Adjustment in EMU: Is Convergence Assured?  
– Preliminary version. Please Do not Cite! –

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Abstract
Using a modified version of the model presented by Belke and Größl (2007), we analyze the stability of adjustment in a currency union. Using econometric estimates for parameter values we check the stability conditions for the 11 original EMU countries and Greece. We found significant instability in the model for a large number of countries. We then simulate the adjustment process for some empirically observed parameter values and find that even for countries with relatively smooth adjustment, the adjustment to a price shock in EMU might take several decades.

Keywords: EMU, convergence, stability, inflation

JEL classification: E32, E61, C32

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1 Introduction

While the introduction of the euro is widely seen as a huge success, in recent years the concern has grown that the countries of the currency union are drifting apart from each other (Gros, 2006, Dullien and Schwarzer, 2005). Investment banks and the financial press have even developed scenarios of countries leaving EMU (Roubini, 2006, Munchau, 2006, Riches-Flores, 2006).

Central to the argument of the sceptics is the lasting divergence in inflation rates as has been widely described.1 As Dullien and Fritsche (2007) and Lane (2006) argue, this divergence in inflation has two different, counteracting effects on the economy: First, via the real interest rate channel, a higher national rate of inflation lowers the relevant real interest rate for the country in question given the uniform nominal interest rate in a monetary union. This boosts investment in housing, consumption and investment in equipment. Second, a national rate of inflation above the average inflation in the currency union dampens growth via the real exchange rate channel: As the country in question loses competitiveness vis-à-vis the rest of the currency union, eventually, export growth will be dampened which should also lower overall GDP growth.

If markets are reasonably flexible and wage growth reacts reasonably strongly to an increase in unemployment, divergence in inflation should eventually reverse: As soon as the dragging effect of the external sector becomes large enough, this will pull down overall GDP growth. With a growing output gap, also inflation should fall, bringing it below the average in the currency union. Inversely to the accelerating effect of a above-average inflation, this would then dampen construction, consumption and investment in equipment, further depressing growth, but also bringing about a real depreciation until the overvaluation is corrected again.

However, sceptics have doubted whether this adjustment takes place reasonably quickly: They argue that the exchange rate channel works much slower than the real interest rate channel. In addition, they argue that there might be some structural national features in wage and price setting which might hinder markets from reacting sufficiently to a strong fall in growth and an increase in unemployment. Dullien and Fritsche (2006) argue that a slow adjustment might even endanger EMU: As a slow adjustment would lead to long periods of sub-trend growth, this might change the incentives of politicians with a high personal discount rate to leave EMU in order to use

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1See i.e. Alvarez et al. (2006), Angeloni et al. (2006), Cecchetti and Debelc (2006) or European Central Bank (2005).
2 Modelling Adjustment Dynamics of a Small Economy

2.1 The Basic Model

In this paper, we intend to analyze if a given structure of output/inflation dynamics – as observable over the last couple of years – is actually in line with a stable adjustment path and how long do these processes last and how much (if any) the model parameters changed over time. To this end, we have chosen to use a simple system of reduced form equations, one for the output gap (IS-curve) and one for the Philips-curve.

We are well aware that most modern macroeconomic analysis today is done within the framework of a dynamic stochastic general equilibrium (DSGE) models, in which reduced form equation for the economic aggregates are deducted from a rigorous micro-foundation with individuals which rationally expect the development of the main variables. In some of these models, it
is attempted to replicate stochastic patterns observed in the real world by introducing specific labour market or product market frictions.

However, while these models have a lot of theoretical appeal, for our current analysis, they have a number of problems:

1. When solving the maximization problem of the individual in DSGE models, unstable rational expectation paths are routinely dismissed. This does seem to be a questionable approach if the goal of the inquiry is whether there might be instabilities in the system.

2. This is especially important as empirically, it is very hard to find empirical evidence in EMU for a forward-looking behaviour, as Angeloni and Ehrmann (2004, p. 13) note.

