Risk of liquidity and contagion of the crisis on the US, UK and Euro Area money markets

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Preliminary draft - Do not quote

Abstract

The financial crisis has produced a generalized rise of the liquidity risk on the money markets. The purpose of this article is to highlight the mechanisms of contagion between the money markets of the United States, the United Kingdom and the Euro Zone. To give an account of these mechanisms, a BEKK model, in which we introduce a structural break (August 9, 2007), is adopted. Unlike many analysts who suggest that the spreading process of strain on liquidity starts in the US, our results show that before and after the crisis, the spillover effect on money markets was manifold.

Code JEL: E4, G15, E58

1 Introduction

The upsurge of the financial crisis stems directly from the solvability crisis of the least well-off American households who took out subprime mortgages. From August 2007, the solvability crisis has turned into a bank liquidity crisis that has contributed to the spillover of international upheaval. The bank liquidity issue has since appeared as the Gordian knot of the financial crisis (suspicion between financial institutions, credit restrictions, etc.).

This paper focuses on the major issue of liquidity on the money markets. It sets out to achieve two purposes. It seeks to analyse the dynamics of bank liquidity problems in the United States, the Euro Zone and the United Kingdom through the evolution of the interest rate spreads between LIBOR (London Interbank Offered Rate) and OIS (Overnight Index Swaps) on the different money markets. It aims to show the growing intensity of difficulties encountered by financial institutions in this matter. In an original way, this paper means to highlight the spreading process of strain on liquidity by testing potential causal connections between the spreads of the three money markets. By extension, it will attempt to reveal the truth about the causes of this liquidity crisis: did it too start in America? Did tensions in American money markets really spread to European markets as suggested by many comments such as those of the Deutsche Bank: "This week a year ago, on 9 August 2007, a French investment house announced that it would suspend redemption of shares held by investors on a few of its enhanced-return euro money market funds. The reason for this move was that the market for some of the paper in which these funds had invested, US-dollar denominated asset-backed commercial paper, had collapsed. With this announcement, the credit crisis—which was to broaden into a fullblown financial crisis—had started in earnest" (Deutsche Bank, Global Economic Perspectives, August 4, 2008)?

To shed light on these questions, the BEKK model, named after the bivariate model proposed by Baba et al. (1989), has been adopted. This model reveals in particular the spillovers mechanisms governing the evolution of variances and covariances in spreads between LIBOR and OIS interest rates over time. The originality of this modelling lies with the introduction of a structural break in the equation of central tendency and equations of conditional variances and covariances. Taking this break into account helps to explicitly test the potential modifications of the spillover phenomenon between money markets.

This article is organized in the following way: the second section analyses the meaning of the spreads of interest used, as well as their evolution between January 2007 and November 2008. The third section presents the adopted econometric model (BEKK) and specifies the results obtained. Finally, the last section provides a conclusion to our study.

2 Amplification of liquidity problems on money markets (2007-2008)

Banks make two types of banking operations on the money market to manage liquidity (or cash-flow): unsecured operations and repurchase transactions. These operations lead to risk-taking: credit risk and liquidity risk.

To deal with these risks, banks can resort to the derivatives market, in particular to the overnight index swaps market (OIS) in order to guard against events unforeseen by LIBOR, i.e. the unexpected degradation of the borrower situation and non-anticipated changes in liquidity demand addressed to the creditor and to the market (Kwan, 2009), as well as CDS (credit default swap) to hedge against credit risk.

Of these two types of risks, credit risk for non-secured operations and overnight

index swaps operations is almost non-existent, these operations being of short maturity. Therefore, spread between LIBOR and OIS hardly reflects the credit risk premium. It thus mainly reveals the liquidity premium amount[1]. Indeed, the OIS is the fixed-leg rate in the coupon swap. The variable rate is an overnight rate; the interests from the variable leg are capitalized and are subjected to maturity payment in the same way as interests from the fixed-leg. The LIBOR is a cash rate in the sense that the creditor immediately releases (at contract signature) liquidities to transfer them to the borrower whereas there is no immediate cash released with the OIS (cash swap occurs at maturity providing there is a gap between rates). Consequently, the spread between LIBOR and OIS primarily reflects the intensity of bank liquidity problems.

In this paper, we will proceed to analyse three daily series of spread between LIBOR and OIS three months apart, in the United States, the Euro Zone[2] and the United Kingdom.

[Figure 1]

During the first semester of 2007 before the financial crisis arose, spreads seemed quite stable, around 5-6 base points for the Euro Zone, 7-8 base points in the United States and 10 base points in the United Kingdom. None of these markets appeared to be the driving force for the overall dynamics of the three markets.

On August 9, 2007, the spread between LIBOR and OIS increased in all the markets. In the United States, it rose by 13 base points on the 8th to 34 on the 9 th and 49 on the 10th. In the Euro Zone, it rose by a smaller proportion by 11 to 18 points and 27 points on the 10th. In the United Kingdom,

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[1] The discrepancy between IBOR and REPO from which is substracted the CDS discrepancy also helps to measure the liquidity premium. The liquidity premium thus obtained is very close to the IBOR-OIS discrepancy that we have adopted in this study. (cf. ECB, 2008, p. 144)

[2] For an analysis of the Euro Area money market we may refer to the article published by the ECB in February 2009.

the increase was even more moderate, from 20 to 25 then 32 points. This rise in liquidity price can be linked to upheavals that were then affecting the pricing system of sophisticated financial products. Indeed, the growing rise of uncertainty (and thus of the liquidity premium) came from the implementation of an increasingly complex securitization system that made risk pricing more and more difficult (Gonzalez-Paramo (2008); Zagaglia (2008)). As a result, the first American defaults warned operators that they were short of information for risks associated with these new financial products: they began to have doubts. On August 9, 2007, uncertainty was so acute that BNP-Paribas suspended the calculation of the net asset values of three of its funds.

