represents e.g. indirect reputational costs from distrust in government institutions, and because of it the welfare-maximizing government can fail to provide the promised withdrawn deposits only when providing them is sufficiently costly. Formally, we define $\hat{F}$ by

(29) $$\hat{F} = \begin{cases} F, & \tau < \tau_{DEP} \\ 0, & \tau \geq \tau_{DEP} \end{cases}$$

We are now in the position to motivate the assumption (2), i.e. $u'(0) < 1$. We conclude from (26) and (25) that this assumption restricts the weight that consumers' utility has in the government's welfare function. In general, a welfare-maximizing government might wish to make social transfers to the depositors of a failed bank even in the absence of any deposit guarantee (simply in order to increase their utility). However, wishing to focus only on government spending which is motivated by the guarantee, we shall exclude the possibility of such transfers from our model. To exclude it, we conclude from (1) that the maximal aggregate utility that a small transfer Δc to m bank depositors could yield is $m(\Delta c)u'(0)$, while the welfare cost of those transfers is $m(\Delta c)$. Hence, the postulate that such transfers are never socially optimal may be formulated as the condition (2), i.e. $u'(0) < 1$.

3 Solving the model

We are now ready to solve the restricted model which describes the events after the “bad” signal $\eta = B$. Solving it consists of finding the 3-tuples (R_2, χ, τ) which correspond to its Nash equilibria. Proceeding by backward induction, we begin by solving the choice of the deposit guarantee payment τ by the government, when the values of R_2 (which is chosen by the banker) and the value of χ (which emerges from the choices of the patient depositors) have been given.

3.1 Choice of the government at T=2

The following remark, which is a straightforward consequence of (2) and (25), states that the government never makes to the depositors payments which would exceed the payments motivated by deposit guarantee; i.e., it makes either the just the promised payment τ_{DEP} or no payment at all.

Remark 2. The transfer τ that a welfare-maximizing government chooses is always either $\tau^* = \tau_{DEP}$ (i.e. the minimal transfer which is compatible with the promised guarantee) or $\tau^* = 0$.

Obviously, the choice $\tau^* = 0$ corresponds to deposit guarantee failure whenever $\tau_{DEP} > 0$. On the other hand, when the bank does not fail, and also when the assets ρ of the failed bank suffice for covering the principal of the remaining χ deposits (i.e. when $\chi \le \rho$), (28) implies that $\tau_{DEP} = 0$. In this case Remark 2 simply states that the government does not make any extra transfers to the remaining depositors of the bank. The following theorem states that deposit guarantee failures can only occur when the revenue from the bank’s investment is sufficiently small.

Theorem 1. If the government lets the deposit guarantee fail for some values of the bank’s revenue ρ, there is a threshold value $\overline{\rho}^*_{GUAR}$ of the revenue ρ which is such that the government lets the deposit guarantee fail when $\rho < \overline{\rho}^*_{GUAR}$ but not otherwise. The value $\overline{\rho}^*_{GUAR}$ is determined by

$$\chi u(1) - \left(\chi - \overline{\rho}^*_{GUAR}\right) = \chi u\left(\frac{\overline{\rho}^*_{GUAR}}{\chi}\right) - F$$

We can conclude from Theorem 1 that

(30) $\overline{\rho}^{*}_{GUAR} \leq \chi$

as it, of course, should be the case (since, as we just noted, the deposit guarantee is not needed when $\rho \geq \chi$).

For the ease of notation, we now define $\overline{\rho}^{*}_{GUAR} = 0$ if it is not welfare-maximizing to let the deposit guarantee fail for any value of the revenue ρ. Given this convention, Theorem 1 implies that the set of revenue values ρ for which the government lets the deposit guarantee fail is always the (possibly empty) interval $\left[0, \overline{\rho}^{*}_{GUAR}\right)$. We shall still present an essential result which is concerned with the comparative statics of $\overline{\rho}^{*}_{GUAR}$.