3. The labour and product markets in EMU are far from the theoretical model of a perfectly functioning market and are empirically exhibiting a number of rigidities, including collective bargaining with principal-agent problems between workers and union representatives, legal regulations of wages for certain groups of workers and complicated and diverse regulation of employment protection. While some DSGE models manage to model labour market imperfection in a way that replicate reasonable well some of the real world data, they are typically not intended at taking strategic interactions of the kind mentioned above into account.

4. There is a growing body of research which questions the empirical validity of rational expectation models for financial markets (i.e. de Grauwe and Grimaldi (2006) or Frydman and Goldberg (2007)) which also might cause doubts on the empirical validity of model-consistent rational expectations for macroeconomic analysis.

Thus, it is possible that a rational expectation DSGE model would point toward stability, whereas in the empirical data we would find strong diverging forces. From a policy maker’s or a financial market participant’s perspective, in this case it seems safer to consider the empirical sound (even if backward-looking) structure of the economy than an empirically shaky (yet theoretically more sound) rational expectation framework: Even if individuals were finally to learn the underlying structure of the economy and start to learn to act forward-looking, there is no guarantee that this will happen before political processes as explained in Dullien and Fritsche (2006) run their course and lead to a single country leaving EMU.
Finally, in the case for which the fundamental backward looking model is used in this paper, the Lucas critique does not apply. In his seminal article, Lucas (1976) has criticised that one cannot take econometrically evaluated parameters as stable when changing conducting economic policy as individuals adjust their behaviour towards policy makers’ actions or changes in the economic policy regime. However, in most of this paper, we are not trying to evaluate any changes in the economic regime or even economic policy variables, but only giving a positive assessment of the structural behaviour of the economy in the absence of changes in the policy environment.

Concerning our approach, there are two works which have also recently tried to get analytical solutions to the stability problem of a small open country in EMU: Belke and Gros (2007) and Geiger and Spahn (2007) have tried to deduce stability conditions using simple two-equations-models in the form of:

\[ y_t - y_{t-1} = -\delta (i - (p_t - p_{t-1})) - \lambda p_t + \varepsilon_{y,t} \]  
\[ p_t - p_{t-1} = \beta (y_t - y_{t-1}) + \varepsilon_{p,t} \]  

Where the usual notation applies: \( y \) stands for output, \( i \) is the nominal interest rate, \( p \) the (national) price level. The first equation describes that the output growth depends on the real interest rate as well as the real exchange rate (which in a monetary union is given by the national price level assuming price stability in the rest of the currency union). The second equation is a standard Philips curve, stating that inflation is positively related to growth, albeit without any forward or backward looking part.

However, Belke and Gros have not included the possibility for a systematic inflation persistence, but only for price persistence, possibly in order to keep the model solution simple and intuitive. As the inflation persistence (and not the price persistence) is one of the central arguments of those who question the stability of EMU (and is well described in empirical analysis), this might not be sufficient to answer the question whether adjustment in EMU happens sufficiently quickly.

Note that the parameter notations have been slightly changed in order to get them into line with the slightly more complicated model used later.

Solving the system under the assumption of price persistence yields a difference equation of second order while solving the system under the assumption of inflation persistence yields a difference equation of third order which is significantly harder to solve.
We thus propose a little augmented system of the equations (1) and (2) with the explicit possibility of a persistence in inflation:

\[ y_t = \gamma y_{t-1} - \delta r_t - \lambda \tau_t + \varepsilon_{yt} \]  

(3)

\[ \pi_t = \alpha \pi_{t-1} + (1 - \alpha) \pi^E + \beta \left( \frac{y_t - \bar{y}}{\bar{y}} \right) \]  

(4)