From August 9, the European Central Bank reacted by injecting liquidities (€94.8 billion) and the Fed injected in turn \$24 billion. On August 13, both banks again injected €48 billion and \$2 billion respectively into banks. These measures had limited impact on the liquidity risk: the spread increased until September 12, 2007.

[Figure 2]

Over this period, the liquidity risk appeared to be more significant in the United States and the United Kingdom than in the Euro Zone, for the situation of banks in the Euro Zone appeared to offer better security against liquidity issues.

Subsequently, the liquidity risk fell in the United Kingdom by about 50 points: this fall could be related to the emergency aid granted on September 14 by the Bank of England to Northern Rock, the fifth largest bank in Great Britain. Spreads stabilized to similar levels between September 21 and November 14, 2007.

Spreads shot up again from November 15: the rise was halted on December 7 for the dollar, December 11 for the pound and December 7 for the euro. This

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reversal could be explained by the concerted support measure from five central banks (the European Central Bank, the Fed, the Bank of England, the Bank of Canada and the Bank of Switzerland) which injected liquidities on December 12. Over this period, as shown by the figure above, the euro spread seemed to lag behind the movements affecting the dollar and pound sterling.

Spreads reached minimum levels on January 29 for the euro and the pound and on January 30 for the dollar. Subsequently, they rose until April 16 for the pound, April 30 for the dollar and May 14 for the euro. Then, until mid-September 2008, spreads stabilized around 70 base points. This relative stability was certainly linked to the support measures from the Fed and the American Treasury to help a number of the organisations in difficulty (Fannie Mae and Freddie Mac on July 13, 2008).

[Figure 3]

September 15, 2008 marked a break in the trajectory of liquidity risk. The announcement of the failure of Lehman Brothers (the fourth largest investment bank on Wall Street) led to a sharp rise in spreads. The "too big to fail" principle that seemed to be governing rescue actions from authorities on an international level was abandoned. It then put an end to the moral hazard induced by systematic rescue. This option made operators think that other institutions were not shielded from a similar sanction. After this episode, interbank loans were considered riskier, rates on money markets shot up and liquidity problems grew worse. Unsecured loans seemed frozen as suspicion became widespread amongst bankers. As shown in figure 3, the LIBOR-OIS spread soared after the failure of Lehman Brothers. It peaked on October 10 with 366 base points for the dollar and 194 base points for the euro. For the pound, the spread kept on increasing until November 6, 2008 (it then reached 298 base points), while it fell in the United States and stabilized in the Euro Zone. During this period of extreme

tension, risk remained lower for the euro.

This description of the evolution of liquidity risk suggests that support from central banks had, at best, helped to stabilize the liquidity premium level rather than sharply reduce it. Each period of tension on money markets implied a long-term and cumulative rise between LIBOR and OIS. This hysteresis effect may be explained by the observed substitution between repurchase transactions and non-secured operations (LIBOR). Indeed, on money markets, repurchase transactions were preferred to non-secured actions as they naturally offer better security. Considering the increase of counterparty risk, it was more precisely repurchased transactions on Treasury bills that were preferred to non-secured transactions (without guaranty). Accordingly, the interest rate on repurchase transactions with Treasury bills security decreased so much that it dropped below the LIBOR or the OIS (Hördahl and King, 2008). The absorption of liquidity on the interbank market led to a rise in the LIBOR in such a way that the spread between the LIBOR and OIS rose.

The liquidity crisis associated with the financial crisis gives food for thought over the existence of contagion mechanisms that could have been set between the different money markets. We would like to determine more particularly, the extent to which the periods of strain on money markets in the Euro Zone and Great Britain together with the subsequent BCE and Bank of England's support actions, may be primarily explained by the rise in the liquidity risk on the American money market or whether the absorption of liquidity was simultaneous on the markets we are studying.

3   Frame of analysis and findings

To highlight the potential links between three variables Y1, Y2 and Y3, representing more specifically the daily variances in spread rates observed in the

United States, the United Kingdom and the Euro Zone over the period from January 3, 2002 to November 28, 2008, the BEKK model (Beirne, Caporale, Schulze-Ghattaz et Spagnolo, 2008) has been adopted. This model helps to evidence propagation mechanisms that govern the evolution of variances and covariances of these variables over time.

Considering the acute instability that existed in money markets after the failure of Lehman Brothers and therefore the unstable character of series of spreads, this period cannot be included in our econometric analysis. Then, we will only focus on the period from January 3, 2002 to September 12, 2008.

Can the three series be directly modelized over this period with a muli-variate autoregressive BEKK-type model? To do so, unit-root tests must conclude with the rejection of H0 (chart 1). Yet, as shown by tests performed on level series (ADF test with constant) and over the period from January 3, 2002 to September 12, 2008, this hypothesis can never be rejected irrespective of the chronicle studied.

[Table 1]

We will next proceed to a unit-root test over the same period from January 3, 2002 to September 12, 2008, introducing a structural break on August 9, 2007, the day the financial crisis began. The technique used is the technique of Westerlund (2006).

i.e. Y_t the period studied. We set:

$$Y_t = \alpha + \tau t + \delta D_t + z_t \tag{1}$$

$$\Delta z_t = \phi z_{t-1} + \sum_{j=1}^{p} \gamma_j \Delta z_{t-j} + e_t \tag{2}$$

where e_t is deemed to be pure noise.

t is a trend variable, z_t a static component if $\phi < 0$ and D_t is a dummy variable equal to 1 from August 9, 2007 or 0 otherwise.

