From global imbalances to excess liquidity: a political economy view

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Abstract

This paper takes the view that the root cause of the global liquidity glut observed in the run-up to the global financial crisis resides in an easy monetary policy by the world's main reserve currency issuer and debtor (the United States), in part induced by cheap imports from its main creditor countries – in particular China – who peg their exchange rate to the US dollar. It explores the welfare effects of two possible alternative exchange rate regimes for China: (i) a free float of China's currency and (ii) an exchange rate peg to a composite of the US dollar and the euro. From a European point of view regime (i) turns out first-best and regime (ii) second-best, while a continuation of China's (de facto) exclusive US dollar peg would be third-best. Obviously these findings rest on stylised assumptions and need to be recalibrated on a broader range of economic and political considerations before they can be translated into policy making.

Key words: exchange rate regimes, global imbalances, excess liquidity

JEL classification: E52, F31, F59

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1 I am grateful for comments by Helge Berger and several colleagues in DG ECFIN on an earlier draft. All errors are mine. The paper reflects my personal views and not those of the European Commission.
The financial system is in distress, monetary policy transmission severely impaired and a
global recession unfolding. On the causes of the financial crisis a consensus is rapidly
taking shape (see e.g. Baily et al. 2008). Among the macroeconomic factors that are most
frequently mentioned are: (i) global imbalances (excess saving in parts of the developing
world financing a large US current account deficit), (ii) excess liquidity and the
associated hunt for yield and carry trades, and (iii) the excessive leveraging of financial
institutions and households and the bubble formation in real estate and commodity
markets that ensued. These macroeconomic factors have arguably been amplified by
microeconomic flaws, including: (iv) the pro-cyclicality of mark-to-market accounting
rules, (v) poor risk management, and (vi) supervisory deficiencies both at national and
global levels.

As to the role of global imbalances to the financial crisis, there are those who believe that
excess saving in the developing economies is the main culprit (Bernanke, 2005) and
those who attach a larger weight to the US current account deficit (Gourinchas and Rey,
2007). Probably both channels are relevant:

- On the one hand, the United States, being the largest economy in the world, issues the
  world's main reserve currency and derives from this a so-called 'exorbitant privilege'.
  Unlike economies whose currencies do not have this privileged status, the United
  States can issue international securities denominated in domestic currency with a
  liquidity premium and afford to sustain a large current account deficit as its creditors
  are happy to keep future claims on US output on their balance sheets (Aizenman and
  Sun 2008). The US current account deficit was the result of a combination of low
  household saving and a large fiscal deficit (Chinn and Ito 2007).

- On the other hand, the emerging economies – in particular China – are naturally
disposed to assume their role as US creditor owing to their large national saving
surpluses – not least owing also to the US' financial maturity, manifested in its open
and deep financial markets (Caballero et al. 2008). The Chinese saving surplus stems
inter alia from: (i) a strategy of export-driven growth, (ii) underdeveloped financial
systems that force domestic businesses to fund their investment primarily through
retained earnings, and (iii) underdeveloped social insurance systems that force
households to maintain high rates of precautionary saving.

This constellation of policy strategies led to massive dollar inflows in the emerging
economies, which were recycled in the global economy and helped finance the US
current account deficit on relatively favourable terms. Thus, the global savings glut
which, while originating in emerging Asian countries, by definition matched the 'saving
draught' in the United States which it has helped financing.

However, while the divergent saving propensities in the US and Asian economies may
explain the observed global imbalances, it does not provide a satisfactory explanation of
the global liquidity glut that accompanied it and that contributed to the ensuing bubbles.
Monetary policy must have played an accommodative role. Had monetary policy been
tighter in the United States than it actually was before the crisis, liquidity creation and
the associated risk of bubbles would have been smaller. Moreover, had monetary policies
in emerging Asia been tighter, their currencies would have appreciated (more) and their
official reserves and recycling of US dollars in financial markets, and the associated risk
of bubble formation, been smaller. The question then becomes: why was monetary policy so accommodative before the crisis?

The following additional elements are necessary to complete the story:

- The emerging economies have been maintaining (de facto) exchange rate pegs to the US dollar. The rationale for this choice has been three-pronged: (i) to support their export-led growth strategy by maintaining a stable exchange rate vis-à-vis the dollar, (ii) to build up large foreign currency (US dollar) 'war chests' in response to the painful experience of the Asian crisis in the late 1990s and to build up foreign exchange reserves by way of 'collateral' to attract foreign direct investment (Dooley et al., 2004), and (iii) to avoid adverse balance sheet effects associated with capital losses on their currency reserves. While it is true that since 2005 China has adopted a more flexible de jure exchange rate regime, there was little change in the de facto dollar peg (Frankel and Wei 2007, Frankel 2009).

- Because the emerging economies (again China is prominent) kept their currencies from appreciating, the US benefited from a disinflation impulse from lower manufacturing prices which allowed it to keep monetary policy easier than it otherwise could. As a result, its money stock expanded rapidly, chasing assets with higher yields (and consequently higher risk) and driving up their prices. This fuelled the dotcom bubble which, after it burst, prompted further US monetary ease. The dollar exchange rate plummeted, and this spilled over into emerging economies' monetary policies via their exchange rate pegs. As a result, global liquidity soared (see for recent evidence Adalid and Detken 2006, Ahrend et al. 2008 and Belke et al. 2008) and eventually produced yet another bubble (this time in real estate and commodity markets) which again burst.

There are blueprints for reforms of the international monetary system being developed to prevent such scenarios from re-occurring, and this is obviously very welcome. But, the root causes of the global liquidity glut are still firmly in place. Global imbalances remain large despite some correction recently in response to the crisis. Liquidity is now being hoarded by financial institutions out of fear of insolvency of their counterparties in the face of the financial crisis, but this does not mean it is no longer there. In fact, with the recent monetary policy easing in response to the crisis and the issuance of new Special Drawing Rights (SDR's), global liquidity will increase further. Once the global economy recovers, this liquidity may well contribute to new bubbles developing.

Europe will need to take a position on this issue, and this paper may provide an input in this regard. The paper will, however, look at the adoption of an alternative exchange rate regime by China. Possible alternatives, such as the conversion of China's currency reserves into SDR's in exchange for a greater number of seats in the International Monetary Fund and leaving it to the Fund to intervene in currency markets and act as its asset manager (Bordo and James, 2009), are beyond its scope. The approach is theoretical and stylised at this stage.

In the first section I will develop a simple two-stage game with two players, representing the United States and China. Each player aims to minimise its welfare losses associated with a classical output-inflation trade-off. The computations suggest that, from the point of view of the economy that represents China, a fixed exchange rate regime is naturally its preferred one, in line with the empirical observations. They also suggest that surprise money creation in the economy that represents the United States spills over into the
Chinese economy and again back to the US economy, giving rise to a 'tug of war' resulting in a larger monetary expansion than each of the two players on their own would have envisaged.