This simple system of equations represents a single economy in EMU. We assume that the EMU economy as a whole is in its steady state equilibrium with an inflation rate coinciding with the target level and a nominal interest rate \(i^E\) which keeps the EMU economy as a whole in equilibrium. Equation (3) represents the equilibrium in the goods market of the single country in EMU with output \(y_t\) being a function of previous GDP \((y_{t-1})\), the current domestic real interest rate \((r_t)\) and the current real exchange rate \((\tau_t)\) defined as the ratio between the EMU price level and the domestic price level. \(\varepsilon_t\) represents an asymmetric demand shock. The domestic interest rate is defined as the difference between the European nominal interest rate \((i^E)\) and expected future domestic inflation which corresponds to its current level \((\pi_t)\):

\[ r_t = i^E - \pi_t \]  

(5)

Equation (4) represents the small economy’s Philips curve. Domestic inflation depends on wage developments in the country which in term depends partly on the backward-looking behavior of wage setters reflected in the first term and partly on the steady-state inflation rate in the rest of monetary union. This term can be read as the forward-looking component in wage setting as in the long run, inflation in a single country cannot deviate from inflation in EMU. The last term of the right-hand side of (4) represents the relative output gap with \(\bar{y}\) standing for the steady state GDP value, and indicates the impact of nominal rigidities. As mentioned before, asymmetric shocks explain deviations of the small economy from the EMU average. A positive demand shock, for example, increases demand above its steady state level. Due to nominal rigidities, output increases inducing a rise in domestic inflation. This in turn will lead to a falling real exchange rate over time according to

\[ \tau_t - \tau_{t-1} = \pi^E - \pi_t \]  

(6)

As is shown in the appendix, this system of equations can be rewritten
in terms of deviation of output and inflation from their steady state levels:

\[
\begin{align*}
\hat{y}_t &= \gamma \hat{y}_{t-1} + \tilde{\delta} \hat{\pi}_t + \tilde{\lambda} \hat{\tau}_t \\
\hat{\pi}_t &= \alpha \hat{\pi}_{t-1} + \beta \hat{y}_t
\end{align*}
\]  

(7)  

(8)

with

\[
\begin{align*}
\hat{y}_t &\equiv \log \left( \frac{Y_t}{Y} \right) \quad (9) \\
\hat{\tau}_t &\equiv \log \left( \frac{T_t}{T} \right) \quad (10) \\
\hat{\pi}_t &= \pi_t - \pi^E \quad (11) \\
\tilde{\delta} &= \delta \frac{Y}{Y} \quad (12) \\
\tilde{\lambda} &= \lambda \frac{\tau}{Y} \quad (13)
\end{align*}
\]

2.2 Stability Conditions of the Small Economy Model

The system of difference equations (7) and (8) can be reduced to a single difference equation of third order in deviations of the real exchange rate from its steady state value. In order to accomplish this task, we substitute equation (7) into (8), use the additional identity \(\hat{\pi}_t = \hat{\tau}_t - 1 - \hat{\tau}_t\) and solve for the current deviation of GDP from its steady state value, to obtain

\[
\hat{y}_t = \frac{1}{\beta} \left[ - (\hat{\tau}_t - \hat{\tau}_{t-1}) + \alpha (\hat{\tau}_{t-1} - \hat{\tau}_{t-2}) \right]
\]  

(14)

We observe that current deviations of GDP from its steady state value are negatively correlated with current changes in the real exchange rate, and positively with past changes in the real exchange rate, the amount of which increases with \(\alpha\). Reformulating (14) delivers

\[
\hat{y}_t = \frac{1}{\beta} \left[ - \hat{\tau}_t + (1 + \alpha) \hat{\tau}_{t-1} - \alpha \hat{\tau}_{t-2} \right]
\]  

(15)

Note that \(\alpha\) determines the magnitude of inflation persistence. From equation (15) we see that the impact of the parameter \(\alpha\) on \(\hat{y}_t\) is ambiguous: The higher the magnitude of \(\alpha\), so much the higher is the positive impact of the past real exchange rate and so much the higher is the negative impact of the the real exchange rate pertaining to \(t - 2\). This information will prove helpful
for the interpretation of stability conditions. Substituting (8) and (15) into the market equilibrium condition we obtain

\[
\frac{1}{\beta} \left[ -\hat{\tau}_t + (1 + \alpha) \hat{\tau}_{t-1} - \alpha \hat{\tau}_{t-2} \right] = (16)
\]

\[
\frac{\gamma}{\beta} \left[ -\hat{\tau}_{t-1} + (1 + \alpha) \hat{\tau}_{t-2} - \alpha \hat{\tau}_{t-3} \right] - \delta (\hat{\tau}_t - \hat{\tau}_{t-1}) + \lambda \hat{\tau}_t = (17)
\]