Coefficients τ and δ are estimated from the equation in difference:

$$\Delta Y_t = \tau + \delta \Delta D_t + \Delta z_t \tag{3}$$

and the estimated value of α is determined as:

$$\widehat{\alpha} = Y_1 - \widehat{\tau} \tag{4}$$

The unit-root test is conducted from the variable t, corrected from the structural break:

$$\widehat{S}_t = Y_t - \widehat{\alpha} - \widehat{\tau}t - \widehat{\delta}D_t \tag{5}$$

Chart 2 reproduces the results of the three tests over the period January 3, 2002 to September 12, 2008 when the reality of the structural break of August 9, 2007 is taken into account.

[Table 2]

The three series are static around a constant if we take the trouble to consider the existence of a break on August 9, 2007.

We then proceed to the multivariate modelling of the three series. The model adopted is a trivariate BEKK model with 1 structural break. In matrix form, this model is written:

Equations of average values:

$$Y_t = \Delta_t \alpha + [I - \Delta_t]\beta + \phi Y_{t-1} + \varepsilon_t$$

Where Yt is a column vector with three components: $Y_t' = \begin{bmatrix} Y_t^1, & Y_t^2, & Y_t^3 \end{bmatrix}$; α (resp. β) is likewise the column vector with three components: constants that

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prevail from August 9, 2007 (resp., before this date); ϕ is the supposed diagonal matrix of autoregressive coefficients to the order of one; finally, Δ_t is an identity matrix of dimension 3 as from August 9, 2007 and nil before this date.

Equations of conditional variances – covariances

We set H_t as the symmetrical matrix of conditional variances and covariances of Δ_t errors:

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{12,t} & h_{22,t} & h_{23,t} \\ h_{13,t} & h_{23,t} & h_{33,t} \end{bmatrix}$$

Generically, we modelize these conditional variances and covariances as follows:

$$
\begin{aligned}
H_t &= C'\Delta_t C + D'[I - \Delta_t]D + \\
&\quad + A'\varepsilon_t \Delta_t \varepsilon_t' A + B'\varepsilon_t[I - \Delta_t]\varepsilon_t' B + \\
&\quad + G'H_{t-1}\Delta_t G + F'H_{t-1}[I - \Delta_t]F
\end{aligned}
\tag{6}
$$

Considering the way the Δ_t matrix is constructed, we note that the C, A and G matrices are those of coefficients governing the evolutions of conditional variances – covariances from the structural break; matrices D, B and F are those prevailing before this break.

Matrices C and D have, whatever the hypothesis made subsequently on the propagation mechanism, a superior triangular structure:

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix} \qquad D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix}$$

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The structure of matrices A, B, F and G varies according to the hypothesis made on the spreading mechanism of volatility levels.

The "complete" model M1 introduces no restrictions on these volatilities propagation mechanisms: the conditional variance in zone i is determined not only by its own lag but also by those of conditional variances of other zones:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \qquad F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

The "triangular" model M2 postulates an oriented propagation Y3 $\longrightarrow$ Y2 $\longrightarrow$ Y1. It is materialized by a restriction of nullity concerning the superior triangle of each A, B, G and F matrices:

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$G = \begin{bmatrix} g_{11} & 0 & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \qquad F = \begin{bmatrix} f_{11} & 0 & 0 \\ f_{21} & f_{22} & 0 \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

According to this scheme, the conditional variance in zone 3 can only depend on its own lags while the one in zone 1 depends both on its own lag and on lags of the conditional variances of zones 2 and 3.

Lastly, **the "diagonal" model** supposes that each variance and covariance never depends only on its own lags and squares or cross-products of corresponding lagged-innovations. The structure of matrices A, B, G and F is of a diagonal type:

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$$

$$G = \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \qquad F = \begin{bmatrix} f_{11} & 0 & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & f_{33} \end{bmatrix}$$

The equations of variances and covariances associated with each of these three models are reproduced in annex 1.

Chart 3 provides the values obtained for the log-likelihood function of the sample associated with the triangular model for each potential scheme of propagation.

[Table 3]

From this table it ensues that if a "triangular"-type model should challenge the complete model, the UK $\longrightarrow$ EuroZone $\longrightarrow$ USA scheme would be favoured. It is thus on the basis of this scheme that several series of tests have been conducted:

- Stability tests focusing on each of the three models (allowing a decision on the reality of the structural break as far as volatilities propagation mechanisms are concerned)

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- Tests on various restrictions that may be added to coefficients and correspond to different propagation schemes.

When these tests involve model M2, we always work with a "triangular" propagation scheme UK $\longrightarrow$ EuroZone $\longrightarrow$ USA.

Chart 4 indicates that whatever the model considered the models's null hypothesis of stability must be rejected in favour of instability around August 9, 2007.

[Table 4]

We will now test a number of restrictions relating to the values of the different coefficients (chart 5). Are tested:

- H0: model M2 versus H_A: model M1

- H0: model M3 versus H_A: model M2

- H0: triangular structure before the break and complete after versus HA: complete model over the whole period

- H0: triangular structure after the break and complete before versus HA: complete model over the whole period.

[Table 5]

We can firmly deduce that the "complete" model must be favoured. Therefore, the results of this valuation are reproduced in annex 2.

Charts 6 and 7 recap for each equation of the variance and for each period (before and after the break) the coefficients connecting the hii t variance.

- To its $h_{ii\ t-1}$ lag

- To those of other conditional variances $h_{jj\ t-1}$

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- To lags of squared innovations $\varepsilon^2_{i\ t-1}$ and $\varepsilon^2_{j\ t-1}$[3].