Remark 3. The threshold value $\overline{\rho}^{*}_{GUAR}$ increases with the number χ of the staying depositors. More rigorously, the deposit guarantee cannot fail if χ is sufficiently small, and $\overline{\rho}^{*}_{GUAR}$ is strictly increasing in χ whenever χ is such that the deposit guarantee can fail.

Summing up, in our model the government makes only transfers which are made necessary by the deposit guarantee. Further, the values of the revenue ρ for which the deposit guarantee fails (if any) are below the threshold value $\overline{\rho}^{*}_{GUAR}$, and the range of such values (if any) gets larger as the number of the staying depositors increases. This is, of course, because of the raising costs that payments to a larger number of depositors cause for the government.

3.2 The choice between staying and withdrawing by the patient depositors

Having found the equilibrium choice by the government at T=2, we now turn to the choice that the patient depositors make at T=1 between staying and withdrawing. While withdrawing always produces the utility $u(1)$, the utility from staying depends on both the interest factor R_2 and the signal η which determines the probability distribution of the revenue of the bank's investment, $h_{\eta}(\rho)$. We shall denote the expected utility from staying (given $\eta = B$ and R_2) by $E_{\rho|B} u_S$.

Assuming that the bad signal $\eta = B$ has been observed, there are four cases to consider when evaluating u_S. Firstly, the bank does not fail if the revenue from the investment, ρ are equal with or larger than its liabilities χR_2. In this case each depositor receives the sum R_2. Secondly, if

$\chi < \rho < \chi R_2$, the bank's assets suffice for paying the guaranteed sum (i.e. 1) to each staying depositor despite of bank failure. In this case the assets of the bank are divided evenly between the staying depositors, so that each of them receives the sum of ρ / χ. Thirdly, if $\overline{\rho}^*_{GUAR} \leq \rho < \chi$, the payments to each staying depositor amount up to the minimum which is compatible with the guarantee, i.e. 1. Finally, if $\rho < \overline{\rho}^*_{GUAR}$, the government fails to honor its promise and each staying depositor receives only the sum ρ / χ which they would receive in the absence of the deposit guarantee. Summing up,

$$(31)\qquad \begin{aligned} E_{\rho|B} u_S &= \int_0^{\overline{\rho}^*_{GUAR}} u\left(\frac{\rho}{\chi}\right) h_B(\rho)\, d\rho + \int_{\overline{\rho}^*_{GUAR}}^{\chi} u(1) h_B(\rho)\, d\rho \\ &+ \int_{\chi}^{\overline{\rho}_{BANK}} u\left(\frac{\rho}{\chi}\right) h_B(\rho)\, d\rho + \int_{\overline{\rho}_{BANK}}^{\infty} u(R_2) h_B(\rho)\, d\rho \end{aligned}$$

We are now in the position to explain why bank runs always remain partial in our model. To see why this is the case, we observe that a partial bank run makes the liabilities of the bank decrease, but due to recapitalization, there is no corresponding decrease in the revenue from the bank's investment. Hence, (as also Remark 3 implies) the bank failure probability must decrease as the number of staying depositors decreases, and a bank run stops when the expected utility from staying has become identical with the utility from withdrawing, i.e. when

$$(32)\qquad E_{\rho|B} u_S = u(1)$$

Theorem 2. The bank run is partial for any interest factor $R_2 > 1$. In other words, when $R_2 > 1$, the equilibrium number χ^* of the staying depositors satisfies $\chi^* > 0$.

The monotonous decrease of bank failure probability implies that the number of the staying depositors has a unique equilibrium value. This result is due to recapitalization, and must remain valid even in the absence of the deposit guarantee.[11] When extra capital is available, the decision of some patient consumers to withdraw is not a reason for the other patient consumers to follow suit; rather, it might be a reason to stay because it reduces the remaining liabilities of the bank.

Theorem 3. Assume that the banker's interest factor choice $R_2 > 1$ is fixed. The subgame which consists of the number staying depositors χ and the government's choice of τ has a unique

[11] More rigorously, the situation in which there is no deposit guarantee may be represented by putting $F = 0$ and $\overline{\rho}^*_{GUAR} = \chi$. The result (31) implies, also when these choices are made, that the attractiveness of staying decreases with χ.

equilibrium. In particular, the number χ^* of the staying depositors is uniquely determined in equilibrium.