In the second section I will expand this model by introducing a third player, representing the euro area. The euro area is assumed to loosely manage its exchange rate against the US dollar while China adopts a basket of the euro and the US dollar. This constellation of exchange rate regimes would still expose the global economy to surprise liquidity expansion by the reserve-currency issuer (United States), which spills over unfettered to the rest of the world economy. In the third section I will, finally, examine which constellation of exchange rate regimes would be welfare-optimal for the euro area. This turns out to be a constellation of fully flexible exchange rate regimes across the three economies as this would remove spill-overs and allow each of them to target their domestic inflation and output objectives. A regime in which China includes the euro in its basket turns out to be second best since it would enhance the effectiveness of monetary policy of the euro area.

1. THE US-CHINA GAME

I set up a two-region economic model, replicating the United States and China. Both regions are assumed to have one policy instrument – monetary policy. The United States' loss function includes the distances of inflation and output from their respective targets. The same holds for the Chinese loss function, but it also contains the distance from its dollar exchange rate target. The emerging economy pre-commits to an exchange rate regime in a first stage of the game where after each player minimises its loss function in an uncoordinated (Nash) game.

1.1. The economies

The model distinguishes two economies: a reserve currency-issuing hegemon representing the United States (denoted as economy 1), and an emerging global player representing China (denoted as economy 2). All variables are defined in terms of deviations from a steady state.

Simple aggregate demand and supply (Phillips curve) equations are specified for each of the two economies (see Annex 1):

\begin{align*}
(1) \quad y_i &= \phi_i (m_i - \pi_i); i = 1,2 \\
(2) \quad \pi_i &= \omega_i y_i; i = 1,2
\end{align*}

The aggregate demand equation (1) relates output to real money creation (i.e. nominal money creation \(m\) less inflation \(\pi\)).\(^2\) Note that 'money growth' in this context may be driven by other forces than the creation of base money. If the financial system extends

\(^2\) While a focus on money aggregates is less straightforward way of gauging monetary policy in the United States, it is a rather common approach for China, given that the interest channel is weak or non-existent. See for example Koivu et al. (2008).
credit beyond what would normally be perceived as 'prudent' at given base money, this would imply a monetary expansion without necessarily involving official monetary policy stimulus (see for empirical evidence of such non-policy driven money supply shocks, Chadha et al. 2008). The Phillips curve (2) relates inflation in each of the economies to their output gap. For simplicity, and without any loss of generality of the results, shock terms are omitted.

The model is completed with an equation linking the (only) exchange rate \( e_{1,2} \) is the exchange rate of the emerging economy against the reserve currency) to the fundamentals:

\[
(3) \quad e_{1,2} + m_2 = \pi_1 + y_1
\]

This equation follows from the assumption that the reserve currency is the only financial asset in this model. Hence the only way the emerging economy can create liquidity is via open market operations in which it exchanges local currency for the reserve currency. The left-hand side of equation (3) therefore represents the demand for the reserve currency in the emerging economy, which in this set-up is equal to the money creation in the emerging economy denominated in the reserve currency. The right-hand side of equation (3) represents the supply of reserve currency by the mature economy, which is equal to the mature economy's current account deficit. The latter is proportional to its nominal output, assuming that trade flows in one direction (from the emerging to the mature economy), and that the ratio of nominal imports to nominal output is constant. The latter assumption is consistent with a Cobb-Douglas utility function to describe the comparative utility derived from home products and imports, as explained in the annex.

There is no capital account, so the exchange rate is driven exclusively by the demand and supply for dollar conversion stemming from bilateral trade.

Solving this set of equations yields:

\[
(4) \quad y_i = \frac{\phi_i m_i}{1 + \phi_i \omega_i}; \quad i = 1, 2
\]

\[
(5) \quad \pi_i = \frac{\phi_i \omega_i m_i}{1 + \phi_i \omega_i}; \quad i = 1, 2
\]

\[
(6) \quad e_{1,2} = \frac{\phi_1 (1 + \omega_1)}{1 + \phi_1 \omega_1} m_1 - m_2
\]

The exchange rate of the emerging economy depreciates if it eases its monetary policy \( (m_2 > 0) \) and appreciates if the mature economy does so \( (m_1 > 0) \). The channels are straightforward. Money creation in the emerging economy entails the purchase by its financial system of reserve currency, whose exchange rate will thus be pushed up. Conversely, money creation in the mature economy increases demand and imports and hence the supply of reserve currency, whose exchange rate, therefore, will be under downward pressure.
1.2. Welfare loss minimisation

The mature economy sets its monetary policy so as to minimise the welfare loss stemming from output slack relative to a target $\hat{y}_i$, and headline inflation:

\begin{equation}
L_i = (y_i - \hat{y}_i)^2 + \beta_1[(1 - \gamma_1)\pi_1 + \gamma_1 (\pi_2 + \epsilon_{1,2})]^2: \hat{y}_i \geq y_i
\end{equation}

Headline inflation is a composite of the inflation originated at home and inflation imported from the emerging economy. The weight $\gamma_1$ of imported inflation in headline inflation reflects the size of the 'global imbalance'. Recall that the only trade flow in the model is that from the emerging to the mature economy, and hence $\gamma_1$ corresponds to the current account deficit of the mature economy as well as the current account surplus of the emerging economy. It also corresponds to the weight of imports in the (Cobb-Douglas) utility function governing the choice between home-made and imported products in the mature economy (see the annex). The global imbalance is therefore exogenous in this model, i.e. independent of the exchange rate. This assumption is at odds with the 'unwinding of global imbalances' rhetoric which predicts a dollar fall to be instrumental in bringing about current account adjustment. However, it is consistent with the alternative view which sees the global imbalances as being rooted in structural features of the respective economies and independent of whatever constellation of nominal exchange rates that happens to prevail; see Chinn and Wei (2008).

From the first order condition for a minimum the following monetary policy reaction function for the mature economy can be derived:

\begin{equation}
\begin{align*}
m_1 &= \Omega_1 \hat{y}_1 + \gamma_1 \beta_1 (\omega_1 + \gamma_1) \left[1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2}\right] \Omega_1 m_2 \\
\Omega_1 &= \frac{1 + \phi_1 \omega_1}{1 + \beta_1 (\omega_1 + \gamma_1)^2} \phi_1 ^{-1} > 0
\end{align*}
\end{equation}

What is important to retain from this result is that a monetary expansion in the emerging economy will trigger a monetary expansion in the mature economy as well. A monetary expansion in the emerging economy will put downward pressure on its exchange rate and since the mature economy is an importer the associated disinflationary impulse will allow its monetary policy to be eased. This channel is obviously predicated on the weight of imports in the mature economy's demand, as gauged by the parameter $\gamma_1$, to be non-zero. In other words, globalisation, to the extent this produces a trade flow from the emerging to the mature world, leads to a spill-over of monetary policy from the emerging to the mature world.