[Tables 6 and 7]

4 Conclusion

This paper, focused on the major issue of liquidity on money markets, has set out to highlight several phenomena and produce several findings. The analysis of the dynamics of bank liquidity issues in the United States, the Euro Zone and the United Kingdom through the evolution of the interest rate spread between LIBOR and OIS in various money markets indicates the growing intensity of the liquidity risk as suspicion grew between financial institutions. From this perspective, the failure of Lehman Brothers marked a break: after September 15, 2008 the price of liquidity shot up. Support actions from central banks only served to contain tensions around liquidity between August 2007 and September 2008 and they seemed overwhelmed by the growing distrust amongst financial institutions thereafter.

Then, in an original and novative manner, the article has highlighted the propagation process of the liquidity crisis by testing, with a trivariate BEKK model in which a break was introduced, the causal links between spreads in the three money markets. Tests could evidence the following results. Over the period of analysis chosen for the econometric study (i.e. from January 3, 2002 to September 12, 2008), we see that before and after the rise of the financial crisis, the spillover between money markets was manifold: interest spreads interacted. On this basis, the bank liquidity crisis would not have come specifically from America and would not have spread from the United States to Europe.

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[3]We deliberately leave aside the interpretation of coefficients associated with covariances and cross-products of past innovations.

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Table 1
Unit-root tests (ADF) for each series of spreads
(From January 3, 2002 to September 12, 2008)

	t-Statistic	Prob.
Spread_us_3M	- 0.940	0.776
Spread_euro_3M	- 1.335	0.615
Spread_bp_3M	- 2.146	0.227

Unit-root tests: the number of lags is automatically determined on the basis of the Min(SIC) criterion. The Prob.column provides the risk threshold from which it becomes possible to reject H0

Table 2

Unit-root tests (ADF) for each series of spreads taking into account structural breaks
(From January 3, 2002 to September 12, 2008)

	t-Statistic	Prob.
Spread_us_3M	-3.23	0.018
Spread_euro_3M	-2.944	0.041
Spread_bp_3M	-3.022	0.033

Unit-root tests: the number of lags is automatically determined on the basis of the Min(SIC) criterion. The Prob.column provides the risk threshold from which it becomes possible to reject H0

Table 3

Log likelihood function of the "triangular" type BEKK model

Propagation scheme	Log L
USA → EURO → UK	14303
USA → UK → EURO	14558
EURO → USA → UK	13468
EURO → UK → USA	14464
UK → EURO → USA	14681
UK → USA → EURO	14480

Table 4

Stability hypothesis tests of conditional variance – covariance equations

Model	Chi2 Stat	P – Value	Restrictions
Complet	658.98	0.0000	$\{c_{ii} = d_{ii}\ \forall i\}\ ;\ \{c_{ij} = d_{ij}\ \forall i, j, j > i\}\ ;\ \{a_{ij} = b_{ij}, f_{ij} = g_{ij}\ \forall i, j\}$
Triangulaire[1]	272.51	0.0000	$\{c_{ii} = d_{ii}\ \forall i\}\ ;\ \{c_{ij} = d_{ij}\ \forall i, j, j > i\}\ ;\ \{a_{ij} = b_{ij}, f_{ij} = g_{ij}\ \forall i, j, i \geq j\}$
Diagonal	856.95	0.0000	$\{c_{ii} = d_{ii}\ \forall i\}\ ;\ \{c_{ij} = d_{ij}\ \forall i, j, j > i\}\ ;\ \{a_{ii} = b_{ii}, f_{ii} = g_{ii}\ \forall i\}$

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[1] For the "triangular" model, we suppose that the propagation scheme starts from the UK zone, passing through the Euro Zone to finally reach the USA zone.

Table 5
Tests on restrictions that may be added to the different models

Null hypothesis	Alternative hyp.	Chi2 – Stat	P – Value	Restrictions : nullity of coefficients
M2 (triangular)	M1 (complete)	144.44	0.0000	$a_{12}, a_{13}, a_{23}, b_{12}, b_{13}, b_{23}, g_{12}, g_{13}, g_{23}, f_{12}, f_{13}, f_{23}$
M3 (diagonal)	M2 (triangular)	136.54	0.0000	$a_{21}, a_{31}, a_{32}, b_{21}, b_{31}, b_{32}, g_{21}, g_{31}, g_{32}, f_{21}, f_{31}, f_{32}$
Triangular then complete	M1 (complete)	109.42	0.0000	$f_{12}, f_{13}, f_{23}, b_{12}, b_{13}, b_{23}$
Complete then triangular	M1 (complete)	34.35	0.0000	$g_{12}, g_{13}, g_{23}, a_{12}, a_{13}, a_{23}$

Table 6

Coefficients connecting conditional variances to their past values and past innovation squares

	$h_{11\,t}$		$h_{22\,t}$		$h_{33\,t}$	
	Before	After	Before	After	Before	After
$h_{11\,t-1}$	f_{11}^{2}	g_{11}^{2}	f_{12}^{2}	g_{12}^{2}	f_{13}^{2}	g_{13}^{2}
$h_{22\,t-1}$	f_{21}^{2}	g_{21}^{2}	f_{22}^{2}	g_{22}^{2}	f_{23}^{2}	g_{23}^{2}
$h_{33\,t-1}$	f_{31}^{2}	g_{31}^{2}	f_{32}^{2}	g_{32}^{2}	f_{33}^{2}	g_{33}^{2}
$\varepsilon_{1\,t-1}^{2}$	b_{11}^{2}	a_{11}^{2}	b_{12}^{2}	a_{12}^{2}	b_{13}^{2}	a_{13}^{2}
$\varepsilon_{2\,t-1}^{2}$	b_{21}^{2}	a_{21}^{2}	b_{22}^{2}	a_{22}^{2}	b_{23}^{2}	a_{23}^{2}
$\varepsilon_{3\,t-1}^{2}$	b_{31}^{2}	a_{31}^{2}	b_{32}^{2}	a_{32}^{2}	b_{33}^{2}	a_{33}^{2}

Note: the coefficient appears in standard character; it must be squared. The test statistics for the hypothesis of coefficient nullity is in brackets and italics.