Equation (17) expresses the time path of deviations of GDP from its steady state value as a function of deviations of the real exchange rate from its steady state value. Since current GDP is positively correlated with GDP of the previous period, we obtain a difference equation in the real exchange rate of order three. Reformulating (17) delivers

\[
\hat{\tau}_t = \frac{\gamma + \alpha + \left(1 - \beta \delta\right)}{\left(1 + \beta \left(\lambda - \delta\right)\right)} \hat{\tau}_{t-1} - \frac{\alpha + (1 + \alpha) \gamma}{\left(1 + \beta \left(\lambda - \delta\right)\right)} \hat{\tau}_{t-2} + \frac{\alpha \gamma}{\left(1 + \beta \left(\lambda - \delta\right)\right)} \hat{\tau}_{t-3} = (18)
\]

Since we have expressed the lagged value of \( \hat{y} \) as a function of real exchange rate dynamics with the past change having a negative and the change between \( t - 2 \) and \( t - 1 \) having a positive effect on \( \hat{y} \), both the impact of inflation persistence as well as GDP persistence on the time path of \( \hat{y} \) as well as of course \( \hat{\tau} \) becomes ambiguous. Moreover we observe that a further component affecting dynamics is related to the issue whether changes of the real interest rate or the real exchange rate have a more significant impact on the demand for domestically produced goods.

For testing for stability, we next have to rewrite equation (18) in the characteristic equation form

\[
\mu_t + a_1 \mu_{t-1} + a_2 \mu_{t-2} + a_3 \mu_{t-3} = 0
\]

with

\[
a_1 = -\frac{\gamma + \alpha + \left(1 - \beta \delta\right)}{\left(1 + \beta \left(\lambda - \delta\right)\right)},
\]

\[
a_2 = \frac{\alpha + (1 + \alpha) \gamma}{\left(1 + \beta \left(\lambda - \delta\right)\right)},
\]

\[
a_3 = -\frac{\alpha \gamma}{\left(1 + \beta \left(\lambda - \delta\right)\right)}
\]
According to Okuguchi and Irie (1990) the following set of conditions are necessary and sufficient to ensure convergence to the steady state:

\begin{align*}
1 + a_1 + a_2 + a_3 > 0 \quad (22) \\
1 - a_1 + a_2 - a_3 > 0 \quad (23) \\
1 - a_2 + a_1a_3 - a_3^2 > 0 \quad (24)
\end{align*}

Substituting (??) into (22), (23) and (24) gives us the two rather simple conditions:

\begin{align*}
1 - \beta \tilde{\delta} + \beta \tilde{\lambda} - 1 - \alpha - \gamma + \beta \tilde{\delta} + \alpha + \alpha \gamma + \gamma - \alpha \gamma > 0 \\
(1 - \beta \tilde{\delta} + \beta \tilde{\lambda}) \\
\iff \frac{\beta \tilde{\lambda}}{1 - \beta \tilde{\delta} + \beta \tilde{\lambda}} > 0 \quad (25)
\end{align*}

\begin{align*}
1 - \beta \tilde{\delta} + \beta \tilde{\lambda} + 1 + \alpha + \gamma - \beta \tilde{\delta} + \alpha + \alpha \gamma + \gamma + \alpha \gamma > 0 \\
(1 - \beta \tilde{\delta} + \beta \tilde{\lambda}) \\
\iff \frac{2 + 2\alpha + 2\alpha \gamma + 2 \gamma + \beta \gamma - 2 \beta \tilde{\delta}}{1 - \beta \tilde{\delta} + \beta \tilde{\lambda}} > 0 \quad (26)
\end{align*}