The emerging economy sets its monetary policy by fixing its policy instrument $m_2$, so as to minimise losses stemming from inflation, output slack and deviations from the exchange rate target. So the emerging economy solves the problem:

\begin{equation}
L_2 = (1 - \gamma_2) \left[(y_2 - \hat{y}_2)^2 + \beta_2 \pi_2^2\right] + \gamma_2 e_{1,2}^2: \hat{y}_2 \geq y_2
\end{equation}

The parameter $\gamma_2$ is the weight the emerging economy attaches to staying close to its exchange rate target relative to achieving its output and inflation objectives. It also
reflects its exchange rate regime. If \( \gamma_2 = 0 \) its currency will be freely floating, if \( 0 < \gamma_2 < 1 \) its exchange rate is managed and if \( \gamma_2 = 1 \) it is fixed. Note that the exchange rate objective is not to minimise short-run exchange rate volatility per se but rather to avoid the exchange rate from exceeding systematically its target or historical value.

The first order condition for a minimum loss for the emerging economy yields the following reaction function:

\[
m_2 = (1 - \gamma_2)\Omega_2 \hat{y}_2 + \gamma_2 \frac{\phi_1 (1 + \omega_1)(1 + \phi_2 \omega_2)}{\phi_2 (1 + \phi_1 \omega_1)} \Omega_2 m_1
\]

(10)

\[
\Omega_2 = \frac{1 + \phi_2 \omega_2}{\phi_2} \left\{ (1 - \gamma_2) (1 + \beta_2 \omega_2^2) + \gamma_2 \left[ \frac{1 + \phi_2 \omega_2}{\phi_2} \right]^2 \right\}^{-1} > 0
\]

The crucial parameter in this relationship is \( \gamma_2 \). If it is zero, there is no spill-over effect of monetary expansion in the mature economy on the emerging economy (since \( \partial m_2/\partial m_1 = 0 \)). Monetary policy is then solely determined by the emerging economy's own output ambitions.

1.3. The Nash equilibrium

The Nash equilibrium can be solved from the reaction functions (8) and (10):

\[
m_1 = \frac{\Omega_1 \hat{y}_1 + \gamma_1 (1 - \gamma_2) \beta_1 (\omega_1 + \gamma_1) \left[ 1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2} \right] \Omega_1 \Omega_2 \hat{y}_2}{\Delta}
\]

(11)

\[
m_2 = \frac{(1 - \gamma_2) \Omega_2 \hat{y}_2 + \gamma_2 \frac{\phi_1 (1 + \omega_1)(1 + \phi_2 \omega_2)}{\phi_2 (1 + \phi_1 \omega_1)} \Omega_1 \Omega_2 \hat{y}_1}{\Delta}
\]

\[
\Delta = 1 - \gamma_1 \gamma_2 \beta_1 \frac{\phi_1 (1 + \omega_1)(1 + \phi_2 \omega_2)}{\phi_2 (1 + \phi_1 \omega_1)} (\omega_1 + \gamma_1) \left[ 1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2} \right] \Omega_1 \Omega_2
\]

This result implies that under a free float money (\( \gamma_2 = 0 \)), money growth in the emerging economy is solely a function of its own growth ambitions. However, if the exchange rate is managed also the growth ambition of the mature economy enters the equation. Conversely, if the emerging economy fixes its exchange rate (\( \gamma_2 = 1 \)), monetary expansion of the mature economy is solely a function of its own growth ambitions. In the opposite case of a free float, monetary expansion motivated by growth ambitions in the emerging economy will drive its exchange rate down and, via imported disinflation, provide room for monetary expansion in the mature economy. It is only in the intermediate case of a managed (but not fixed) exchange rate (\( 0 < \gamma_2 < 1 \)) that the transmission of money growth works in both directions.

It is possible to trace graphically how the Nash equilibrium will change as a result of changing growth ambitions in the mature economy. Figure 1 depicts the reaction functions of the mature economy (R1) and the emerging economy (R2). Note that if the emerging economy fixes its exchange rate (\( \gamma_2 = 1 \)), R2 will cross the origin, but this does
not alter the basic results. The initial equilibrium is determined by the intersection of the reaction functions in point A. If the mature economy increases its output target, the equilibrium shifts to point B, which indicates that monetary policy will be eased not only in the mature economy, but also in the emerging economy.

Figure 1  The impact on liquidity of an increase in the output target of the mature economy

1.4.  The choice of exchange rate regime

It is possible to consider the Nash equilibrium as the outcome of stage 2 of a two-stage game, whereby in stage 1 the emerging economy pre-commits to its exchange rate regime. I will assume that the emerging economy constrains monetary policy by choosing a money growth target, as has been the practice in Chinese monetary policy. It will choose its exchange rate regime so as to strike a balance between its macroeconomic policy objectives of supporting economic growth and low inflation and its money target. I solve the game backwards, taking the (expected) Nash equilibrium in stage 2 as given, and then for stage 1 substituting the results back in the loss function of the emerging economy to determine the optimal value of $\gamma_2$. So the loss function reads:
\[ L^*_2 = \left[ E(y_2) - E(\hat{y}_2) \right]^2 + \beta_2 E(\pi_2)^2 + \chi_2 E(m_2)^2; \]
\[ \gamma_2 \]
\[ E(\hat{y}_2) \geq (y_2) \]

where:

\[ E(y_2) = \frac{\phi_2 E(m_2)}{1 + \phi_2 \omega_2} \]
\[ E(\pi_2) = \frac{\phi_3 \omega_3 E(m_2)}{1 + \phi_2 \omega_2} \]
\[ E(m_2) = \frac{(1 - \gamma_2)\Omega_2 E(\hat{y}_2) + \gamma_2 \frac{\phi_1 (1 + \omega_1) (1 + \phi_2 \omega_2)}{\phi_2 (1 + \phi_1 \omega_1)} \Omega_1 \Omega_2 E(\hat{y}_1)}{\Delta} \]

\( E \) refers to expectations and a weight \( \chi_2 \) is attached to the money target. It is rather cumbersome to compute \( \gamma_2 \) analytically, so I will revert to a numerical approach. Figure 2 reports the welfare loss for a range of feasible values for \( \gamma_2 \) (starting at 0 and ending at 1) and \( \chi_2 \). The other assumed parameter values are:

\[ \omega_1 = \omega_2 = \frac{1}{3}; \phi_1 = \phi_2 = \frac{1}{4}; \beta_1 = \beta_2 = 1; \gamma_1 = \frac{1}{10}; E(\hat{y}_1) = E(\hat{y}_2) = 1 \]

The expected (log) output targets are normalised to unity. Under this assumption, without intervention in foreign exchange markets, the emerging economy's exchange rate will tend to depreciate as the money creation necessary to achieve its output and inflation objectives will exceed its current account surplus. The computations (Figure 2) show that, under these assumptions and assuming that the emerging economy attaches some weight to its money target (\( \chi_2 > 0 \)) it will want to pick a fully fixed exchange rate regime (\( \gamma_2 = 1 \)).