Table 7

Values and significativity of coefficients linking conditional variances to their past values and past innovation squares before and after the break of August 9, 2007

(1 = USA, 2 = Euro Zone, 3 = UK)

	$h_{11\,t}$		$h_{22\,t}$		$h_{33\,t}$	
	Before	After	Before	After	Before	After
$h_{11\,t-1}$	0.877	0.904	0.0103	0.014	0.149	0.004
	(94.59)	(88.34)	(2.19)	(4.07)	(6.16)	(0.56)
$h_{22\,t-1}$	-0.017	-0.008	0.845	0.909	0.017	-0.001
	(-1.27)	(-0.38)	(113.64)	(104.31)	(0.433)	(-0.05)
$h_{33\,t-1}$	0.035	-0.007	0.018	0.020	0.810	0.718
	(3.81)	(-0.28)	(3.42)	(1.32)	(26.81)	(45.95)
$\varepsilon_{1\,t-1}^{2}$	0.464	0.437	-0.030	0.006	-0.247	0.008
	(28.04)	(17.26)	(-3.99)	(0.69)	(-5.06)	(0.71)
$\varepsilon_{2\,t-1}^{2}$	0.024	-0.152	0.486	0.319	0.089	-0.049
	(0.92)	(-4.29)	(30.43)	(12.46)	(1.33)	(-2.78)
$\varepsilon_{3\,t-1}^{2}$	-0.019	0.057	0.016	0.028	0.397	0.481
	(-1.88)	(1.67)	(4.33)	(1.53)	(15.25)	(21.62)

Note: the coefficient appears in standard character; it must be squared. The test statistics for the hypothesis of coefficient nullity is in brackets and italics.

Figure 1:
Spreads between the LIBOR and OIS over the whole period

Figure 2:

Spreads between the LIBOR and OIS between August 1, 2007 and September 12, 2008

Figure 3:
Spreads between the LIBOR and OIS between September 15, 2008 and November 28, 2008

Annex 1: conditional variances and covariances in the different models

"Complete" model M1

$$h_{11\,t} = \delta_{1\,t}^2\, c_{11}^2 + \delta_{2\,t}^2\, d_{11}^2 + \delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{11}\, (\delta_{2\,t}\, f_{11}\, h_{11\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{13\,t\text{-}1}) + g_{11}\, (\delta_{1\,t}\, g_{11}\, h_{11\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{13\,t\text{-}1})$$
$$+ f_{21}\, (\delta_{2\,t}\, f_{11}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{22\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{23\,t\text{-}1}) + g_{21}\, (\delta_{1\,t}\, g_{11}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{22\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{23\,t\text{-}1})$$
$$+ f_{31}\, (\delta_{2\,t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{31}\, (\delta_{1\,t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{22\,t} = \delta_{1\,t}^2\, c_{12}^2 + \delta_{1\,t}^2\, c_{22}^2 + \delta_{2\,t}^2\, d_{12}^2 + \delta_{2\,t}^2\, d_{22}^2 + \delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{12}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2\,t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2\,t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2\,t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{12}\, (\delta_{2\,t}\, f_{12}\, h_{11\,t\text{-}1} + \delta_{2\,t}\, f_{22}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{32}\, h_{13\,t\text{-}1}) + g_{12}\, (\delta_{1\,t}\, g_{12}\, h_{11\,t\text{-}1} + \delta_{1\,t}\, g_{22}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{32}\, h_{13\,t\text{-}1})$$
$$+ f_{22}\, (\delta_{2\,t}\, f_{12}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{22}\, h_{22\,t\text{-}1} + \delta_{2\,t}\, f_{32}\, h_{23\,t\text{-}1}) + g_{22}\, (\delta_{1\,t}\, g_{12}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{22}\, h_{22\,t\text{-}1} + \delta_{1\,t}\, g_{32}\, h_{23\,t\text{-}1})$$
$$+ f_{32}\, (\delta_{2\,t}\, f_{12}\, h_{13\,t\text{-}1} + \delta_{2\,t}\, f_{22}\, h_{23\,t\text{-}1} + \delta_{2\,t}\, f_{32}\, h_{33\,t\text{-}1}) + g_{32}\, (\delta_{1\,t}\, g_{12}\, h_{13\,t\text{-}1} + \delta_{1\,t}\, g_{22}\, h_{23\,t\text{-}1} + \delta_{1\,t}\, g_{32}\, h_{33\,t\text{-}1})$$