And one more complicated condition:

\begin{align*}
1 - \frac{\alpha^2 \gamma^2}{(1 - \beta \tilde{\delta} + \beta \tilde{\lambda})^2} + \frac{\alpha \gamma (1 + \alpha + \gamma - \beta \tilde{\delta})}{(1 - \beta \tilde{\delta} + \beta \tilde{\lambda})^2} - \frac{\alpha + \gamma + \alpha \gamma}{1 - \beta \tilde{\delta} + \beta \tilde{\lambda}} > 0 \quad (27)
\end{align*}

Taking a closer look, (25) and (26) can be further simplified: As long as \( \beta \) and \( \tilde{\lambda} \) have normal signs (a positive reaction of the inflation to the output gap and a negative reaction of the output gap to a real appreciation), \( 1 - \beta \tilde{\delta} + \beta \tilde{\lambda} > 0 \) needs to be fulfilled in order for (25) and (26) to be met.

However, as we will see in our empirical section 3, it is not clear that \( \beta \) and \( \tilde{\lambda} \) actually have normal signs. Thus, we retain all three conditions.

Independent from the question whether these conditions are fulfilled, from a policy perspective, it might be interesting to see in how far a change in the parameters would actually increase stability (by moving conditions (25) and (26) away from 0. Unfortunately, however, the first derivatives especially of
condition (26) is not a monotonically increasing or monotonically decreasing function in the parameters in question. Figures 1 and 2 illustrate this point. The figures show the value of the left-hand-side of condition (26) for values of $\lambda$ and $\beta$ between 0 and 2 for the (a priori realistic) parameters $\alpha = 0.8$, $\delta = 0.5$ and $\gamma = 0.5$. Interestingly, for both parameters $\lambda$ and $\beta$, there are areas in which an increase in the parameter in question increases stability and other areas in which an increase in the parameter decreases stability of the system.

From a policy maker’s perspective, this is an extremely important result: If we assume that economic reforms in one single market (i.e. the labour market in one EMU country) usually change only one of the parameters at a time, there is a real risk that a reform which in other circumstances (and other countries) might lead to a stabilization of the country’s situation in EMU might actually lead to a destabilization. This would call into question the notion to give all EMU countries the same reform prescription in order to improve the stability of the currency union.

3 How Stable is EMU? A First Assessment Using the Small Economy Model

Against the background of conditions (22) to (24), it is now an empirical question whether the conditions are fulfilled in real-life EMU. In order to gauge whether single EMU countries can be expected to show a long-run convergence towards an equilibrium real exchange rate and thus a stable real exchange rate development in a monetary union, we estimated the parameters $\alpha$, $\beta$, $\gamma$, $\bar{\lambda}$ and $\bar{\delta}$ in different settings. We tried to keep the equations to be estimated as close as possible to the theoretical model, to stay as close as possible to a theory-guided view on the data.

The data set consist of annual data for the EU 12 and was taken from the AMECO data base. All data are denominated in euro. The real GDP was detrended applying a Hodrick-Prescott filter on the log-level data and subtracting the trend from the data. As a proxy for price level as well as the calculation of inflation rates, the GDP deflator was used. Real interest

\footnote{Follow the link}
rates were obtained by subtracting the current inflation rate from short-term nominal interest rates. If necessary, data were rescaled to have the same dimension. All data were compared to the respective Euro area data. National output gap, inflation rate and real interest rate numbers are calculated as deviations from the respective Euro area data (in percentage points). The national relative price level is the percentage deviation from the Euro area price level – measured by the GDP deflator (all indexed to 2000 = 100) and calculated as log difference.