The intuition is that the emerging economy, to respect its money supply target, will have to accumulate foreign reserve at a pre-committed rate and this, in turn, implies a pre-commitment also to an exchange rate target. Only if no weight is attached to the money target (\( \chi_2 = 0 \)), the emerging economy will want to pick a flexible exchange rate regime (\( \gamma_2 = 0 \)).

To sum up, if the emerging economy wants to keep its money growth under control, it will be inclined to fix its exchange rate against the reserve currency. Paradoxically, this exchange rate regime implies that its own and global liquidity will respond more strongly to a surprise monetary expansion by the reserve-currency issuer.

These results are able to capture a number of stylised real-world developments: in the early days of its conversion to capitalism in the 1980s and the early-1990s China saw its exchange rate against the US dollar weaken and money growth soaring. Since then it has pursued both a money target and an exchange rate target (against the US dollar). But since it stuck to its exchange rate target also in the face of surprise liquidity creation in the United States (in response to the dotcom slump) this tended to amplify the initial monetary impulse both in the United States and China.
2. A THREE-REGION GAME -- ADDING IN THE EURO AREA

I will now assume that a third player enters the scene, which represents the euro area. Moreover, I assume that China allows the euro to enter the exchange rate basket to which it will peg its currency. The euro area is an exporter to the reserve currency issuer and an importer from the emerging economy, but it does not issue itself a reserve currency. I will allow for the possibility euro area manages its exchange rate against the US dollar.

2.1. The economies

The specifications of the supply and demand equations are identical for all three economies (the third player is denoted as economy 3):

\[ y_i = \phi_i (m_i - \pi_i); i = 1,2,3 \]  

\[ \pi_i = \omega_i y_i; i = 1,2,3 \]

The equation for the bilateral exchange rate of the emerging economy vis-à-vis the reserve currency is now more complex because there are now two sources of reserve currency inflows: exports to the reserve currency issuer and exports to the other mature economy (assuming that the latter are invoiced in the reserve currency). The equation reads (see Annex 2):

\[ e_{1,2} + m_2 = \lambda (\pi_1 + y_1) + (1 - \lambda)(e_{1,3} + \pi_3 + y_3) \]
The accumulation of reserve currency in the emerging economy (left-hand side of the equation) is equal to the weighted sum of the imports by the reserve currency issuer and by the other mature economy (both expressed in the reserve currency). These are again proportional to nominal income in the respective economies in line with the Cobb-Douglas preferences assumption (see annex). The bilateral exchange rate of the emerging economy against the reserve currency will appreciate if either of the two mature economies sees its nominal output expand or if the bilateral exchange rate of the other mature economy against the reserve currency appreciates. The latter occurs because it implies a terms of trade gain for the other mature economy, which translates into an increase in import demand from the emerging economy.

The equation for the bilateral exchange rate of the other mature economy vis-à-vis the reserve currency is analogous to that for the bilateral exchange rate in the two-region model:

\[
e_{1,3} + m_3 = \pi_1 + \gamma_1
\]

This is based on the assumption that import flows to the reserve currency issuer from the other mature economy (and not the other way around) and that this flow is proportional to nominal income of the former economy (reflecting Cobb-Douglas preferences).

The exchange rate of the emerging economy vis-à-vis the other mature economy, finally, is determined as a residual:

\[
e_{3,2} = e_{1,2} - e_{1,3}
\]

The solution of the model is analogous to that of the two-region version of the model:

\[
y_i = \frac{\phi_i m_i}{1 + \phi_i \omega_i}; i = 1,2,3
\]

\[
\pi_i = \frac{\phi_i \omega_i m_i}{1 + \phi_i \omega_i}; i = 1,2,3
\]

\[
e_{1,2} = \frac{\phi_1 (1 + \omega_1)}{1 + \phi_1 \omega_1} m_1 - (1 - \lambda) \left[ 1 - \frac{\phi_3 (1 + \omega_3)}{1 + \phi_3 \omega_3} \right] m_3 - m_2
\]

\[
e_{1,3} = \frac{\phi_1 (1 + \omega_1)}{1 + \phi_1 \omega_1} m_1 - m_3
\]

\[
e_{3,2} = \left[ 1 - (1 - \lambda) \frac{1 - \phi_3}{1 + \phi_3 \omega_3} \right] m_3 - m_2
\]

A number of observations are worth making. First, the exchange rate of the emerging economy vis-à-vis the reserve currency (equation (21)) appreciates if more reserve currency is issued \((m_1 > 0)\), if the other mature economy tightens its monetary policy \((m_3 < 0)\) -- because this will reduce the demand for reserve/invoicing currency -- and if the emerging economy tightens its monetary policy \((m_2 < 0)\). Second, the exchange rate of the
other mature economy vis-à-vis the reserve currency (equation 22) appreciates if more reserve currency is issued \((m_1 > 0)\) or if the other mature economy tightens its monetary policy \((m_3 < 0)\). Third, the exchange rate of the emerging economy vis-à-vis the currency of the other mature economy (equation 23) appreciates if the former tightens its monetary policy \((m_2 < 0)\), or if the latter eases it \((m_3 > 0)\). If the reserve currency issuer is the emerging economy’s only importer \((\lambda = 1)\), equation (21) collapses into equation (6) and equation (23) collapses into \(e_{3,2} = m_3 - m_2\).