To add further intuition to Theorem 3, one should note that when the government decides whether to make deposit guarantee payments, the costs of recapitalization are already *sunk costs*. However, the earlier bank run reduces the costs that are caused by the guarantee for the remaining deposits. Hence, the bank run serves as *commitment device*, as it increases the government's incentives to keep its promise and the remaining depositors' expected utility from staying, and this makes the bank run stop at a uniquely determined point. Also the following plausible result is valid.

Remark 4. In a partial bank run equilibrium the equilibrium number of staying depositors increases with the bank's interest factor R_2. In other words, $d\chi^* / dR_2 > 0$ when the bank run is partial.

3.3 The choice by the banker

The first move of the three-move game after the "bad" signal is made by the bank, and it consists of choosing R_2. The banker aims at maximizing his expected profit while choosing it, and the expected profit is according to (23) and (24) given by

$$E_{\rho|\eta}\pi_{BANKER} = E_{\rho|\eta} \max\left\{\rho\left(\frac{\chi+\gamma-1}{\gamma}\right) - R_2\chi, 0\right\}$$

Defining $\bar{\rho}_{BANKER}$ as the threshold value which satisfies

(33) $$\bar{\rho}_{BANKER} = \frac{\gamma R_2 \chi}{\chi+\gamma-1}\ \rho$$

we may express the banker's profit also in the form

(34) $$E_{\rho|\eta}\pi_{BANKER} = \int_{\bar{\rho}_{BANKER}}^{\infty}\left[\rho\left(\frac{\chi+\gamma-1}{\gamma}\right) - R_2\chi\right] h_B(\rho)\, d\rho$$

Theorem 3 implies that when the interest factor value R_2 has been fixed, there is a unique value of the bank stability χ which corresponds to an equilibrium. Finding the expected-profit-maximizing value of χ is a difficult task despite of this uniqueness result. In general, there are three kinds of cases to consider.

Firstly, we remember that according to Remark 3, the deposit guarantee never fails when the number of staying depositors is sufficiently small. We let χ_M represent the threshold value which separates the χ values for which the deposit guarantee can and cannot fail. It must be the case that $\overline{\rho}^*_{GUAR} = 0$ when $\chi = \chi_M$, and we can infer from Theorem 1 that χ_M is also characterized by

$$(35) \qquad \chi_M \left(1 - u(1)\right) = F$$

We now observe that as R_2 approaches the minimum $R_2 = 1$ from above, the number of staying depositors must according to (31) and (32) approach χ_M. (Intuitively, the interest $R_2 - 1$ is a compensation for the loss that the depositor suffers when the deposit guarantee fails, and in equilibrium this compensation approaches zero when the risk of deposit guarantee failure approaches zero.) The profit which corresponds to this limiting case is

$$E_{\rho|\eta}\pi_{BANKER} = E_{\rho|\eta} \max\left\{\rho\left(\frac{\chi_M + \gamma - 1}{\gamma}\right) - \chi_M, 0\right\}$$

In this case the banker takes no action to stop the bank run which is caused by the bad signal and relies completely on the government's promise as a tool for stopping it.

Secondly, considering larger values of R_2, the maximization problem might have an internal solution for which the derivative of (34) is zero, i.e. for which

$$(36) \qquad \frac{dE_\rho \pi_{BANKER}}{dR_2} = \int_{\overline{\rho}}^{\infty} \left[-\chi + \left[\frac{\rho}{\gamma} - R_2\right]\frac{d\chi}{dR_2}\right] h_B(\rho)\, d\rho = 0$$

Thirdly, there is another corner solution to be considered: it might be possible and optimal for the banker to increase the interest factor R_2 until there is no bank run, i.e. until $\chi = 1$. We denote the smallest value of the interest factor (if any) which suffices for this purpose by $R_{2,M}$.