$$h_{33\,t} = \delta_{1\,t}^2\, c_{13}^2 + \delta_{1\,t}^2\, c_{23}^2 + \delta_{1\,t}^2\, c_{33}^2 + \delta_{2\,t}^2\, d_{13}^2 + \delta_{2\,t}^2\, d_{23}^2 + \delta_{2\,t}^2\, d_{33}^2$$
$$+ \delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2\,t}\, b_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2\,t}\, b_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2\,t}\, b_{13}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{23}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{33}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{13}\, (\delta_{2\,t}\, f_{13}\, h_{11\,t\text{-}1} + \delta_{2\,t}\, f_{23}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{33}\, h_{13\,t\text{-}1}) + g_{13}\, (\delta_{1\,t}\, g_{13}\, h_{11\,t\text{-}1} + \delta_{1\,t}\, g_{23}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{33}\, h_{13\,t\text{-}1})$$
$$+ f_{23}\, (\delta_{2\,t}\, f_{13}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{23}\, h_{22\,t\text{-}1} + \delta_{2\,t}\, f_{33}\, h_{23\,t\text{-}1}) + g_{23}\, (\delta_{1\,t}\, g_{13}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{23}\, h_{22\,t\text{-}1} + \delta_{1\,t}\, g_{33}\, h_{23\,t\text{-}1})$$
$$+ f_{33}\, (\delta_{2\,t}\, f_{13}\, h_{13\,t\text{-}1} + \delta_{2\,t}\, f_{23}\, h_{23\,t\text{-}1} + \delta_{2\,t}\, f_{33}\, h_{33\,t\text{-}1}) + g_{33}\, (\delta_{1\,t}\, g_{13}\, h_{13\,t\text{-}1} + \delta_{1\,t}\, g_{23}\, h_{23\,t\text{-}1} + \delta_{1\,t}\, g_{33}\, h_{33\,t\text{-}1})$$

$$h_{12\,t} = \delta_{1\,t}^2\, c_{11}\, c_{12} + \delta_{2\,t}^2\, d_{11}\, d_{12} + \delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{12}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{12}\, (\delta_{2\,t}\, f_{11}\, h_{11\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{13\,t\text{-}1}) + g_{12}\, (\delta_{1\,t}\, g_{11}\, h_{11\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{13\,t\text{-}1})$$
$$+ f_{22}\, (\delta_{2\,t}\, f_{11}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{22\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{23\,t\text{-}1}) + g_{22}\, (\delta_{1\,t}\, g_{11}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{22\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{23\,t\text{-}1})$$
$$+ f_{32}\, (\delta_{2\,t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{32}\, (\delta_{1\,t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{13\,t} = \delta_{1\,t}^2\, c_{11}\, c_{13} + \delta_{2\,t}^2\, d_{11}\, d_{13} + \delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1\,t}\, a_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1\,t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2\,t}\, b_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2\,t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2\,t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2\,t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{13}\, (\delta_{2\,t}\, f_{11}\, h_{11\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{13\,t\text{-}1}) + g_{13}\, (\delta_{1\,t}\, g_{11}\, h_{11\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{13\,t\text{-}1})$$
$$+ f_{23}\, (\delta_{2\,t}\, f_{11}\, h_{12\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{22\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{23\,t\text{-}1}) + g_{23}\, (\delta_{1\,t}\, g_{11}\, h_{12\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{22\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{23\,t\text{-}1})$$
$$+ f_{33}\, (\delta_{2\,t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2\,t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2\,t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{33}\, (\delta_{1\,t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1\,t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1\,t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{23\,t} = \delta_{1\,t}^2\, c_{12}\, c_{13} + \delta_{1\,t}^2\, c_{22}\, c_{23} + \delta_{2\,t}^2\, d_{12}\, d_{13} + \delta_{2\,t}^2\, d_{22}\, d_{23}$$
$$+ \delta_{1\,t}\, a_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1\,t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1\,t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1\,t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$

$$+ \delta_{1t}\, a_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1t}\, a_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{13}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{23}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{12}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{13}\, (\delta_{2t}\, f_{12}\, h_{11\,t\text{-}1} + \delta_{2t}\, f_{22}\, h_{12\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{13\,t\text{-}1}) + g_{13}\, (\delta_{1t}\, g_{12}\, h_{11\,t\text{-}1} + \delta_{1t}\, g_{22}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{13\,t\text{-}1})$$
$$+ f_{23}\, (\delta_{2t}\, f_{12}\, h_{12\,t\text{-}1} + \delta_{2t}\, f_{22}\, h_{22\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{23\,t\text{-}1}) + g_{23}\, (\delta_{1t}\, g_{12}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{22}\, h_{22\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{23\,t\text{-}1})$$
$$+ f_{33}\, (\delta_{2t}\, f_{12}\, h_{13\,t\text{-}1} + \delta_{2t}\, f_{22}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{33\,t\text{-}1}) + g_{33}\, (\delta_{1t}\, g_{12}\, h_{13\,t\text{-}1} + \delta_{1t}\, g_{22}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{33\,t\text{-}1})$$

"Triangular" model M2

$$h_{11\,t} = \delta_{1t}^{2}\, c_{11}^{2} + \delta_{2t}^{2}\, d_{11}^{2} + \delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{11}\, (\delta_{2t}\, f_{11}\, h_{11\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{12\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{13\,t\text{-}1}) + g_{11}\, (\delta_{1t}\, g_{11}\, h_{11\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{13\,t\text{-}1})$$
$$+ f_{21}\, (\delta_{2t}\, f_{11}\, h_{12\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{22\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{23\,t\text{-}1}) + g_{21}\, (\delta_{1t}\, g_{11}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{22\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{23\,t\text{-}1})$$
$$+ f_{31}\, (\delta_{2t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{31}\, (\delta_{1t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{22\,t} = \delta_{1t}^{2}\, c_{12}^{2} + \delta_{1t}^{2}\, c_{22}^{2} + \delta_{2t}^{2}\, d_{12}^{2} + \delta_{2t}^{2}\, d_{22}^{2} + \delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1}) + \delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{22}\, (\delta_{2t}\, f_{22}\, h_{22\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{23\,t\text{-}1}) + g_{22}\, (\delta_{1t}\, g_{22}\, h_{22\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{23\,t\text{-}1})$$
$$+ f_{32}\, (\delta_{2t}\, f_{22}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{33\,t\text{-}1}) + g_{32}\, (\delta_{1t}\, g_{22}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{33\,t\text{-}1})$$