### 2.2. Welfare loss minimisation

The loss function of the reserve currency issuer is essentially the same as in the two-region model, except that headline inflation now also includes inflation imported from economy 3:

\[
L_1 = (y_1 - \hat{y}_1)^2 + \beta_1 \left[ (1 - \gamma_1)\pi_1 + \gamma_1 \left( (1 - \delta_1)\left( \pi_2 + e_{1,2} \right) + \delta_1 \left( \pi_3 + e_{1,3} \right) \right) \right]^2;
\]

\(\hat{y}_1 \geq y_1\)

\((24)\)

The first order condition for a minimum yields the following reaction function:

\[
m_1 = \Omega'_1 \hat{y}_1 + \gamma_1 \beta_1 \left( \omega_1 + \gamma_1 \right) \left( 1 - \delta_1 \right) \left( 1 - \phi_2 \omega_2 \right) \Omega'_1 m_2 + \gamma_1 \beta_1 \left( \omega_1 + \gamma_1 \right) \left( 1 - \delta_1 \right) \left( 1 - \phi_2 \omega_2 \right) \Omega'_1 \left( 1 - \phi_2 \omega_2 \right) \Omega'_1 m_2 + \gamma_1 \beta_1 \left( \omega_1 + \gamma_1 \right) \left( 1 - \delta_1 \right) \left( 1 - \phi_2 \omega_2 \right) \Omega'_1 m_2
\]

\(\Omega'_1 = \frac{1 + \phi_1 \omega_1}{\left( 1 + \beta_1 \left( \omega_1 + \gamma_1 \right) \left( 1 - \gamma_1 \right) \omega_1 + \gamma_1 \left( 1 - \delta_1 \right) \left( 1 + \omega_1 \right) \right) \phi_1} > 0\)

\((25)\)

As in the two-region model, more reserve currency will be issued if the hegemon sets a higher output target and if the emerging economy creates money. The latter channel is again due to the downward pressure on the emerging economy’s exchange rate against the reserve currency. A similar channel is now also operational with regard to the impact of money creation in the other mature economy: its exchange rate against the reserve currency would depreciate and hence imported disinflation provides room for the reserve currency issuer to expand its money stock.

The loss function of the emerging economy reads:

\[
L_2 = (1 - \gamma_2) \left( y_2 - \hat{y}_2 \right)^2 + \beta_2 \pi_2^2 + \gamma_2 \left( (1 - \delta_2) e_{1,2}^2 + \delta_2 e_{3,2}^2 \right) \hat{y}_2 \geq y_2
\]

\((26)\)

The first order condition for a minimum yields the following reaction function:
\[ m_2 = (1 - \gamma_2)\Omega_2' \hat{y}_2 + \gamma_2 (1 - \delta_2) \phi_1 (1 + \omega_1) (1 + \phi_2 \omega_2) \phi_2 (1 + \phi_1 \omega_1) \Omega_2' m_1 \]

(27) \[-\gamma_2 [(1 - \lambda)(1 - \phi_3) - \delta_2 (1 + \phi_3 \omega_3)] \frac{1 + \phi_2 \omega_2}{\phi_2 (1 + \phi_3 \omega_3)} \Omega_2' m_3 \]

\[ \Omega_2' = \frac{1 + \phi_2 \omega_2}{\phi_2} \left\{ (1 - \gamma_2) (1 + \beta_2 \omega_2) + \gamma_2 \left[ \frac{1 + \phi_2 \omega_2}{\phi_2} \right]^2 \right\}^{-1} \]

Money creation in the hegemon economy again spills over into money creation in the emerging economy, who eases its monetary policy to force its exchange rate against the reserve currency back down. By contrast, money creation in the other mature economy leads to monetary contraction in the emerging economy. The reason is that monetary expansion in the other mature economy increases demand for the reserve/invoicing currency, whose exchange rate increases. As a result, to maintain its exchange rate peg, the emerging economy will need to tighten its monetary policy.

The loss function of the other mature economy (euro area), finally, reads:

\[ L = \left(1 - \gamma_3\right) \left(\gamma_3 - \hat{y}_3\right)^2 + \beta_3 \left(1 - \delta_3\right) \pi_3 + \delta_3 (\pi_2 + e_{3,2}) \right)^2 + \gamma_3 e_{1,3}^2; \]

\[ \hat{y}_3 \geq y_3 \]

The reaction function reads:

\[ m_3 = (1 - \gamma_3)\Omega_3' \hat{y}_3 + \gamma_3 \frac{1 + \phi_3 \omega_3}{\phi_3} \phi_1 (1 + \omega_1) \Omega_3' m_1 \]

\[ + (1 - \gamma_3) \Gamma \beta_3 \delta_3 \left[ 1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2} \right] \Omega_3' m_2 \]

(28) \[ \Omega_3' = \frac{1 + \phi_3 \omega_3}{\phi_3} \left\{ (1 - \gamma_3) (1 + \beta_3 \Gamma^2) + \gamma_3 \left[ \frac{1 + \phi_3 \omega_3}{\phi_3} \right]^2 \right\}^{-1} \]

\[ \Gamma = \left[ (1 - \delta_3) \omega_3 + \delta_3 \frac{1 + \phi_3 \omega_3}{\phi_3} \left[ 1 - (1 - \lambda) \frac{1 - \phi_3}{1 + \phi_3 \omega_3} \right] \right] \]

The interesting observation is that monetary policy in the other mature economy is insulated from monetary expansion by the reserve currency issuer if it adopts a flexible exchange rate (\( \gamma_3 = 0 \)). Money creation in the emerging economy, by contrast, does always spill over to money creation in the other mature economy via the exchange rate channel (imports get cheaper, hence there is more room for monetary stimulus).
2.3. The Nash equilibrium

Given that there are three reaction functions with three endogenous variables $m_i$, $i=1,2,3$ it is possible to compute a Nash equilibrium. It reads:

\[ m_1 = \frac{1 + \gamma_2 (1 - \gamma_3) \Theta_{2,3} \Theta_{3,2} \Omega'_2 \Omega'_3}{\Delta'} \Omega'_1 \hat{y}_1 + \frac{\gamma_1 (1 - \gamma_2) (1 - \gamma_3) \Theta_{1,2} + (1 - \gamma_3) \Theta_{1,3} \Theta_{3,2} \Omega'_3}{\Delta'} \Omega'_1 \Omega'_2 \hat{y}_2 \]

\[ + \frac{\gamma_1 (1 - \gamma_3) \Theta_{1,3} - \Theta_{1,2} \gamma_2 \Theta_{2,3} \Omega'_2}{\Delta'} \Omega'_1 \Omega'_3 \hat{y}_3 \]

\[ (29) \]

\[ m_2 = \frac{\Theta_{2,1} - \gamma_3 \Theta_{2,3} \Theta_{3,1} \Omega'_3}{\Delta'} \Omega'_1 \Omega'_2 \hat{y}_1 + (1 - \gamma_2) \frac{1 - \gamma_1 \gamma_3 \Theta_{1,3} \Theta_{3,1} \Omega'_1 \Omega'_3}{\Delta'} \Omega'_2 \hat{y}_2 \]

\[ - (1 - \gamma_3) \gamma_2 \frac{\Theta_{2,3} - \gamma_1 \Theta_{1,3} \Theta_{2,1} \Omega'_1}{\Delta'} \Omega'_2 \Omega'_3 \hat{y}_3 \]

\[ (30) \]

\[ m_3 = \frac{\Theta_{2,1} \Omega'_2 + \gamma_3 \Theta_{3,1} \Omega'_3}{\Delta'} \Omega'_1 \hat{y}_1 + (1 - \gamma_2) \frac{(1 - \gamma_3) \Theta_{3,2} + \gamma_1 \gamma_3 \Theta_{1,2} \Theta_{3,1} \Omega'_1}{\Delta'} \Omega'_2 \Omega'_3 \hat{y}_2 \]