However, calculating the deviation of the real exchange rate from its steady state value posed a number of methodological problems. A priori it is not clear how and whether to detrend the real exchange rate time series. On the one hand, a Balassa-Samuelson type argument would call for a detrending: If some countries are experiencing a catch-up process, it could be expected that their inflation is higher than that of the other countries. However, detrending the time series on the other hand poses the risk that some pathological development away from the steady-state equilibrium is considered to be a normal development: Assume that a small country (say Portugal) is experiencing a continuous real appreciation. This could both reflect a Balassa-Samuelson type of adjustment toward a changed steady-state equilibrium as well as a permanent move away from the steady state in an unstable system. As the question cannot be easily solved empirically, we have decided to run three sets of estimations: We first used the long-term average of the real exchange rate as the steady-state value. In a second set of estimations, we used a HP filter with the standard parameter of $\lambda = 100$ for detrending the time series. However, as this setting assumes cycles to last for less than 10 years and we wanted to allow for the possibility of a slower adjustment, we ran a final set of estimations for which we computed the steady-state value for the real exchange rate by using a HP filter with the parameter setting of $\lambda = 1000$.

To get a first impression about the data properties, we estimated the aggregate demand (equation 7) and Phillips curves (equation 12) for each country over the time span 1960 to 2005. Preliminary stability tests (recursive coefficients, CUSUM, CUSUM$^2$) reveals the most influential structural instability in a number of cases around the early 1980. Therefore, we have in a second step reduced the sample size to 1980 to 2005 to reduce the error.

The parameters of the model of interest were estimated by several methods: OLS, seemingly unrelated regressions (a system of two equations for each country since it can reasonably be argued that inflation and output gap shocks are correlated) and also a state space formulation of the system of
two equations for each country where the coefficients follow a random walk – to consider possible regime shifts. Here we report the SUR results only because they revealed the most efficient estimates. In general, due to the short time span, asymptotic evidence is quite weak. However, in order to get a first approximation of possible stability properties for the EMU countries, we have taken these point estimates and evaluated the estimated coefficients according the stability conditions \((22)\) to \((24)\).

– Insert table 1 to 3 about here –

The results are quite interesting and conclusive. Taking the results of table 1 and 3 as a benchmark, the stability conditions are not fulfilled for Spain, France and Finland no matter how which method we apply to measure the steady-state real exchange rate. While France and Finland violate the first stability criterion \((25)\), Spain violates the third criterion \((27)\). Moreover, there are a number of countries which violate the first stability condition at least under one of the alternative methods to measure the steady-state real exchange rate. Greece violates the stability condition if we measure the steady-state real exchange rate by any form of the HP filter; Italy violates the stability condition when applying either the mean or the HP filter with \(\lambda = 1000\) for measuring the steady-state real exchange rate. Ireland violates the stability condition if we apply the mean, Belgium if we apply the HP filter with \(\lambda = 100\).

Moreover, even outside these countries which show an outright violation of the stability criteria, it is hard to be sure about the stability properties. For Germany, the stability criterion 1 is very close to 0 no matter which method we apply. The same holds true for Ireland and Italy for the methods of measuring steady-state real exchange rates in which these countries fulfill the stability criteria.

Hence, using this simple model to gauge the stability of the euro area leaves us with rather frightening results: For a number of countries in EMU, a smooth adjustment to external shocks does not seem to be guaranteed. However, one has to keep in mind that EMU has only existed for a very short time and structural changes brought about by the introduction of the common currency might only be slow to materialize.
4 Visualizing Adjustment of Small Economies in EMU

However, even if adjustment in EMU eventually takes place, this might not be enough to guarantee political stability of the currency union. Politicians with a high personal discount rate might have an incentive to leave EMU to prevent from a long and potentially painful adjustment period even if long-term costs are high (Dullien and Fritsche, 2006).

In order to judge the relevance of this argument, it would be interesting to see how long adjustment takes. To this end, we have simulated two adjustment processes for parameter constellations which we empirically found and which guarantee stability and have compared those with two empirically observed adjustment processes of countries with parameter constellations which point towards instability. We have chosen the parameter estimates for Netherlands in the case of steady-state real exchange rate being computed with the HP-100 filter (as a case of smooth adjustment), the case of Germany in the case of steady-state real exchange rate being computed as a mean (as a case of slow adjustment), the case of Spain in the case of steady-state real exchange rate being computed with the HP-100 filter (as a case of no adjustment because of violation of stability criterion 3) and the case of Italy in the case of steady-state real exchange rate being computed as a mean (as a case of no adjustment because of violation of stability criterion 1). The parameter values are provided in table 4.