$$h_{33\,t} = \delta_{1t}^{2}\, c_{13}^{2} + \delta_{1t}^{2}\, c_{23}^{2} + \delta_{1t}^{2}\, c_{33}^{2} + \delta_{2t}^{2}\, d_{13}^{2} + \delta_{2t}^{2}\, d_{23}^{2} + \delta_{2t}^{2}\, d_{33}^{2} + \delta_{1t}^{2}\, a_{33}^{2}\, \varepsilon_{3\,t\text{-}1}^{2}$$
$$+ \delta_{2t}^{2}\, b_{33}^{2}\, \varepsilon_{3\,t\text{-}1}^{2} + \delta_{2t}\, f_{33}^{2}\, h_{33\,t\text{-}1} + \delta_{1t}\, g_{33}^{2}\, h_{33\,t\text{-}1}$$

$$h_{12\,t} = \delta_{1t}^{2}\, c_{11}\, c_{12} + \delta_{2t}^{2}\, d_{11}\, d_{12} + \delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{22}\, (\delta_{2t}\, f_{11}\, h_{12\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{22\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{23\,t\text{-}1}) + g_{22}\, (\delta_{1t}\, g_{11}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{22\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{23\,t\text{-}1})$$
$$+ f_{32}\, (\delta_{2t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{32}\, (\delta_{1t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{13\,t} = \delta_{1t}^{2}\, c_{11}\, c_{13} + \delta_{2t}^{2}\, d_{11}\, d_{13} + \delta_{1t}\, a_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{1t}\, a_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{11}\, \varepsilon_{1\,t\text{-}1} + \delta_{2t}\, b_{21}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{31}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{33}\, (\delta_{2t}\, f_{11}\, h_{13\,t\text{-}1} + \delta_{2t}\, f_{21}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{31}\, h_{33\,t\text{-}1}) + g_{33}\, (\delta_{1t}\, g_{11}\, h_{13\,t\text{-}1} + \delta_{1t}\, g_{21}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{31}\, h_{33\,t\text{-}1})$$

$$h_{23\,t} = \delta_{1t}^{2}\, c_{12}\, c_{13} + \delta_{1t}^{2}\, c_{22}\, c_{23} + \delta_{2t}^{2}\, d_{12}\, d_{13} + \delta_{2t}^{2}\, d_{22}\, d_{23} + \delta_{1t}\, a_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{1t}\, a_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{1t}\, a_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ \delta_{2t}\, b_{33}\, \varepsilon_{3\,t\text{-}1}\, (\delta_{2t}\, b_{22}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, b_{32}\, \varepsilon_{3\,t\text{-}1})$$
$$+ f_{33}\, (\delta_{2t}\, f_{22}\, h_{23\,t\text{-}1} + \delta_{2t}\, f_{32}\, h_{33\,t\text{-}1}) + g_{33}\, (\delta_{1t}\, g_{22}\, h_{23\,t\text{-}1} + \delta_{1t}\, g_{32}\, h_{33\,t\text{-}1})$$

"Diagonal" model M3

$$h_{11\,t} = \delta_{1t}^{2}\, c_{11}^{2} + \delta_{2t}^{2}\, d_{11}^{2} + \delta_{1t}^{2}\, a_{11}^{2}\, \varepsilon_{1\,t\text{-}1}^{2} + \delta_{2t}^{2}\, b_{11}^{2}\, \varepsilon_{1\,t\text{-}1}^{2} + \delta_{2t}\, f_{11}^{2}\, h_{11\,t\text{-}1} + \delta_{1t}\, g_{11}^{2}\, h_{11\,t\text{-}1}$$

$$h_{22\,t} = \delta_{1t}^{2}\, c_{12}^{2} + \delta_{1t}^{2}\, c_{22}^{2} + \delta_{2t}^{2}\, d_{12}^{2} + \delta_{2t}^{2}\, d_{22}^{2} + \delta_{1t}^{2}\, a_{22}^{2}\, \varepsilon_{2\,t\text{-}1}^{2} + \delta_{2t}^{2}\, b_{22}^{2}\, \varepsilon_{2\,t\text{-}1}^{2} + \delta_{2t}\, f_{22}^{2}\, h_{22\,t\text{-}1} + \delta_{1t}\, g_{22}^{2}\, h_{22\,t\text{-}1}$$

$$h_{33\,t} = \delta_{1t}^{2}\, c_{13}^{2} + \delta_{1t}^{2}\, c_{23}^{2} + \delta_{1t}^{2}\, c_{33}^{2} + \delta_{2t}^{2}\, d_{13}^{2} + \delta_{2t}^{2}\, d_{23}^{2} + \delta_{2t}^{2}\, d_{33}^{2} + \delta_{1t}^{2}\, a_{33}^{2}\, \varepsilon_{3\,t\text{-}1}^{2} + \delta_{2t}^{2}\, b_{33}^{2}\, \varepsilon_{3\,t\text{-}1}^{2}$$
$$+ \delta_{2t}\, f_{33}^{2}\, h_{33\,t\text{-}1} + \delta_{1t}\, g_{33}^{2}\, h_{33\,t\text{-}1}$$

$$h_{12\,t} = \delta_{1t}^{2}\, c_{11}\, c_{12} + \delta_{2t}^{2}\, d_{11}\, d_{12} + \delta_{1t}^{2}\, a_{11}\, a_{22}\, \varepsilon_{1\,t\text{-}1}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}^{2}\, b_{11}\, b_{22}\, \varepsilon_{1\,t\text{-}1}\, \varepsilon_{2\,t\text{-}1} + \delta_{2t}\, f_{11}\, f_{22}\, h_{12\,t\text{-}1} + \delta_{1t}\, g_{11}\, g_{22}\, h_{12\,t\text{-}1}$$