4. The welfare effects of a change in deposit guarantee reliability

In our model the reliability of the deposit guarantee is represented by the cost *F*. As *F* represents the inability of the government to make binding commitments, the search for the optimal (welfare-maximizing) value of *F* does not seem very meaningful; after all, *F* cannot, by definition, be freely adjusted by the government. Nevertheless, we shall address the question how expected welfare (relative to the probability distribution of ρ, given the signal $\eta = B$) would be affected by changes in *F*.

Considering the expectation value of our welfare function (25), it is easy so see that the expected consumer utility $\tilde{U}$ is a constant, since in equilibrium the utility of each consumer is according to (32) always $u(1)$. This is because the risks that bank failure or deposit guarantee failure might cause to the depositors are always compensated by interest payments in equilibrium. Hence, we may write expected welfare as

(37) $$E_{\rho|B}W(R_2,\chi) = u(1) + E_{\rho|B}(\xi\pi_{BANKER} + \pi_{GOV}) - (1-\chi) - E_{\rho|B}(\chi\tau + \hat{F})$$

Since we measure the reliability of the deposit guarantee by *F*, i.e. by the cost of breaking it, an improvement in its reliability has a direct negative welfare effect when the guarantee breaks down and which in accordance with (29) shows up as an increased value of $\hat{F}$. This negative welfare effect has no counterpart in the traditional bank run models in which the guarantee is always perfectly reliable and often a promise that one never needs to keep.

The rest of the terms depend on (37) the reliability parameter *F* indirectly, because of its influence on bank stability, as measured by χ. In addition, the final payoff from the bank – which is divided into the banker's profit π_{BANKER} and government's final payoff π_{GOV} – depends also on the interest factor R_2 that the banker chooses, which is affected by *F*.

In a discussion of the aggregate effect on expected welfare there are three cases to consider. Beginning with the easiest case, we consider the situation in which the banker eliminates the bank run altogether by choosing the smallest interest factor $R_2 = R_{2,M}$ which suffices for preventing it. In this case there is no recapitalization, the banker's profit is identical with the final payoff from the bank, and $\chi = 1$ so that (37) becomes

$$E_{\rho|B}W(R_{2,N},\chi) = u(1) + E_{\rho|B}(\xi\pi_{BANKER}) - E_{\rho|B}(\tau + \hat{F})$$

In the no-bank-run equilibrium the increased deposit guarantee reliability will, according to Theorem 1, decrease ρ^{*}_{GUAR}, and in accordance with (32) and (31) this effect must be compensated by a decrease in the interest factor $R_{2,M}$. Intuitively, as the government takes care of improving the stability of the banking system, the bank can make its depositors stay also with a lowered interest factor. Now the positive welfare effect of the improved guarantee consists solely in increased profits of the banker.

In the other corner solution $R_2 = 1$, and the bank run stops only when there are so few staying depositors that the government guarantee never fails. In this limiting case the number of the staying consumers has the value χ_M which is determined by (35). Now an improvement in the reliability of the guarantee leads to greater bank stability (i.e, greater χ_M) and greater profits for the banker. It also decreases the amount of new capital which is needed at T=1 (i.e, $1-\chi_M$) and, accordingly, the part of the bank's profit that the banker is obliged to give to the government at T=2. Assuming that recapitalization is on the whole costly to the government, the combined welfare effect of the last two changes is positive. Again, the positive effects must be weighted against the increased welfare cost *F* that emerges in case of actual deposit guarantee failure.

The above analysis becomes much more complicated when one considers the internal solution in which (36) is valid. It is clear that in the internal solution the interest factor R_2 and stability χ are between the values that they have in the two corner solutions, i.e. that in the internal solution $1 < R_2 < R_{2,M}$ and $\chi_M < \chi < 1$. While it is also obvious that – keeping the interest factor R_2 fixed – an increase in the reliability of the deposit guarantee improves bank stability χ, it is not obvious how the derivative $d\chi / dR_2$, which according to (36) affects the expected-profit-maximizing choice of R_2 by the banker, changes as a result of a change in χ. It is even conceivable that a small improvement in the deposit guarantee reliability might motivate the banker to lower the deposit interest factor to an extent which would *increase* the size of the bank run χ. If one wanted to exclude this implausible case, one would have to introduce more specific assumptions concerning the probability distribution $h_B(\rho)$ of the return from the bank's investment, which connects R_2 and χ in equilibrium in accordance with (31).