\[ + (1 - \gamma_3) \frac{1 - \gamma_1 \gamma_2 \Theta_{1,2} \Theta_{2,1} \Omega'_2}{\Delta'} \Omega'_3 \hat{y}_3 \]

\[ (31) \]

in which:

\[ \Delta' = 1 + \gamma_2 \Theta_{2,3} \Theta_{3,2} \Omega'_2 \Omega'_3 \]

\[ - \gamma_2 \gamma_3 \Theta_{2,3} \Theta_{3,2} \Omega'_2 \Omega'_3 \]

\[ - \gamma_1 \gamma_2 \Theta_{2,1} \Omega'_1 \Omega'_2 \left[ \Theta_{1,2} + (1 - \gamma_3) \Theta_{1,3} \Theta_{3,2} \Omega'_3 \right] \]

\[ - \gamma_1 \gamma_3 \Theta_{3,1} \Omega'_1 \Omega'_3 \left[ \Theta_{1,3} - \gamma_2 \Theta_{1,2} \Theta_{2,3} \Omega'_2 \right] \]

and
\[
\Theta_{1,2} = \beta_1(\omega_1 + \gamma_1)(1 - \delta_1)\left(1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2}\right)
\]

\[
\Theta_{1,3} = \beta_1(\omega_1 + \gamma_1)\left(1 - \lambda\right)\left(1 - \frac{\phi_3 (1 + \omega_3)}{1 + \phi_3 \omega_3}\right) + \delta_1\left(1 - \frac{\phi_3 \omega_3}{1 + \phi_3 \omega_3}\right)
\]

\[
\Theta_{2,1} = (1 - \delta_2)\frac{\beta_1 (1 + \omega_1)(1 + \phi_2 \omega_2)}{\phi_2 (1 + \phi_1 \omega_1)}
\]

\[
\Theta_{2,3} = \left[\left(1 - \lambda\right)(1 - \phi_3) - \delta_2 (1 + \phi_3 \omega_3)\right]\frac{1 + \phi_2 \omega_2}{\phi_2 (1 + \phi_3 \omega_3)}
\]

\[
\Theta_{3,1} = \frac{1 + \phi_3 \omega_3}{\phi_3} \frac{\beta_1 (1 + \omega_1)}{1 + \phi_1 \omega_1}
\]

\[
\Theta_{3,2} = \Gamma \beta_3 \delta_3 \left[1 - \frac{\phi_2 \omega_2}{1 + \phi_2 \omega_2}\right]
\]

From this result can be inferred that an increase in the output ambitions of the reserve-currency issuer will boost liquidity in all three economies. This is illustrated by Figure 3 in which the Nash equilibrium is represented by the intersections marked A. The upper right quadrant corresponds to Figure 1 and shows the reaction functions in the \(m_1 - m_2\) space. An upward shift in the reaction function of the emerging economy results in more liquidity in both the emerging economy and the reserve currency issuer. The lower right and upper left quadrants depict that liquidity in the other mature economy would also increase. So, liquidity expands in all three economies.

From the results can also be inferred that if all economies adopt a free float (\(\gamma_2 = \gamma_3 = 0\)), the Nash equilibrium collapses to:

\[
m_1 = \Omega_1' \hat{y}_1 + \gamma_1 \left(\Theta_{1,2} + \Theta_{1,3} \Theta_{3,2} \Omega_3'\right) \Omega_2' \hat{y}_2 + \gamma_1 \frac{\Theta_{1,3}}{\Delta'} \Omega_1' \Omega_3' \hat{y}_3
\]

\[
m_2 = \Omega_2' \hat{y}_2
\]

\[
m_3 = \Theta_{3,2} \Omega_2' \Omega_3' \hat{y}_2 + \Omega_3' \hat{y}_3
\]

In this situation the emerging economy can fully gear its money stock to its own growth ambitions, whereas the other mature economy remains exposed to spill-over effects from the emerging economy and the reserve-currency issuer remains exposed to spill-over effects from both other economies. These spill-overs will also disappear if \(\gamma_1 = \delta_3 = 0\), i.e. if net imports from the emerging economy are zero. This goes to show that it is the combination of 'global imbalances' (persistent net imports from the emerging world) and managed exchange rates that produces monetary policy spill-over effects across the globe.
3. **TOWARDS A NEW CONSTELLATION OF EXCHANGE RATE REGIMES**

There is empirical evidence to suggest that China has gradually moved towards an exchange rate basket in which both the US dollar and the euro are represented, while the euro area has occasionally intervened to maintain the euro-dollar exchange rate within reasonable bounds (this happened for example when the euro plummeted against the US dollar in the early years of European Economic and Monetary Union). The question is legitimate if this arrangement is optimal from the European point of view or whether it should pursue alternatives. This problem can be solved mathematically by minimising a loss function in which the euro area has power to influence China's choice of exchange rate regime, as follows:
\[
\begin{align*}
L_3 &= 
\gamma_2 \beta_2 \gamma_3 \\
(1 - \gamma_3) \left[ E(y_3) - E(\hat{y}_3) \right]^2 + \beta_3 \left( (1 - \delta_3)V'(\pi_3) + \delta_3 \left[ E(\pi_2) + E(e_{3,2}) \right] \right)^2 \\
+ \gamma_3 \epsilon_{1,3}^2 \\
E(\hat{y}_3) &\geq E(y_3) \\
E(y_3) &= \frac{\phi_2 E(m_3)}{1 + \phi_2 \omega_3} \\
E(\pi_2) &= \frac{\phi_2 \omega_2 E(m_2)}{1 + \phi_2 \omega_2} \\
E(\pi_3) &= \frac{\phi_3 \omega_3 E(m_3)}{1 + \phi_3 \omega_3} \\
E(e_{1,3}) &= \frac{\phi_1 (1 + \omega_1)}{1 + \phi_1 \omega_1} E(m_1) - E(m_3) \\
(35) \\
E(e_{3,2}) &= \left[ 1 - (1 - \lambda) \frac{1 - \phi_3}{1 + \phi_3 \omega_3} \right] E(m_3) - E(m_2)
\end{align*}
\]