$$h_{13\,t} = \delta_{1t}^{2}\, c_{11}\, c_{13} + \delta_{2t}^{2}\, d_{11}\, d_{13} + \delta_{1t}^{2}\, a_{11}\, a_{33}\, \varepsilon_{1\,t\text{-}1}\, \varepsilon_{3\,t\text{-}1} + \delta_{2t}^{2}\, b_{11}\, b_{33}\, \varepsilon_{1\,t\text{-}1}\, \varepsilon_{3\,t\text{-}1} + \delta_{2t}\, f_{11}\, f_{33}\, h_{13\,t\text{-}1} + \delta_{1t}\, g_{11}\, g_{33}\, h_{13\,t\text{-}1}$$

$$h_{23\,t} = \delta_{1\,t}{}^2 c_{12} c_{13} + \delta_{1\,t}{}^2 c_{22} c_{23} + \delta_{2\,t}{}^2 d_{12} d_{13} + \delta_{2\,t}{}^2 d_{22} d_{23} + \delta_{1\,t}{}^2 a_{22} a_{33} \varepsilon_{2\,t-1} \varepsilon_{3\,t-1} + \delta_{2\,t}{}^2 b_{22} b_{33} \varepsilon_{2\,t-1} \varepsilon_{3\,t-1} + \delta_{2\,t} f_{22} f_{33} h_{23\,t-1} + \delta_{1\,t} g_{22} g_{33} h_{23\,t-1}$$

Annex 2: estimation of the complete model (1 = USA, 2 = EURO, 3 = UK)

	Coefficient	Std. Error	z-Statistic	Prob.
C1(1)	0.002301	0.001102	2.087082	0.0369
D1(1)	0.003088	0.000242	12.76605	0.0000
A1(1)	0.436948	0.025318	17.25871	0.0000
A2(1)	-0.152294	0.035490	-4.291212	0.0000
A3(1)	0.056764	0.034024	1.668353	0.0952
B1(1)	0.464620	0.016569	28.04094	0.0000
B2(1)	0.023861	0.025997	0.917846	0.3587
B3(1)	-0.018778	0.010001	-1.877543	0.0604
F1(1)	0.877383	0.009275	94.59528	0.0000
F2(1)	-0.017475	0.013659	-1.279401	0.2008
F3(1)	0.035517	0.009307	3.815932	0.0001
G1(1)	0.904504	0.010238	88.34540	0.0000
G2(1)	-0.008332	0.021806	-0.382087	0.7024
G3(1)	-0.006805	0.024472	-0.278088	0.7809
C1(2)	-0.000963	0.001144	-0.841498	0.4001
C2(2)	0.002015	0.000741	2.719864	0.0065
D1(2)	3.28E-05	0.000181	0.180762	0.8566
D2(2)	0.001498	0.000102	14.68051	0.0000
A1(2)	0.006395	0.009233	0.692630	0.4885
A2(2)	0.319416	0.025638	12.45866	0.0000
A3(2)	0.028422	0.018578	1.529901	0.1260
B1(2)	-0.029601	0.007424	-3.987024	0.0001
B2(2)	0.485696	0.015962	30.42873	0.0000
B3(2)	0.015883	0.003664	4.335251	0.0000
F1(2)	0.010353	0.004726	2.190507	0.0285
F2(2)	0.844896	0.007435	113.6437	0.0000
F3(2)	0.018598	0.005430	3.424898	0.0006
G1(2)	0.014414	0.003543	4.068507	0.0000
G2(2)	0.909865	0.008723	104.3116	0.0000
G3(2)	0.020138	0.015230	1.322256	0.1861
C1(3)	0.001868	0.002791	0.669134	0.5034
C2(3)	-0.001700	0.002560	-0.664289	0.5065
C3(3)	0.007684	0.000555	13.84670	0.0000
D1(3)	-0.005688	0.001368	-4.159302	0.0000
D2(3)	-0.010311	0.001807	-5.705053	0.0000
D3(3)	-0.003115	0.004764	-0.653954	0.5131
A1(3)	0.008455	0.011921	0.709252	0.4782
A2(3)	-0.049595	0.017853	-2.777896	0.0055
A3(3)	0.481515	0.022271	21.62087	0.0000
B1(3)	-0.246799	0.048736	-5.064038	0.0000
B2(3)	0.089035	0.066819	1.332480	0.1827
B3(3)	0.397261	0.026051	15.24955	0.0000

F1(3)	0.148947	0.024155	6.166347	0.0000
F2(3)	0.017447	0.040254	0.433410	0.6647
F3(3)	0.809700	0.030202	26.80922	0.0000
G1(3)	0.003631	0.006464	0.561728	0.5743
G2(3)	-0.000869	0.017060	-0.050939	0.9594
G3(3)	0.717851	0.015621	45.95497	0.0000
ALFA(1)	-0.002139	0.000600	-3.562974	0.0004
BETA(1)	-0.000828	0.000173	-4.772610	0.0000
PHI(1)	-0.376594	0.014401	-26.15135	0.0000
ALFA(2)	-0.000154	0.000404	-0.381695	0.7027
BETA(2)	-0.000577	7.91E-05	-7.289147	0.0000
PHI(2)	-0.386478	0.011338	-34.08760	0.0000
ALFA(3)	8.20E-05	0.000359	0.228298	0.8194
BETA(3)	-0.007639	0.000540	-14.13751	0.0000
PHI(3)	-0.248709	0.014444	-17.21825	0.0000
Log likelihood	14516.94	Akaike info criterion		-16.58250
Avg. log likelihood	8.323931	Schwarz criterion		-16.40391
Number of Coefs.	57	Hannan-Quinn criter.		-16.51647