Against these parameters, we have introduced a price shock in period one, say triggered by some strong wage increase. This price shock then is translated towards a fall in the real interest rate in the country in question which leads to increased demand and further price pressure. Figures 3 to 10 show the adjustment paths for the output gap and the real exchange rate.

The results of this exercise are quite interesting: In the case of Spain, the instability comes in the form of ever-growing amplitudes of cyclical fluctuations while the parameter constellation of Italy leads to a straight divergence away from the steady state for output gap and real exchange rate.

However, even for the Netherlands, a country which according to the parameter estimates fulfills all stability conditions with quite some safety margin, adjustment is far from quick: It takes more than 20 periods (remember that estimations have been made with annual data!) until the output...
gap has endogenously closed in EMU. In the case of the German parameters (which result in a stability condition 1 only slightly above 0), adjustment takes even longer: After 50 periods, the effects of the shock are still felt.

5 Conclusion

Our paper contributes to the literature on the stability of the EMU under a common monetary policy in the following way:

First, we present a simple and reasonably tractable model to analyze the stability of the adjustment processes for a small economy against a steady-state state situation in EMU. This model is easily enough to allow some interpretation but – in contrast to previous attempts – allows for inflation persistence, a feature typically found in the European data (Alvarez et al., 2006). The model can be transformed into a difference equation of third-order.

Second, taking the analytical stability conditions of the model as a benchmark for a small enough economy, we estimate the parameters using OLS, SUR and state space models, each with three different approaches for determining the steady-state real-exchange rate fluctuations. The results are quite imprecise – which is not astonishing since our inference is based on only a few observations – but interesting in that sense that under the point estimates some countries do not show convergence towards a European business cycle over the sample under investigation. This is especially true for Spain, Finland and France, but also doubts about stability for Italy, Portugal, Greece and Ireland arise.

Third, the simulation reveals that even if adjustment eventually takes place the time periods involved might be extremely long. This might pose political problems as long periods of sub-trend growth might cause opposition against the euro.

All in all, this supports the view that the actual setting of economic policy in the EMU is not necessarily stability-oriented and calls for some action. One path of investigation would be in how far fiscal transfer mechanisms as proposed by ? might help to shorten the adjustment process. Another path for further investigation would be in how far deregulations of labour and product markets might help to improve the adjustment mechanism.
References


Tables and Figures

Figure 1: Sample plot stability condition I

Figure 2: Sample plot stability condition II
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<td>Criterion 2</td>
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</tr>
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Table 3: Stability conditions, using deviation from mean for steady-state estimate of $\tau$

<table>
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<tr>
<th>Country</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>criterion 1</th>
<th>criterion 2</th>
<th>criterion 3</th>
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<tbody>
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<td>0.213</td>
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<td>Finland</td>
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<td>-0.167</td>
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<td>4.509</td>
<td>0.076</td>
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Table 4: Parameter estimates for simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Netherlands (HP=100)</th>
<th>Spain (HP=100)</th>
<th>Germany (Mean)</th>
<th>Italy (Mean)</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>0.383</td>
<td>-0.102</td>
<td>0.348</td>
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<td>$\beta$</td>
<td>0.137</td>
<td>0.970</td>
<td>0.028</td>
<td>0.544</td>
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<tr>
<td>$\tilde{\delta}$</td>
<td>-0.153</td>
<td>0.244</td>
<td>-0.137</td>
<td>0.044</td>
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<tr>
<td>$\gamma$</td>
<td>0.793</td>
<td>0.939</td>
<td>0.757</td>
<td>0.67</td>
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<tr>
<td>$\lambda$</td>
<td>0.103</td>
<td>0.128</td>
<td>0.043</td>
<td>-0.024</td>
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</table>
Figure 3: Output adjustment, Netherlands