The expected development of money aggregates is described by the following set of reduced form equations:

\[
E(m_1) = \frac{1 + \gamma_2 (1 - \gamma_3) \Theta_{2,3,2} \Omega_2 \Omega_3}{\Delta'} \Omega_1 E(\hat{y}_1) \\
+ \gamma_1 (1 - \gamma_2) \frac{\Theta_{1,2} \Theta_{1,3,2} \Omega_2 \Omega_3'}{\Delta'} \Omega_1 \Omega_2 E(\hat{y}_2) \\
+ \gamma_1 (1 - \gamma_3) \frac{\Theta_{1,3} - \Theta_{1,2} \gamma_2 \Theta_{2,3} \Omega_2}{\Delta'} \Omega_1 \Omega_3 E(\hat{y}_3)
\]

\[
E(m_2) = \gamma_2 \frac{\Theta_{2,1} - \gamma_3 \Theta_{2,3,2} \Omega_3'}{\Delta'} \Omega_1 \Omega_2' E(\hat{y}_1) \\
+ (1 - \gamma_2) \frac{1 - \gamma_1 \gamma_3 \Theta_{1,3,2} \Omega_1 \Omega_3'}{\Delta'} \Omega_2 E(\hat{y}_2) \\
- (1 - \gamma_3) \frac{\Theta_{2,3} - \gamma_1 \Theta_{1,3} \Theta_{2,1} \Omega_1 \Omega_3'}{\Delta'} \Omega_2 \Omega_3 E(\hat{y}_3)
\]
\[ E(m_3) = \frac{\gamma_2 \Theta_{2,1} \Omega_2^2 + \gamma_3 \Theta_{3,1} \Omega_3^1}{\Delta'} \Omega_1^1 E(\hat{y}_1) \]

\[ + (1 - \gamma_2) \frac{\left(1 - \gamma_3\right) \Theta_{3,2} + \gamma_1 \gamma_3 \Theta_{1,2} \Theta_{3,1} \Omega_2}{\Delta'} \Omega_2^2 \Omega_2' E(\hat{y}_2) \]

\[ + (1 - \gamma_3) \frac{1 - \gamma_1 \gamma_2 \Theta_{1,2} \Theta_{2,1} \Omega_1 \Omega_2'}{\Delta'} \Omega_3^3 E(\hat{y}_3) \]

Again the analytical solution is 'messy', but it is possible to compute numerical solution, based on the following assumed parameter values:

\[ \omega_1 = \omega_2 = \omega_3 = \frac{1}{3}; \phi_1 = \phi_2 = \phi_3 = \frac{1}{4}; \beta_1 = \beta_2 = \beta_3 = 1; \gamma_3 = 0 \]

\[ \gamma_1 = \delta_3 = \frac{1}{10}; \gamma_2 = 1; \lambda = \frac{1}{2}; \delta_2 = 0; E(\hat{y}_1) = E(\hat{y}_2) = E(\hat{y}_3) = 1 \]

Figure 4 reports the computed welfare losses for different combinations of exchange rate flexibility (\( \gamma_2 \)) and weights of the euro in the exchange rate basket of the emerging economy (\( \delta_2 \)), assuming that the other mature economy itself has adopted a fully flexible exchange rate regime (\( \gamma_3 = 0 \)). This suggests that for the other mature economy a first-best solution in this case would be for the emerging economy to also adopt a flexible exchange rate regime. The explanation is rather straightforward: if all exchange rates are flexible, the other mature economy is 'isolated' from monetary policy responses to its own monetary policy by the reserve currency issuer and can fully gear its monetary policy to its own growth and inflation objectives.

If, however, the emerging economy cannot be deterred from managing its exchange rate, a second-best solution from the point of view of the other mature economy would be for the emerging economy to include the currency of the other mature economy in its basket (\( \delta_2 = 1 \)). The intuition is that under that scenario the other mature economy is shielded from monetary policy decisions by the reserve currency issuer (since the emerging economy would align its monetary policy with that of the other mature economy). So, the other mature economy would again benefit in terms of monetary policy autonomy, even though its monetary policy would be less effective as compared with the case of fully flexible exchange rates.

These results are generally robust to modifications in the numerical values of the parameters within reasonable ranges. The exception is when the assumption that \( \gamma_3 = 0 \) is relaxed, i.e. when it is assumed that the other mature economy also manages its exchange rate to the reserve currency to some extent. In that case, the second-best solution becomes first-best. This means that if the other mature economy manages its exchange rate against the reserve currency it is optimal from its point of view if the emerging economy adopts a basket in which both other currencies figure. This is illustrated in Figure 5 for the case of \( \gamma_3 = 0.1 \).
Figure 4. Welfare loss in the other mature economy for different exchange rate regimes in the emerging economy

Figure 5. Welfare loss in the other mature economy for different exchange rate regimes in the emerging economy (case of $\gamma_3 = 0.1$)
4. CONCLUSIONS

From this stylised theoretical exercise two sets of conclusions can be inferred:

(1) The global liquidity glut may be explained at least to some extent by the presence of pre-committed fixed exchange rate regimes, whereby emerging economies, such as notably China, target the US dollar. This is not by itself inflationary, but it becomes so in case of a surprise liquidity expansion in the economy that issues the global reserve currency, i.e. the United States. So, it could be argued that the United States, by expanding liquidity in the aftermath of the dotcom slump led the emerging economies to follow suit so as to be able to respect their exchange rate pre-commitments. Moreover, since as a result the dollar exchange rate held up well in the face monetary easing in the United States, US inflation staid low and stronger US monetary policy stimulus was feasible than otherwise would have been the case.

(2) It would be in the interest of both the euro area and China, if the latter adopted a flexible exchange rate regime. This would allow each of the economies to pursue their own growth and inflation objective without having to absorb spillovers from monetary policy in the United States. It is however not obvious that China is ready to adopt a flexible exchange rate at this stage, but may be forced to once its capital account is open and it becomes much more costly for it to manage its exchange rate, as suggested by the 'trilemma' literature (Bordo 2003). In the interim, a possible alternative exchange rate regime for China would be to peg its currency to a composite of the US dollar and the euro. But obviously these findings rest on stylised assumptions and need to be recalibrated on a broader range of economic and political considerations before they can be translated into policy making.

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Annex 1

A SIMPLE TWO-REGION MODEL

The world is assumed to consist of two interdependent economies, a mature economy (economy 1) and an emerging economy (economy 2). The mature economy issues the reserve currency and therefore can borrow from the emerging economy in its own currency. The emerging economy in turn is prepared to provide such funding on a sustained basis. For simplicity I assume that the mature economy imports from the emerging economy but the emerging economy not from the mature economy. All variables are defined as deviations from an unknown steady state.