We may, however, observe that the three equilibria approach each other when *F* approaches the value for which the deposit guarantee never fails (not even when $\chi = 1$ and all depositors stay). We conclude from (35) that this will be the case when *F* is at least

$$F_N = 1 - u(1)$$

Considering the limit in which *F* approaches F_N, we observe that in the no-bank-run equilibrium (in which $\chi = 1$) the deposit interest factor $R_{2,M}$ approaches 1 from above, and in the maximal-bank-run equilibrium (in which $R_2 = 1$) the bank stability χ_M approaches 1 from below. In the limit in which $F = F_N$ one reaches the trivial equilibrium which occurs also after the good signal $\eta = G$, and in which there is no bank run although the interest factor is 1 and the depositors do not get interest for their deposits.

5. The effects of EDIS on bank stability

We now apply the insights from our new framework to EDIS. The natural field of application of our framework is a crisis which is sufficiently large to make the assets of deposit insurance funds insufficient, implying that reimbursing deposits may involve a political decision to provide additional funding for the reimbursement. In the case of a national deposit insurance scheme, the decision would normally be made by the government, while in the case of EDIS the counterparts of the "government" of our model would be the Single Resolution Fund and – should the single Resolution Fund be unable to fulfill its task – its backstop. The decision to make use of the backstop would be a political decision and quite analogous with the decision that the government makes at T=2 in our model. More specifically, in the Commission proposal the backstop would only be deployed if the decision to deploy it was backed by 85 % of the votes of the member countries (European Commission 2017b, p. 6).

Our model allows us to give precise formulations to two opposite effects of a shared deposit insurance scheme. Firstly, consider a crisis which is restricted in size, such as a financial crisis in a single eurozone country, or the crisis of a single large bank. Our framework leads to the conclusion that in the case of a restricted crisis, the shared deposit insurance scheme tends to improve the stability of the banking sector (measured by the size of bank runs). This conclusion is normally supported by referring to the better diversification that a larger insurance company or fund provides. However, in our model the shared scheme is a "diversification device" in a more abstract sense.

As already discussed earlier, the government's costs from a deposit guarantee breakdown are in our model indirect (as they consist of reputational costs and e.g. reduced trust in government institutions) but the costs from reimbursing depositors of a failed bank are direct. The indirect costs grow when the deposit guarantee area grows, which can be represented as growth of the guarantee failure cost F in our framework, while the direct costs are not affected by the size of the deposit guarantee area. In other words, in case of a regional bank crisis we may argue that the costs from a deposit guarantee breakdown are increased by the shared deposit insurance scheme (since the "reputational" cost is now faced by the whole EU Banking Union) without a corresponding increase in costs from reimbursing deposits. In our model this should make a deposit guarantee breakdown less likely and reduce or altogether eliminate partial bank runs. (In reality it might, of course, also happen that the national deposit insurance fund is insufficient for the needed reimbursements, while a shared deposit insurance fund suffices for them, in which case a shift to EDIS would altogether eliminate the government decision which occurs in our model.)

On the other hand, the stability effects of introducing EDIS might be ambiguous in a systemic crisis which affects the whole Banking Union and leads to the use of the backstop of the Single Resolution Fund. In our model there is just a single bank, and a natural way to apply it to a crisis of the whole deposit guarantee area would be to think of the bank as a representative "average" bank and of the cost F as the reputational cost of deposit guarantee failure, divided by the number of banks in which such failures occur. Under this interpretation a change of scale would not by itself cause any changes in the above analysis, if the aggregate reputational cost F grew in proportion to the size of the deposit guarantee area. In other words, one would arrive at the conclusion that the changing size of the deposit guarantee area (e.g. shifting from a national deposit insurance scheme to EDIS) is irrelevant, when a severe, systemic crisis hits the whole area equally.