Figure 4: Real exchange rate adjustment, Netherlands
Figure 5: Output adjustment, Germany

![Output Gap Graph](image)

Figure 6: Real exchange rate adjustment, Germany

![Real Exchange Rate Graph](image)
Figure 7: Output adjustment, Spain

Figure 8: Real exchange rate adjustment, Spain
Figure 9: Output adjustment, Italy

Figure 10: Real exchange rate adjustment, Italy
Appendix

Writing the model as deviation from the steady state

In steady state, domestic and EMU inflation coincide

$$\pi = \pi^E$$  \hspace{1cm} (28)

with $\pi$ standing for the steady state value of domestic inflation. Furthermore the real exchange rate determined such that domestic demand and the steady state GDP equal:

$$y = -\delta r + \lambda \tau \quad \hspace{1cm} (29)$$

with $r = i^E - \pi^E$ and $\tau$ representing steady state values for the real interest rate and the real exchange rate, respectively.

In the next step, we express $y_t$ and $\tau_t$ as percentage deviations from their steady state value.

We first subtract (29) from (3). This yields

$$Y_t - \bar{Y} = \gamma \left( Y_{t-1} - \bar{Y} \right) - \delta \left( \pi^E - \pi_t - \bar{Y} \frac{E}{E} + \bar{Y} \pi^E \right) + \lambda \left( \tau_t - \bar{Y} \right) \quad \hspace{1cm} (30)$$

Next we divide both sides by $\bar{Y}$ and expand the term $(\tau_t - \bar{Y})$ by $\tau_t$:

$$\frac{Y_t - \bar{Y}}{\bar{Y}} = \gamma \left( \frac{Y_t - \bar{Y}}{\bar{Y}} \right) - \delta \frac{\hat{\pi}_t}{\bar{Y}} + \lambda \frac{\hat{\tau}_t}{\bar{Y}} \quad \hspace{1cm} (31)$$

Recalling that for any variable $x$ we have

$$\log \left( \frac{x_t}{\bar{x}} \right) = \log \left( 1 + \frac{x_t - \bar{x}}{\bar{x}} \right) \approx \frac{x_t - \bar{x}}{\bar{x}} \quad \hspace{1cm} (32)$$

and defining

$$\hat{y}_t \equiv \log \left( \frac{y_t}{\bar{y}} \right)$$

$$\hat{\tau}_t \equiv \log \left( \frac{\tau_t}{\bar{Y}} \right)$$

$$\hat{\pi}_t = \pi_t - \pi^E$$

we can reformulate equation (31) as:

$$\hat{y}_t = \gamma \hat{y}_{t-1} + \delta \hat{\pi}_t + \lambda \hat{\tau}_t$$

25
Next, we rewrite (4) as
\[ \pi_t - \pi^E = \alpha \pi_{t-1} + (1 - \alpha) \pi^E - \pi^E \beta \hat{y}_t \] (33)
by using the definition of \( \hat{y}_t \) and subtracting \( \pi^E \) on both sides. Using definition (11), we now get
\[ \hat{\pi}_t = \alpha \hat{\pi}_{t-1} + \beta \hat{y}_t \] (34)

In order to rewrite this equation in terms of \( \hat{\tau} \), we start with
\[ \hat{\tau}_t - \hat{\tau}_{t-1} = \log \left( \frac{\tau_t}{\pi} \right) - \log \left( \frac{\tau_{t-1}}{\pi} \right) \] (35)
This is equivalent to
\[ \frac{\hat{\tau}_t - \hat{\tau}_{t-1}}{\hat{\tau}_{t-1}} = \log \left( \frac{\tau_t}{\tau_{t-1}} \right) \approx \frac{\tau_t - \tau_{t-1}}{\tau_{t-1}} = \pi^E - \pi_t = -\hat{\pi}_t \]