The economic agents in the mature economy maximise the utility $U_1$ they derive from goods produced at home $Y_1$ and goods exported by the emerging economy $X_2$:

(A1) $U_1 = \log Y_1 + \alpha \log X_2$

The demand for the respective goods, multiplied by their respective prices, is constrained by nominal income earned in the previous period $Z_1$ and money creation in the current period $M_1$:

(A2) $M_1 + Z_1 = P_1 Y_1 + P_2 E_{1,2} X_2$

In this identity $P_1$ is the output price level in the mature economy and $P_2$ in the emerging economy. The (nominal) exchange rate of the emerging economy vis-à-vis the mature economy is $E_{1,2}$. The first order conditions read:

(A3) $Y_1 = \frac{1}{1 + \alpha} M_1 + Z_1 \overline{P_1}$

(A4) $X_2 = \frac{\alpha}{1 + \alpha} \frac{M_1 + Z_1}{P_2 E_{1,2}}$

Equation (A3) can be interpreted as a very simple aggregate demand equation. Equation (A4) describes the current account deficit as a function of money creation and past income.

Money in the emerging economy is created via the exchange of foreign for domestic currency. So money creation in the emerging economy is equal to the exports of the emerging economy denominated in its own local currency:

(A5) $M_2 = X_2 P_2$

Combining equations (3), (4) and (5) then yields a solution for the exchange rate:

(A6) $E_{1,2} = \frac{\alpha P_1 Y_1}{M_2}$
This equation says that the emerging economy will see its exchange rate appreciating if demand or prices in the mature economy rise or if its own money creation slows.

The emerging economy produces goods of quantity $Y_2$, which is allocated over exports $X_2$ and domestic consumption $Y_2 - X_2$:

(A7) \[ Y_2 = X_2 + (Y_2 - X_2) \]

A constraint similar to that in the mature economy applies:

(A8) \[ M_2 + Z_2 = (Y_2 - X_2)P_2 \]

Combining equations (A5), (A7) and (A8) then yields the following aggregate demand equation for the emerging economy:

(A9) \[ Y_2 = \frac{2M_2}{P_2} + \frac{Z_2}{P_2} \]

The multiplier 2 in the first right-hand term simply reflects that money created in the emerging economy is spent twice, first on exports and next to finance domestic demand.

Aggregate supply is fixed on the basis of a very simple profit maximisation scheme. Profits are determined as:

(A10) \[ \Pi_i = (P_i - C_i)Y_i - \frac{\theta_i}{2} Y_i^2, i = 1,2 \]

So profits are simply the difference between the price and variable cost $C$ times output $Y$ plus a mark-up to cover the adjustment cost of output. This adjustment cost is quadratic to reflect that adjustment is likely to raise cost progressively as output approaches the economy's potential. If the price level is given from the point of view of suppliers (full competition), the first order condition for profit maximisation reads:

(A11) \[ P_i = \theta_i Y_i + C_i, i = 1,2 ; \]

In the main text I use linearised versions of equations (A3, A6, A9 and A11), taking log differences.

Annex 2

A SIMPLE THREE-REGION MODEL

It is possible to expand the above model with a third economy, say another mature economy. The economic agents in the reserve currency issuing mature economy maximise the utility $U_1$ they derive from goods produced at home $Y_1$ and goods exported by the emerging economy $X_{1,2}$ and good exported by the other mature economy $X_{1,3}$:

(A12) \[ U_1 = \log Y_1 + \alpha_{1,1} \log X_{1,2} + \alpha_{1,2} \log X_{1,3} \]
This maximisation is subject to the liquidity constraint. The demand for the respective goods, multiplied by their respective prices, is constrained by expected nominal income $Z$ and (pre-announced) money creation $M$: 

(A13) \[ M_1 + Z_1 = P_1 Y_1 + P_2 E_{1,2} X_{1,2} + P_3 E_{1,3} X_{1,3} \]

The first order conditions read:

(A14) \[ Y_1 = \frac{1}{1 + \alpha_{1,1} + \alpha_{1,2}} \frac{M_1 + Z_1}{P_1} \]

(A15) \[ X_{1,2} = \frac{\alpha_{1,1}}{1 + \alpha_{1,1} + \alpha_{1,2}} \frac{M_1 + Z_1}{P_2 E_{1,2}} \]

(A16) \[ X_{1,3} = \frac{\alpha_{1,2}}{1 + \alpha_{1,1} + \alpha_{1,2}} \frac{M_1 + Z_1}{P_3 E_{1,3}} \]

The economic agents in the other mature economy maximise the utility $U_3$ they derive from goods produced at home $Y_3$ (less exports $X_{1,3}$) and goods exported by the emerging economy $X_{3,2}$:

(A17) \[ U_3 = \log(Y_3 - X_{1,3}) + \alpha_3 \log X_{3,2} \]

subject to:

(A18) \[ M_3 + Z_3 = P_3 (Y_3 - X_{1,3}) + P_2 \frac{E_{1,2}}{E_{1,3}} X_{3,2} \]

The first order conditions read:

(A19) \[ Y_3 - X_{1,3} = \frac{1}{1 + \alpha_3} \frac{M_3 + Z_3}{P_3} \]

(A20) \[ X_{3,2} = \frac{\alpha_3}{1 + \alpha_3} \frac{M_3 + Z_3}{P_2 E_{1,2} / E_{1,3}} \]

In the emerging economy the budget constraint reads:

(A21) \[ M_2 + Z_2 = (Y_2 - X_{1,2} - X_{3,2}) P_2 \]

This yields the following aggregate demand equation:

(A22) \[ Y_2 - X_{1,2} - X_{3,2} = \frac{M_2 + Z_2}{P_2} \]
Money in both the emerging and other mature economy is created via the exchange of foreign for domestic currency:

\[ M_2 = P_2 \left( X_{1,2} + X_{3,2} \right); \quad M_3 = P_3 X_{1,3} \]

The demand equations (A19) and (A22) can be further reduced by eliminating the export variables using (A23):

\[ Y_2 = 2 \frac{M_2}{P_2} + \frac{Z_2}{P_2}; \quad Y_3 = \left[ 1 + \frac{1}{1 + \alpha_3} \right] \frac{M_3}{P_3} + \frac{1}{1 + \alpha_3} \frac{Z_3}{P_3} \]

Finally, the exchange rates must be determined. Substituting the export equations (A15), (A16) and (A20) in (A23) yields:

\[ E_{1,2} M_2 = (\alpha_{1,1} - \alpha_3 \alpha_{1,2}) P_1 Y_1 + \alpha_3 E_{1,3} P_3 Y_3; \quad E_{1,3} M_3 = \alpha_{1,2} P_1 Y_1 \]

Hence the exchange rates are determined as follows:

\[ E_{1,2} = \frac{(\alpha_{1,1} - \alpha_3 \alpha_{1,2}) P_1 Y_1 + \alpha_3 E_{1,3} P_3 Y_3}{M_2}, \quad E_{1,3} = \frac{\alpha_{1,2} P_1 Y_1}{M_3} \]

Note that if \( \alpha_3 = 0 \) the exchange rate equation for the emerging economy collapses to that for the two-region model.

The aggregate supply equations are assumed to be identical to those in the two-region model.