However, the "reputational cost" F represents also the depositors' trust in the deposit insurance scheme, and such trust – as the example of the Greek "bank jog" in 2009-2012 shows - is not identical in all the countries of the eurozone. If the reputational cost and the corresponding depositor trust reflected under EDIS some weighted average of member countries' national levels of trust before the introduction of the joint scheme, we could conclude that EDIS tends to decrease the danger of partial bank runs in the countries in which there is less trust in the national deposit insurance than in the eurozone on the average. However, the opposite might be the case in the countries in which national institutions are trusted more highly.

In addition, it might be excessively optimistic to view the trust that depositors feel for EDIS as an average. After all, trust depends also on the ability of our model's "government" (which is in the literal sense a government in the national deposit insurance schemes, and the backstop and other EU institutions in EDIS) to make fast decisions. Such decisions might be more difficult for EU institutions than for national institutions in a systemic crisis e.g. because of the required 85 % majority. One policy implication hence is that in order for the EDIS to achieve its full potential stability benefits, the backstop should be designed to be as credible as possible.

6. Concluding Remarks

We have considered bank runs which are caused by the suspicion that, in spite of its promises, a government might not protect deposits during a severe future crisis. In this setting bank runs are quite different from those in more traditional models, in which they occur in the absence of a deposit guarantee and are caused by the fear that a shortage of liquidity might lead to an immediate bank failure. In the absence of a deposit guarantee traditional models of bank runs (e.g. Diamond and Dybvig, 1983) have two equilibria: the one in which no one has an incentive to withdraw his deposits (except for immediate consumption needs) because other depositors do not withdraw theirs, and the other in which all depositors withdraw simultaneously. In contrast, we have assumed that the government always bails out banks by providing recapitalization if banks have a liquidity shortage in the absence of a crisis. Nonetheless, as the government may break its deposit guarantee in a severe crisis, bank runs may still occur.

Our model provides a simple explanation for why in the presence of a government deposit guarantee bank runs are gradual and partial as has been recently often observed; e.g., in the euro area. As deposits are withdrawn during a bank run, the government's future liability of guaranteeing the remaining deposits is gradually reduced. This increases the government's incentive to honor its promise because the cost of breaking its guarantee (which might be caused by e.g. reputational concerns) does not diminish like the remaining payments. This in turn decreases the remaining depositors' incentive to withdraw. Eventually, there is a unique point when the bank run stops. This point (i.e., amount of remaining deposits) depends on the common signal that the depositors receive concerning the future state of the economy, and the government's reputational costs. As an application of our model, we contrasted EDIS with national deposit guarantee schemes and concluded that while EDIS probably tends to improve bank stability (measured by the size of bank runs) in bank crises of a restricted size, the opposite could also be the case in a systemic crisis which affects the whole eurozone. The effects of introducing EDIS might also differ in different countries, depending on whether the citizens have more trust in national than union-level institutions or vice versa.

From the point of view of economic theory, it is worth emphasizing that this mechanism turns the equilibrium of our model unique, although we do not make use of the mathematically essentially more complicated global games framework (cf. Goldstein and Pauzner, 2005). Our analysis could be extended in a variety of directions, one of which is the following. In our model the government reduces the future cost of its own deposit guarantee liability when it provides liquidity to a bank so that the bank can weather a partial run on deposits. In this way, liquidity provision, or recapitalization, serves as a *commitment device*, which makes a deposit guarantee breakdown less likely. This works out because we have assumed that the cost of the government's liquidity provision before a crisis is a *sunk cost*. A possible extension to our analysis would be to assume that liquidity provision is not a sunk cost completely, but increases sovereign debt and contributes to the government's financial distress when the deposit guarantee is in danger of breaking down. This would most likely reduce bank stability in the setting of our model.

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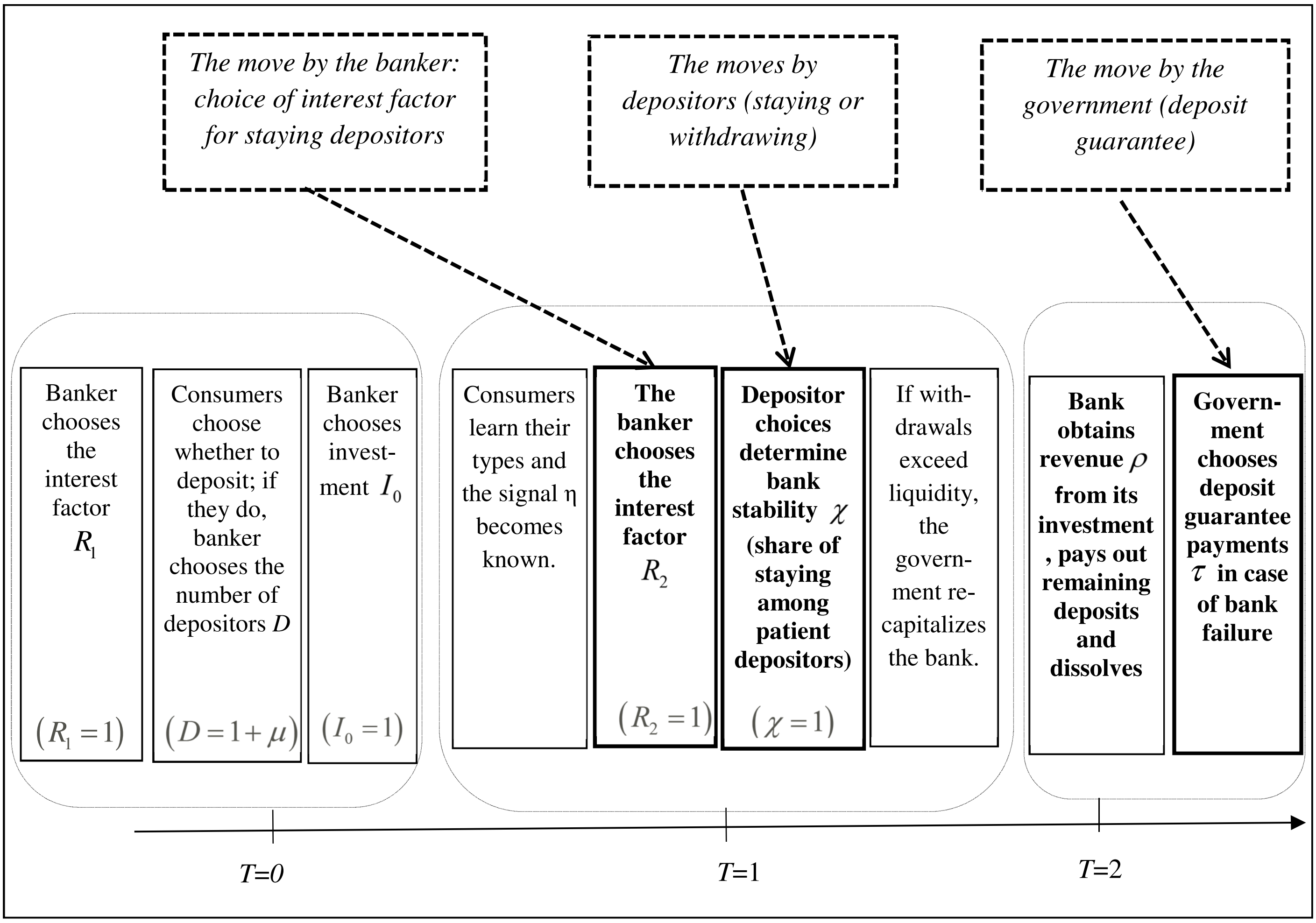

Figure 1. Time line of the model. The values in parantheses correspond to the trivial equilibrium with the "good" signal $\eta = G$. The choices which are made before observing the signal are identical in case of the "bad" signal $\eta = B$. The choices within the restricted game after the signal (which may differ for the two signals) are indicated in bold.